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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXVIII.

I. Revision of the Genus *Sabazia*. By B. L. Robinson and J. M. Greenman.

II. Revision of the Mexican and Central American Species of *Trixis*. By B. L. Robinson and J. M. Greenman.

III. Revision of the Mexican and Central American Species of *Hieracium*. By B. L. Robinson and J. M. Greenman.

IV. Synopsis of the Mexican and Central American Species of *Alnus*. By M. L. Fernald.

V. Diagnoses and Synonymy of Mexican and Central American Spermatophytes. By J. M. Greenman.

VI. Some New Species of Mexican and Nicaraguan Dicotyledons. By M. L. Fernald.

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY, NEW SERIES. — NO. XXVIII.

Presented by B. L. Robinson, February 10, 1904. Received February 27, 1904.

I. — REVISION OF THE GENUS SABAZIA.

B. L. Robinson and J. M. Greenman.

Sabazia is a small and exclusively American genus of the *Helianthoideae.* It is chiefly Mexican in its distribution, although one of its species ranges as far south as Ecuador. It is circumscribed here as by Bentham and Hooker, f., Gen. Pl. ii. 194, and by Hoffmann in Engl. & Prantl. Nat. Pflanzenf. iv. Ab. 5, 228, 233, the leading characteristics and literature of the genus being as follows: —

SABAZIA, Cass. (Name of obscure origin.) — Heads 1 to many, 6 mm. to 4 cm. in diameter (including the rays); involucral scales ovate, mostly obtuse, subequal, 2–3-seriate; disk convex or conical, chaff-bearing. Flowers all fertile. Corollas of the disk-flowers cream-colored to light yellow, shortly 5-toothed; ligules of the ray-flowers 2 to 20 mm. long, white to (externally) deep purple, 3-toothed. Achenes narrowly obovate, 4–5-angled, rounded at the summit; pappus none or of a few minute caducous awns. — Branching annual or perennial herbs, leafy-stemmed and often decumbent. Leaves opposite, ovate, entire or serrate. — Dict. Sci. Nat. xlvi. 480 (1827); DC. Prodr. v. 496; Hemsl. Biol. Cent.-Am. Bot. ii. 160. *Baziasa,* Steud. Nom. ed. 2, i. 192 (1840). *Sabbazia,* Steud. l. c. ii. 489 (1841). — A genus of plants which are without doubt rather nearly related, but which still may not have had a very recent common origin. In habit the species approach on the one hand *Jaegeria* and on the other *Tridax* and *Galinsoga.*

* Annuals.

+ Achenes hispidulous.

1. **S. urticaefolia,** DC. Hispid-pubescent throughout, glandular about the inflorescence: heads several to many, small, (including the 5 short roseate rays) only 6 mm. in diameter. — Prodr. v. 497 (1836); Hemsl. Biol. Cent.-Am. Bot. ii. 161. *Galinsoga urticaefolia,* Benth. in

Oerst. Vidensk. Meddel. 1852, 102. *Wiborgia urticaefolia*, HBK. Nov. Gen. et Spec. iv. 257, t. 389 (1820). — SOUTH MEXICO: plains of Quilo, *Jurgensen*, no. 260, acc. to Hemsl. l. c. NICARAGUA: Grenada, *Oersted*. COSTA RICA: San José, *Oersted*, *Tonduz*, no. 10,143 (hb. Gr.); pastures at Copey, *Tonduz*, no. 11,757 (hb. Gr.). ECUADOR: *Humboldt & Bonpland*, *Couthouy* (hb. Gr.).

+ + Achenes glabrous throughout.

2. **S. microglossa**, DC. Leafy-stemmed, hirsute-pubescent, branching: peduncles glabrous; heads numerous, small, 6 mm. in diameter: rays 5, white or roseate, scarcely exceeding the involucre. — Prodr. v. 497 (1836), including the var. PUBERULA, with pubescent peduncles (the prevailing form); Hemsl. l. c. 160. *Baziasa microglossa*, Steud. Nom. ed. 2, i. 192 (1840). — CHIHUAHUA: humid places, Pilares, *Hartman*, no. 747 (hb. Gr.); near St. Julian, alt. 2150 to 2460 m., *Nelson*, no. 4922 (hb. Gr.). COAHUILA: near Saltillo, *Palmer*, no. 790, in part (coll. of 1898). SAN LUIS POTOSI: 22° N. Lat., alt. 1850 to 2469 m., *Parry & Palmer*, with no. 492 (hb. Gr.). STATE OF MEXICO: mountains, St. Augustin, *Berlandier* (hb. Gr.); Pedrigal, *Pringle*, no. 7345 (hb. Gr.); on walls, *Pringle*, nos. 7903, 8231 (both in hb. Gr.).

3. **S. humilis**, CASS. Suberect, 3 dm. high or more often branching from the base and widely spreading, pubescent: heads few to many, 1.4 cm. broad including the conspicuous well exserted externally roseate-purple rays: disk-flowers numerous, pale yellow. — Dict. Sci. Nat. xlvi. 481 (1827); DC. Prodr. v. 496; Hemsl. l. c. ii. 160. *Eclipta humilis*, HBK. Nov. Gen. et Spec. iv. 264, t. 394 (1820). *Baziasa humilis*, Steud. Nom. ed. 2, i. 192 (1840). — MICHOACAN: near Ario and Patzcuaro, alt. 1850 m., *Humboldt & Bonpland*. STATE OF MEXICO: fields near San Nicolas, *Bourgeau*, no. 958 (hb. Gr.); valley of Toluca, *Berlandier*, no. 1215 (hb. Gr.); moist meadows, Nevado de Toluca, alt. 3080 m., *Pringle*, no. 4245 (hb. Gr.); in mountains, *Schaffner*, no. 296 (hb. Gr.).

* * Perennials.

+ Stem prostrate, strongly repent.

4. **S. sarmentosa**, LESS. Stem elongated, 6 dm. or more in length, rooting at the lower nodes, the internodes 2 to 8 cm. long: leaves ovate, acute, 3 to 4 cm. long, half as broad, hirsute on both sides, narrowed to slender petioles: heads solitary, terminal, long-peduncled, 2.2 cm. broad including the rays; peduncles densely appressed-pubescent and more or less glandular: rays mostly 8, about 7 mm. long. — Linnaea, v. 148

(1830); DC. Prodr. v. 496; Hemsl. l. c. 160. *Baziasa sarmentosa*, Steud. l. c. — VERA CRUZ (?): "Serro Colorado," *Schiede*, no. 324 (hb. Roy. Bot. Mus. Berlin, tracing and fragment in hb. Gr.). OAXACA: northwest slope of Mt. Zempoaltepec, alt. 2460 to 3080 m., *Nelson*, no. 680 (hb. Gr.).

+ + Stem erect or merely decumbent, not repent.

5. **S. Liebmannii**, KLATT. Decumbent, 3 to 4 dm. high; stem subappressed-pubescent, branching; branches mostly simple, curved-ascending, bearing at the summit solitary long-peduncled showy heads: leaves lanceolate or elliptic-ovate, 3-nerved, acute, remotely glandular-toothed: heads 2.7 cm. broad (including the rays); involucral scales broadly elliptical, obtuse, the inner subscarious, erubescent: rays rather large, internally white, externally purplish with deep violet veins: achenes of both the ray- and disk-flowers entirely glabrous. — Leopoldina, xxiii. 90 (1887). *Tridax Liebmannii*, Sch. Bip. acc. to Klatt, l. c. — MEXICO: without exact locality, *Liebmann*, no. 694 (hb. Copenhagen, tracing in hb. Gr.). OAXACA: Sierra de San Felipe, alt. 3080 m., *Pringle*, no. 4921 (hb. Gr.), alt. 1800 m., *Conzatti & González*, no. 395 (hb. Gr.), alt. 2900 to 3100 m., *Nelson*, no. 1126 (hb. Gr.).

Var. **heterocarpa**, n. var. Achenes of the disk-flowers pubescent, of the ray-flowers glabrous: leaves slightly more elliptic-ovate and less distinctly narrowed to a petiole. — *Calea multiradiata*, Seaton, Proc. Am. Acad. xxviii. 120 (1893), in part. *Sabazia* sp., Robinson & Greenman, Proc. Am. Acad. xxxii. 22 (1897). — VERA CRUZ: pine woods, Mt. Orizaba, alt. 3050 m., *Seaton*, no. 167, in part (hb. Gr.).

6. **S. michoacana**, ROBINSON. Similar in habit to the preceding, but erect from a short thickish lignescent rootstock; pubescence of the stem rather coarse and stiff, widely spreading: lower leaves cuneately narrowed to slender petioles: involucral bracts strongly ciliated, sometimes pubescent on the dorsal surface: heads much as in the preceding: achenes of the disk-flowers pubescent. — Proc. Am. Acad. xxvii. 173 (1892); F. N. Williams, Bull. Herb. Boiss. ser. 2, ii. 1020, as to Pringle's no. 4099, but not as to no. 4921, nor as to *Abasoloa*.[1] —

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[1] Mr. F. N. Williams, l. c. 1019–1021, expresses the opinion that *Sabazia michoacana*, Robinson, represented by Pringle's nos. 4099 and 4921, is the long uncertain *Abasoloa Taboarda*, La Llave & Lex. With this view we find it impossible to agree. Notwithstanding some vagueness in the original description of *Abasoloa* (La Llave & Lex, Nov. Veg. Desc. fasc. 1, 11, 1824), enough is said to show clearly that it cannot have related in anywise to *Sabazia michoacana*. In the first place the

Michoacan: mountains near Patzcuaro, *Pringle*, nos. 4099, 4264 (both in hb. Gr.).

Excluded Species.

S. occidentalis, DC. Prodr. v. 497 (1836), is a clerical error for *Siegesbeckia occidentalis*, Walt. (not L.), *i. e. Verbesina occidentalis*, Walt. Fl. Car. 213.

S. glabra, Wats. Proc. Am. Acad. xxiii. 277 (1888), is *Jaegeria petiolaris*, Robinson, Proc. Am. Acad. xxxv. 316 (1900).

S. portoricensis, DC. Prodr. v. 497 (1836), is a clerical error for *Siegesbeckia portoricensis*, Bert.

S. subnuda, Robinson & Seaton, Proc. Am. Acad. xxviii. 108 (1893), is unquestionably the long unidentified *Selloa plantaginea*, HBK. Nov. Gen. et Spec. iv. 266, t. 394 (1820). The genus *Selloa* appears from this new material to be very close to *Sabazia*, but it may be distinguished by its filiform chaff and acaulescent habit, as well as by the presence of a pappus of 2 to 6 filiform awns so caducous as to have escaped detection in the original examination of Mr. Pringle's plant.

II. REVISION OF THE MEXICAN AND CENTRAL AMERICAN SPECIES OF TRIXIS.

B. L. Robinson and J. M. Greenman.

TRIXIS, P. Browne. (Name from τριξός, threefold, from the three-cleft corollas.) The Mexican and Central American species are shrubs (except *T. michuacana*) with alternate to ovate-oblong rarely linear entire or dentate pinnately veined leaves and loosely paniculate to densely corymbose rarely thyrsoid or subsolitary heads; involucre mostly double, of 1 to 5 unequal linear to ovate more or less foliaceous outer

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disk-flowers are said to be 4-toothed, while they are 5-toothed in the *Sabazia;* the leaves are described as linear-lanceolate, although in the *Sabazia* they are ovate or ovate-lanceolate; the rays are said to be numerous and capillary, a mode of description which surely no one would apply to the some ten flat elliptic-oblong conspicuously toothed rays of the *Sabazia*; the involucre is said to be equal, while in the *Sabazia* the scales are very unequal. It seems to us much more likely that Baillon was right in referring *Abasoloa* to *Eclipta* with which in essentials the description corresponds far better. It should be remarked that Pringle's no. 4921 has on re-examination proved to be *Sabazia Liebmannii*, Klatt.

bracts and 5 to 10 equal erect linear-oblong scales: corollas yellow, strongly bilabiate, the upper erect lip 2-cleft, the lower spreading, 3-toothed or subentire: anthers sagittate and bicaudate at the base: achenes cylindrical or slightly flask-shaped, slender, finely papillose, the summit more or less expanded into a circular disk bearing copious simple roughish white or tawny pappus-bristles. — Hist. Jamaic. 312 (1756); Lag. Amen. Nat. i. 35; La Llav. & Lex. Nov. Veg. Desc. fasc. 1, 27; Spreng. Syst. iii. 501; D. Don, Trans. Linn. Soc. xvi. 186, 297; DC. Prodr. vii. pt. 1, 67; Benth. & Hook. f. Gen. ii. 501; Hemsl. Biol. Cent.-Am. Bot. ii. 257; Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 5, 350; Hook. f. & Jacks. Ind. Kew. ii. 1130, which see for generic synonymy. *Perdicium*, L. Pl. Afr. Rar. 22 (1762), in part. About 35 species, ranging from Arizona to Brazil and Chili.

§ 1. Inflorescence an elongated conical thyrsus: herbaceous.

1. **T. michuacana**, La Llav. & Lex. Viscid-resinous and with a heavy odor; root fibrous, yellowish; stem simple, tomentose, 1 dm. high, erect: leaves sessile, punctate, villous below, the floral gradually reduced: heads borne in a large conic-oblong thyrsus; outer bracts of the involucre 5, ample, ovate, the inner involucre prismatic, of 8 erect scales. — Nov. Veg. Desc. fasc. 1, 28 (1824); DC. Prodr. vii. pt. 1, 68; Hemsl. Biol. Cent.-Am. Bot. ii. 258. — MICHOACAN: near Vallisoletum, flowering in January, *Lexarza*. If the above (compiled) description is correct, this is a marked species, not yet rediscovered.

§ 2. Inflorescence corymbose-paniculate: shrubs or undershrubs.

* Leaves sessile by a clasping shortly decurrent base.

2. **T. decurrens**, DC. Stems several, sparingly branched toward the summit: leaves crowded, ovate-lanceolate, acutely acuminate, entire: heads terminal, subsolitary; outer involucre of 5 ovate-lanceolate bracts, the inner of 8 scales: corollas yellow. — Prodr. vii. pt. 1, 68 (1838), whence the foregoing description; Hemsl. Biol. Cent.-Am. Bot. ii. 258; A. DC. Calques des Dess. t. 515. *Perdicium decurrens*, Sessé & Mociño, acc. to DC. l. c. (1838), & Pl. Nov. Hisp. ed. 2, 130. — MORELOS: Ayacapixtla, flowering in November, *Sessé & Mociño*.

* * Stem provided at least at some stages of its development with broad and conspicuous herbaceous wings.

+ Outer bracts of the involucre lanceolate, much shorter than the inner; the latter glandular.

3. **T. mexicana**, La Llav. & Lex. A meter or more high: leaves lanceolate, 5 to 13 cm. long, 1 to 4 cm. broad, acuminate, acute, entire

or denticulate, narrowed below to a somewhat petiole-like base, decurrent upon the stem, sparingly pubescent above, tomentulose and glandular-atomiferous beneath, at length somewhat glabrate: inflorescence loose; heads 2 cm. high, about 20-flowered. — Nov. Veg. Desc. fasc. 1, 27 (1824), not Sessé & Mociño. — MICHOACAN: near Vallisoletum, flowering in October and November, *Lexarza*. GUERRERO: vicinity of Acapulco, *Palmer*, no. 144 (hb. Gr.).

+ + Outer bracts of the involucre elliptical or spatulate, two-thirds as long as the inner; the latter glandless.

4. **T. pterocaulis**, n. sp. Stems much branched; ultimate branchlets subappressed-pubescent: leaves thin, ovate to ovate-lanceolate. 3 to 10 cm. long, 1.5 to 4 cm. broad, acute, mucronate, entire or denticulate, sessile and decurrent upon the stem or narrowed to a short petiole-like base, glabrous or nearly so above, sparingly pubescent beneath: inflorescence loose and comparatively few-headed; heads 1.7 to 2 cm. high, 12–14-flowered; bracts of the proper (inner) involucre 8, linear-oblong, obtusish or abruptly pointed at the tip, appressed-pubescent. — COLIMA: Manzanillo, December, 1890, *Palmer*, no. 897 (hb. Gr.).

+ + + Outer bracts of the involucre 5, very large, ovate-lanceolate, copiously glandular.

5. **T. alata**, D. DON. Stems erect, rigid, branching; branches terete, foliaceous-winged: leaves oblong-ovate, 5 to 8 cm. long, 2.5 to 4 cm. broad, acute, mucronate, denticulate, papillose-setose on both surfaces: bracts of the outer involucre 2.5 to 4 cm. long, about 12 mm. broad, longer than the proper involucre: heads 24-flowered. — Trans. Linn. Soc. xvi. 192; DC. Prodr. vii. pt. 1, 68; Hemsl. Biol. Cent.-Am. Bot. ii. 258. — MEXICO: without locality, *Sessé & Mociño*.

* * * Leaves subsessile or narrowed to a shortly petioled base: stem wingless (sometimes very narrowly and inconspicuously winged in *T. rugulosa*).

+ Bracts of the outer involucre 5, large, foliaceous, ovate, usually exceeding the inner.

6. **T. longifolia**, D. DON. Stems erect, leafy; branches striate, subappressed tawny-pubescent: leaves crowded, sessile or subsessile, lanceolate, 3 to 8 cm. long, 1 to 2 cm. broad, acute or acuminate, entire to shallowly dentate, sparingly pubescent above, more or less appressed-villous beneath, somewhat glabrate, usually conduplicate: inflorescence dense, many-headed, conspicuously glandular; heads large, 2 to 2.5 cm. high, 18–20-flowered. — Trans. Linn. Soc. xvi. 191 (1838); DC. Prodr.

vii. pt. 1, 68; Hemsl. Biol. Cent.-Am. Bot. ii. 258. *T. obvallata*, Hook. & Arn. Bot. Beech. Voy. 300, t. 65 (1840); Walp. Rep. ii. 682; Seem. Bot. Herald. 314; Rose, Contr. U. S. Nat. Herb. i. 106, 338. *T. conferta*, Benth. Pl. Hartw. 289 (1848); Walp. Ann. ii. 950. *Perdicium longifolium*, Sessé and Mociño, acc. to Don, l. c. — MEXICO: without special locality, *Sessé & Mociño*; between San Luis Potosi and Tampico, *Palmer*, no. 1121 (coll. of 1878–1879); Sierra Madre, northwest Mexico, *Seemann* (hb. Gr.). SONORA: Alamos, *Palmer*, no. 290 (hb. Gr.). GUANAJAUTO: Guanajauto, *Berlandier*, no. 1327 (hb. Gr.); Leon and Guanajuato, *Hartweg*, no. 1609 (hb. Kew, acc. to Hemsl., l. c). JALISCO: hills near Guadalajara, *Pringle*, nos. 2173, 2431 (both in hb. Gr.); *Dugès*, MM. (hb. Gr.). COLIMA: *Palmer*, no. 1235 (hb. Gr.). GUERRERO: Acapulco, *Hinds*.

Var. **sericea**, n. var. Stem 6 to 9 dm. high: upper leaves (the only ones shown in the specimen at hand) densely and permanently silvery-sericeous beneath. — ? *T. involucrata*, Don. Trans. Linn. Soc. xvi. 193 (1838). — HIDALGO: hills near Tula, alt. 2300 m., *Pringle*, no. 8051 (hb. Gr.).

Var. **platyphylla**, n. var. Leaves large, obovate, flat, entire or nearly so, sparingly soft-pubescent beneath, 1 dm. long, 3.5 to 4 cm. broad, sharply acuminate: inflorescence more open than in either of the other forms; the bracts subtending its branches ovate, glanduliferous. — GUERRERO: near Acapulco, *Palmer*, no. 522, October, 1894 to March, 1895 (hb. Gr.).

+ + Outer bracts of the involucre linear to narrowly ovate, mostly much shorter than the inner.

++ Inner bracts of the involucre 5; flowers 5 to 7.

7. **T. oligantha**, n. sp. Shrub; stems covered with a brown or grayish brown bark, at first pubescent, later glabrate: leaves lanceolate, 3 to 9 cm. long, 7 to 25 mm. broad, acuminate, mucronately acute, entire or mucronulate-denticulate, narrowed below into a subpetiolate base, at first pubescent above, later more or less glabrate, appressed-sericeous-tomentose beneath: heads 12 to 18 mm. high, 5–7-flowered, disposed in rather close clusters borne in a corymbose panicle; outer bracts of the involucre lanceolate or lance-ovate, sometimes nearly or quite equalling the lance-linear acuminate appressed-pubescent inner scales, the latter 10 to 12 mm. long. — OAXACA: Monte Alban, 27 November, 1894, *Pringle*, no. 5826; Cerro de San Felipe, alt. 1700 m., 29 March, 1896, *Conzatti & González*, no. 70.

↔ ↔ Inner bracts of the involucre 8; flowers 10 to 25.

= Leaves strongly discolorous, silvery-sericeous beneath.

8. **T. hyposericea**, Wats. Slender branching shrub with smooth light brown terete branchlets and narrowly lanceolate entire attenuate-acuminate willow-like leaves 4 to 6 cm. long, 6 mm. broad, green, slightly rugose, and inconspicuously villous above, white and silky-villous beneath: pedicels slender, finely appressed-pubescent; heads small, 1.4 cm. long; outer involucral bracts small, lanceolate, green, 3 to 4 mm. long, the inner about 1 cm. long. — Proc. Am. Acad. xxv. 157 (1890). — Jalisco: in a barranca near Guadalajara, *Pringle*, no. 1741 (hb. Gr.).

9. **T. Pringlei**, n. sp. Shrub; stems below covered with a light gray cortex, glabrous; branchlets subappressed-tomentulose and sericeous: leaves lanceolate, 3 to 7 cm. long, 6 to 12 mm. broad, acuminate, acute, entire, revolute-margined, narrowed below into a subpetiolate base, subhirtellous above, silvery-tomentose and sericeous beneath: inflorescence loose; heads 15 to 18 mm. high, 12–15-flowered; inner scales of the involucre about 1 cm. in length and as well as the outer glandular-puberulent. — Oaxaca: Tomellin Cañon, alt. 925 m., 22 December, 1894, *Pringle*, no. 5894 (hb. Gr.).

10. **T. Haenkei**, Sch. Bip. Shrub; stems covered with a brown cortex, at first sericeous-tomentose, later glabrate: leaves lanceolate to lance-oblong, 3 to 12 cm. long, 1 to 3.5 cm. broad, acuminate, acute, entire or denticulate, subsessile or narrowed at the base to a short petiole (8 mm. or less in length), at first pubescent above but soon glabrate, silky-tomentose beneath: inflorescence dense; heads 1.5 to 2 cm. high, 12–14-flowered; inner bracts of the involucre 8, about 14 mm. in length, appressed-pubescent often with gland-tipped hairs intermixed, usually longer than the outer bracts. — Sch. Bip. in Seem. Bot. Herald, 314 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 258; Mueller, Ann. v. 314. *T. angustifolia*, Vasey & Rose, Contr. U. S. Nat. Herb. i. 73 (1790), not DC. — Mexico: without locality, *Liebmann*, no. 404 (fragment and tracing in hb. Gr.); in the Sierra Nevada of northwestern Mexico, *Seemann* (hb. Gr.). Jalisco: roadside between Mascota and San Sebastian, *Nelson*, no. 4068. Lower California, La Paz, *Palmer*, no. 7 (coll. of 1890).

= = Leaves tomentulose beneath.

11. **T. rugulosa**, n. sp. Shrub, 10 to 16 dm. high; branches light brown, sometimes inconspicuously winged, the wings narrow, scarcely herbaceous, decurrent in pairs from the base of the petiole: leaves

lanceolate, 3 to 8 cm. long, 7 to 17 mm. broad, acuminate, acute, entire, narrowed at the base, dark green, sparingly pubescent and rugose above, paler and finely tomentose beneath: inflorescence an open flat-topped panicle; pedicels appressed-pubescent; bracts of the outer involucre 7 to 8 mm. long, the inner narrowly oblong, obtusish, 10 to 12 mm. long; heads 1.5 to 2 cm. high, about 18-flowered. MEXICO: without locality, *Gregg*, no. 840 (hb. Gr.). GUANAJUATO: alt. 2100 m., *C. C. Dean*, no. 175 (hb. Gr.). QUERETARO: in the plains near the city, *Berlandier*, no. 1292 (hb. Gr.); between Queretaro and Mexico, *Berlandier*, no. 1228 (hb. Gr.).

12. **T. angustifolia**, DC. Shrub, about 1 m. high, fasciculately branched, grayish green: leaves linear or lance-linear, attenuate at each end, 3 to rarely 9 cm. long, usually about 4 (rarely as much as 15) mm. broad, hirsute-tomentose beneath, mostly erect or appressed, the uppermost shorter but otherwise similar; margins strongly revolute: heads about 2 cm. long, subsolitary or aggregated into fascicles; scales of the inner involucre 8, lance-linear, attenuate. — Prodr. vii. pt. 1, 69 (1838); Hemsl. l. c. 258; Gray, Syn. Fl. i. pt. 2, 409, in part. *T. rosmarinifolia*, Nees, Linnaea, xx. 699 (1847); Walp. Ann. i. 460. — MEXICO: without further locality, *Gregg*, no. 566; in the Sierra Nevada of northwest Mexico, *Seemann*, no. 2046 (hb. Gr.). DURANGO: shady hillsides near the city, alt. 1850 m., *Palmer*, no. 30, *Nelson*, no. 4599. ZACATECAS: hills about the city, *Pringle*, no. 180. SAN LUIS POTOSI: *Berlandier*, nos. 1284, 1353 (both in hb. Gr.), alt. 1800 to 2500 m., *Parry & Palmer*, no. 549 (hb. Gr.), *Schaffner*, no. 285 (hb. Gr.); Penasco, *C. & E. Seler*, no. 1138.

= = = Leaves glandular-dotted beneath.

13. **T. californica**, KELLOGG. A much branched shrub, 0.3 to 1 m. or more in height; branches and branchlets appressed-puberulent, glabrate: leaves lanceolate, 2.5 to 10 cm. long, 0.5 to 2 cm. broad, acute or acuminate, entire or mucronulate-dentate, sessile or narrowed below to a subpetiolate base, sparingly appressed-puberulent on both surfaces to nearly or quite glabrous, densely glandular-atomiferous beneath, usually pale green: heads 1.5 to 2 cm. high, commonly 12–14-flowered; inner involucral scales 8 to 10 in number, 10 to 15 mm. in length, thickened and somewhat obtusely keeled at the base, slightly exceeding the lance-linear to subobovate outer bracts, glandular-pubescent with a few subappressed non-glandular hairs intermixed. — Proc. Calif. Acad. ii. 182, fig. 53 (1862). *T. suffruticosa*, Wats. Bot. Calif. ii. 459 (1880).

T. angustifolia, Gray, Pl. Wright, i. 128 (1852), & ii. 102 (as to nos. 413 & 1299), also Syn. Fl. i. pt. 2, 409, in part, not DC. *T. angustifolia*, var. *latiuscula*, Gray, l. c. 410. *T. frutescens*, Gray, Bot. Mex. Bound. 103 (1859), vars.; not Sch. Bip. in Seem. Bot. Herald. 314; not P. Browne. — CALIFORNIA: San Diego County, Tantillas Cañon, *Palmer* (coll. of 1875); Colorado Desert, *Lemmon & Parry*, no. 1185, *Wright*, no. 196; Agua Caliente, *Parish*, no. 353 (hb. Gr.). LOWER CALIFORNIA: Cedros Island, *Veatch*, *Greene* (hb. Gr.), *Palmer*, no. 694 (hb. Gr.), *Anthony*, nos. 49, 318 (both in hb. Gr.); Island of San Pedro Martin, *Palmer*, no. 408 (hb. Gr.); Magdalena Island, *Brandegee* (hb. Gr.); Cañons Cantillas, *Orcutt*, no. 1102. ARIZONA: rocky cañons of the Santa Catalina Mountains, *Pringle* (hb. Gr.); Camp Grant, *Palmer*, no. 143 (hb. Gr.); Tempe, *Ganong & Blaschka* (hb. Gr.); without locality, *Lemmon* (hb. Gr.); Tucson, *Greene*, no. 1099 (hb. Gr.); Grand Cañon, *Gray* (hb. Gr.). NEW MEXICO: Manyos Cañon, *Greene*, no. 101 (hb. Gr.); Bear Mountains, *Rusby*, no. 203 (hb. Gr.); Organ Mountains, *Wooton*, no. 107 (hb. Gr.). TEXAS: western part, *Havard*, no. 74 (hb. Gr.). SONORA: ravines near Santa Cruz, *Wright*, no. 1299 (hb. Gr.). CHIHUAHUA: among rocks, Bachimba, *Thurber*, no. 830 (hb. Gr.); hills near the city, *Pringle*, no. 30 (hb. Gr.); Santa Eulalia Mountains, *Pringle*, no. 525 (hb. Gr.); near Lake Santa Maria, *Nelson*, no. 6407 (hb. Gr.). COAHUILA: Soledad, *Palmer*, no. 2089 (hb. Gr.); Parras, *Palmer*, no. 745 (hb. Gr.); Torreon, *Palmer*, no. 786 (hb. Gr.). SAN LUIS POTOSI: near Opito, *Gregg* (hb. Gr.).

14. **T. silvatica**, n. sp. Branching shrub with light brown branchlets; cortex tending to exfoliate: leaves thin, ovate-oblong, acuminate, cuneate at the sessile base, sharply but rather finely dentate, green, slightly pubescent and obscurely glandular above, scarcely paler, finely pubescent and distinctly glandular-atomiferous beneath, 5 to 8 cm. long, 2 to 3 cm. broad: heads few or many at the ends of the branches in rounded corymbose panicles, about 12-flowered, 2 cm. long in fruit; outer involucral bracts thin, lanceolate or oblanceolate, acuminate, glandular-atomiferous, 12 to 15 mm. long, 3 to 5 mm. broad, the inner 8, linear, short-acuminate, about 12 mm. long, glandular-pubescent: mature achenes 7 to 9 mm. long; pappus slightly tawny, about 8 mm. long. — OAXACA: in dry wooded gorges on the River Tehuantepec, *C. & E. Seler*, no. 1633 (hb. Gr.); without locality, *Liebmann*, no. 405 (hb. Gr.). — Distinguished from the preceding species by its thinner larger leaves, the spreading villous pubescence of the inflorescence, and by the shorter pappus.

= = = = Leaves smoothish, finely reticulate-veiny beneath.

15. **T. frutescens**, P. Br. Shrub, much branched, 1 to 1.5 m. high; branchlets glabrous or sparingly appressed-pubescent: leaves narrowly lanceolate to lance-ovate, 3 to 10 cm. long, 1 to 3.5 cm. broad, more or less acuminate and mucronate, subentire to distinctly dentate, sessile or narrowed into a subpetiolate base, glabrous or nearly so on both surfaces: inflorescence a many-headed somewhat leafy corymbose panicle; heads 1.5 to 2 cm. high, usually about 12-flowered; inner involucral scales 8 (or rarely 10) in number, 10 to 12 mm. long, the outer linear or lance-linear, usually half to two-thirds as long as the inner, rarely quite as long. — P. Br. acc. to Spreng. Syst. iii. 501 (1826); DC. Prodr. vii. pt. 1, 68; Hemsl. l. c. 258; Less. Linnaea, v. 33; Benth. Pl. Hartw. 88; Klatt, Bull. Soc. Bot. Belg. xxxv. pt. 1, 293. *T. radiale*, Lag. acc. Hook. f. & Jacks. Ind. Kew. ii. 1131, but Lagasca seems never to have employed the combination, although he referred the Linnaean species to *Trixis*, as had P. Browne many years before (Hist. Jam. 312). *T. laevigata*, Lag. acc. to Spreng. Syst. iii. 501 (1826). *T. divaricata*, Klatt, Bull. Soc. Bot. Belg. xxxi. pt. 1, 215 (1892) not Spreng. *Perdicium radiale*, L. Sp. ed. 2, 1248 (1763); Lag. Amen. Nat. i. 36; Lam. Ill. t. 677, f. 2. *P. laevigatum*, Berg. Act. Holm. 228, t. 7 (1772); Lam. Dict. v. 179, & Ill. t. 677, f. 1. *Inula Trixis*, L. Amoen. Acad. v. 406 (1760). *Tenorea Berteri*, Colla, Hort. Ripul. 137 (1824). *T. calyculata*, Bert. acc. to Colla, l. c. *Prenanthes fruticosa*, Willd. acc. to Less. Linnaea, v. 33 (1830) & DC. Prodr. vii. pt. 1, 69. — TAMAULIPAS: near Matamoras, *Berlandier*, nos. 690, 2100 (both in hb. Gr.), and near Victoria, *Berlandier*, nos. 810, 2230 (both in hb. Gr.); near Jimenez, *Nelson*, no. 6660 (hb. Gr., hb. U. S. Nat. Mus). The last five numbers represent a form with the outer accessory involucral bracts equalling the inner proper involucre. SAN LUIS POTOSI: alt. 1850 to 2500 m., *Parry & Palmer*, no. 550 (hb. Gr.); limestone hills, Las Palmas, alt. 120 to 160 m., *Pringle*, nos. 5766, 8005 (both in hb. Gr.). TEPIC: Santiago and Tepic, *Lamb*, no. 552. VERA CRUZ: Valley of Cordova, *Bourgeau*, no. 2035 (hb. Gr.); Mirador, *Sartorius* (hb. Gr.); near the city of Vera Cruz, *Pringle*, no. 5771 (hb. Gr.), *Linden*, no. 1208 (hb. Gr.). OAXACA: between San Carlos and San Bartolo, alt. 1000 to 1500 m., *Nelson*, no. 2567 (hb. Gr.); on wooded slopes of the Tehuantepec below Tolopan, growing under trees, *C. & E. Seler*, no. 1702 (hb. Gr.). CHIAPAS: between Tonala and Tuxtla in mountain woods near Cuesta San Fernando, *C. & E. Seler*, no. 1932 (hb. Gr.):

Barranca de Consoquitla, *Liebmann*, no. 108 (hb. Gr.). GUATEMALA: Valley of Guatemala, *Hartweg* (hb. Gr.); alt. 1550 m., *J. D. Smith*, no. 2363 (hb. Gr.); Volcan de Fuego, alt. 1550 m., *J. D. Smith*, no. 2856 (hb. Gr.); Canizal alt. 1550 m., *Heyde & Lux*, no. 4202 of J. D. Smith's distrib. (hb. Gr.). YUCATAN: Merida, *Valdez*, no. 10; without locality, *Gaumer*, no. 397. HONDURAS: San Pedro Sula, alt. 250 m., *Thieme*, no. 5236 of J. D. Smith's distrib. (hb. Gr.); Gulf of Fonseca, *Sinclair* (hb. Gr.). COSTA RICA: Cartago, *Cooper*, no. 5810 of J. D. Smith's distrib.; in woods near Terraba, alt. 260 m., Pittier, no. 3705 (hb. Gr.). PANAMA: Veraguas, *Seemann* (hb. Gr.).

16. **T. Wrightii**, n. sp. A much-branched shrub; branchlets subappressed-pubescent: leaves lanceolate, 1.5 to 7 cm. long, 0.5 to 2 cm. broad, entire or inconspicuously denticulate, narrowed below into a sessile or subsessile base, more or less decurrent on the stem, thickish, appressed-puberulent on both surfaces to essentially glabrous, reticulate-veiny beneath: heads about 1.5 cm. high, loosely disposed in corymbose panicles, 11–13-flowered; bracts of the outer involucre spatulate to obovate, 5 to 10 mm. long, 2 to 6 mm. broad; inner involucre of 8 appressed-pubescent scales 10 to 11 mm. in length. — SINALOA: near Mazatlan, January, 1889, *W. G. Wright*, no. 1224 (hb. Gr.), December, 1894, *Lamb*, no. 248 (hb. Gr.). TRES MARIAS ISLANDS: Maria Madre Island, 3 to 25 May, 1897, *Nelson*, no. 4191 (hb. Gr.).

III. — REVISION OF THE MEXICAN AND CENTRAL AMERICAN SPECIES OF HIERACIUM.

B. L. ROBINSON AND J. M. GREENMAN.

THROUGH a lack of accurate knowledge of the Mexican Hawkweeds there has been for many years a tendency to crowd unlike forms into certain of the older and vaguely characterized species. Some years ago the writers made a preliminary revision of the Mexican and Central American species of this genus, but from the uncertainty which still obscured the exact application of some of the earlier specific names, it seemed best to withhold the paper from publication pending further investigation. Of late, however, it has been possible to gain considerable new light on the subject, first from numerous specimens, tracings, drawings, and authentic fragments obtained by the Gray Herbarium through the purchase of the collection of the late Dr. F. W. Klatt, and second from some recently secured tracings and fragments of Lessing's species preserved in the Royal Botanical Museum, at Berlin. From these new sources and a very full suite of excellent modern specimens collected by Messrs. Pringle, Palmer, Nelson, L. C. Smith, Conzatti, González, Rose, Pittier, Brandegee, and others, it is now possible to offer the following synopsis with a reasonable confidence that it will prove a useful basis for further work in its particular field. For the sake of comparison a few species, which are at present known only in the southwestern United States but are likely to extend within the boundaries of Mexico, are here included.

HIERACIUM, Tourn. (Name from ἱεράξ, a hawk.) — A large and in Europe exceedingly difficult genus. The Mexican and Central American species erect perennial herbs with scapiform or leafy stems, alternate lance-linear to oblong or obovate, dentate or entire pinnately veined leaves and corymbose or paniculate heads; involucral bracts unequal, narrow, unaltered after anthesis, usually pubescent to bristly hirsute dorsally: corollas orange to lemon yellow, greenish white or rarely flesh-colored: achenes slender and cylindrical or attenuate from near the base toward the summit; pappus copious, bright white to sordid, tawny, or rufous, of simple subequal bristles. — Inst. 469, t. 267 (1700); L. Syst. (1735), & Spec. 799 (1753); Fries, Symbol. Hierac. (1848), & Epicrisis Hierac. (1862); Sch. Bip. Ueber die Hieracien Amerika's, Bonplandia, ix. 172 (1861); Benth. & Hook. f. Gen. Pl. ii. 516 (1873); Hemsl. Biol. Cent.-Am. Bot. ii. 259 (1881); Arvet-Touvet, Spicilegium Hierac. (1881), Rev. des Éperviêres & Elenchus, Ann. Conserv. Genèv. i. 68 (1897); Gray, Proc. Am. Acad. xix. 65 (1884), & Syn. Fl. i. pt. 2, 424 (1884); Peter in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 375 (1894); Hook. f. & Jacks. Ind. Kew. i. 1149 (1893), which see for generic synonymy.

* Pubescence of the leaves scanty to very copious, sometimes silky or hirsute, but not woolly or matted.

+ Achenes fully 5 mm. in length, conspicuously tapering almost from the base to the summit.

1. **H. Fendleri**, Sch. Bip. Stems 1 to 3, erect, 26 to 52 cm. high, hirsute with long fine widely spreading or slightly reflexed bristles, or glabrate: basal leaves 3 to 14, oblanceolate to obovate, acute to rounded and apiculate at the apex, cuspidate-serrulate, attenuate below, 5 to 12 cm. long, 17 to 60 mm. broad, covered on both surfaces and at the margin with long sparse hairs (bronze color in dried specimens); cauline leaves

1 or 2, much smaller, lance-oblong, acute: heads 3 to 40, rather large: inner scales of the involucres 10 to 12 mm. long: achenes at length reddish black; pappus sordid white. — Bonplandia, ix. 173 (1861); Gray, Proc. Am. Acad. xix. 69, & Syn. Fl. i. pt. 2, 429, including the poorly marked var. *discolor*. *H. nigrocollinum*, Wats. Proc. Am. Acad. xxv. 133 (1890). *Crepis ambigua*, Gray, Pl. Fendl. 114 (1848), & Pl. Wright, ii. 106 (not i. 129). — NEW MEXICO: banks of Santa Fe Creek, *Fendler*, no. 488 (hb. Gr.); Organ Mountains, *Wright*, no. 1427 (hb. Gr.); Mogollon Mountains, *G. R. Vasey*. ARIZONA: Santa Rita Mountains, alt. 1825 to 2450 m., *Pringle* (hb. Gr.); Tanner's Cañon near Ft. Huachuca, *Lemmon*, no. 2804 (hb. Gr.); Willow Spring, *Palmer*, no. 532 (hb. Gr.); vicinity of Flagstaff, *MacDougal*, no. 19 (hb. Gr.). CHIHUAHUA: Sierra Madre, *Pringle*, no. 1536 (hb. Gr.); pine forest near Chuchuichupa, *Hartman*, no. 698 (hb. Gr.); near Colonia Garcia, *Townsend & Barber*, no. 23 (hb. Gr.). LOWER CALIFORNIA: La Chuparosa, *Brandegee* (hb. Gr.). Extending northward to Dakota.

2. **H. brevipilum**, GREENE. Stem erect, 47 cm. high, leafy only near the base, pilose with slender spreading or slightly reflexed hairs: leaves narrowly oblong, rounded to acute at the apex, remotely and obscurely glandular-denticulate, pilose upon both surfaces: heads about 8, subracemose; peduncles and long narrow involucral scales glandular-hispid: corollas greenish yellow: achenes dark red; pappus white or nearly so. — Bull. Torr. Club, ix. 64 (1882). *H. Fendleri*, var. *mogollense*, Gray, Proc. Am. Acad. xix. 69 (1884), & Syn. Fl. i. pt. 2, 429. *H. erythrospermum*, Greene acc. to Gray, ll. cc. — NEW MEXICO: Mogollon Mountains, *Rusby* (hb. Gr.).

3. **H. prionophilum**, n. sp. Stem stout, erect, green, striate, 12 dm. high, leafy, covered with long spreading hairs (tawny in dried specimens): basal leaves wanting at anthesis, the cauline about 7, thin, oblong-lanceolate, acutish, strongly dentate, sparsely hirsute upon both surfaces, scarcely paler beneath, 10 to 18 cm. long, 2 to 5 cm. broad, the uppermost reduced, caudate-attenuate: corymbose panicle about 50-headed; its branches curved upwards, tapering; pedicels slender, spreading, tomentulose and covered with minute spreading dark glandular bristles: corollas greenish white: achenes reddish black; pappus bright white. — CHIHUAHUA: shaded ledges, Arroyo Aucho, Sierra Madre, *Pringle*, no. 1314 (hb. Gr.). SAN LUIS POTOSI: alt. 1825 to 2450 m., *Parry & Palmer*, no. 551 (hb. Gr.).

+ + Achenes 3.5 to 4.5 mm. long, columnar or very gradually attenuate toward the summit.

++ Cauline leaves none, the basal rosulate, very short.

4. H. junceum, Fries. Stem filiform, 13 to 22 cm. high, loosely 1-several-branched: basal leaves subsessile, obovate, hirsute upon both surfaces, obtuse: heads 3 to 5, few-flowered, on elongated flocculent-pubescent peduncles; involucres slender, the inner scales elongated, linear, with a few dorsal glanduliferous hairs: achenes long and slender, brownish black, not conspicuously attenuate; pappus reddish white. — Symbol. Hierac. 144 (1848), & Epicris. Hierac. 149; Hemsl. Biol. Cent.-Am. Bot. ii. 259. — Oaxaca: in pine woods, Cambre de Ixlepec, alt. 3300 m., *Liebmann* (sketch in hb. Gr.).

Var. angustifolium, Fries. Leaves narrower: achenes more evidently attenuated. — Epicris. Hierac. 149 (1862). — "Sierra de Moucayo" (hb. Martius, acc. to Fries, l. c.). Not seen.

++ ++ Cauline leaves present, lance-linear, entire, 12 times as long as wide.

= Peduncles glabrous or sparingly hispid.

5. H. carneum, Greene. Stem nearly or quite glabrous from the base, glaucous, branched from the middle: basal leaves elliptical, subsessile, thickish, densely covered with long white hairs (rufous in dried specimens); stem-leaves long and relatively narrow, acute, nearly or quite glabrous: involucral scales green and glabrous: corollas deep flesh-colored. — Bot. Gaz. vi. 184 (1881); Gray, Proc. Am. Acad. xix. 69, & Syn. Fl. i. pt. 2, 430. — New Mexico: oak woods, south base of Pinos Altos Mountains, *Greene* (hb. Gr.).

Var. chihuahuense, n. var. Basal leaves attenuate below to a narrow petiole-like base; lower cauline leaves, peduncles, and involucral scales sparingly pubescent: corollas white. — Chihuahua: foothills of the Sierra Madre, *Pringle*, no. 1315 (hb. Gr.); near Colonia Garcia in the Sierra Madre, *Nelson*, no. 6192 (hb. U. S. Nat. Mus.).

= = Peduncles tomentulose.

6. H. prionobium, n. sp. Stems 1 to 3, erect, 6 to 8 dm. high, covered below with rather copious spreading or slightly deflexed hairs: basal leaves spatulate or oblanceolate, 15 to 20 cm. long, 10 to 25 mm. wide, obscurely denticulate, rounded at the apex, erect, hairy on both surfaces; the cauline leaves lance-linear, sessile, acute, essentially entire, 6 to 20 cm long, 3 to 15 mm. broad, sparingly pubescent: heads about 12, in an open loose corymbose panicle; peduncles curved-ascending, 1 to 3 cm. long, finely tomentulose and setulose; heads narrowly campanulate, about 30-flowered; scales of the involucre about 14, linear, acute, nigrescent, dorsally hirtellous and glanduliferous, the inner 8 mm. long: corollas yellow: achenes reddish brown, very slightly attenuate, 4 mm. long; pappus pure white. — Chihuahua: shaded slopes of the Sierra Madre, 18 September, 1887, *Pringle*, no. 1313 (hb. Gr.), distributed as *H. Schultzii*; ? near Colonia Garcia, alt. 2,280 m., *Townsend & Barber*, no. 254 (hb. Gr.), distributed as *H. Rusbyi*, immature and doubtful; in the Sierra Madre, *Nelson*, no. 6128 (hb. U. S. Nat. Mus.).

++ ++ ++ Cauline leaves oblong, broadly cordate at the sessile clasping base, 2 to 8 times as long as broad.

7. **H. crepidispermum**, Fries. Stem usually solitary, erect, 3 to 5 dm. high, covered with long slender widely spreading hairs (becoming bronze color after drying): basal leaves narrowly obovate, 4 to 13 cm. long, 15 to 35 mm. broad, rounded, or obtuse at the apex, cuspidate-denticulate, loosely villous on both surfaces, especially upon the midnerve beneath; cauline leaves 3 to 8, oblong, acute, 2 to 4 cm. broad, cordate-clasping at the often but slightly narrowed base: heads 7 to 40, at first crowded, later in an open corymb; peduncles tomentulose and covered with short dark glandular bristles: achenes reddish black, 3.5 to 4 mm. long, slightly attenuated toward the base as well as toward the summit; pappus bright white. — Symb. Hierac. 146 (1848), & Epicris. Hierac. 152; Wats. Proc. Am. Acad. xviii. 110; Gray, Proc. Am. Acad. xix. 70; Hemsl. Biol. Cent.-Am. Bot. ii. 259. *Crepis stenotheca*, Fries, Symb. Hierac. 146 (1848). *Heteropleura crepidisperma*, Sch. Bip. Flora, xlv. 435 (1862). *H. tolucanum*, Arv.-Touv. Ann. Conserv. Genèv. i. 92 (1897). — Coahuila: east and south of Saltillo, *Palmer*, no. 758 (coll. of 1880), hb. Gr. Durango: Real del Monte, *Ehrenberg*, acc. to Sch. Bip., l. c. Territory of Tepic: near Santa Teresa, *Rose*, nos. 3405, 2080 in part (hb. U. S. Nat. Mus.). State of Mexico: hillsides, valley of Toluca, *Pringle*, no. 4183 (hb. Gr.); *Holway*, no. 3204 (hb. Gr.); pine woods, Eslava, valley of Mexico, alt. 2400 m., *Pringle*, no. 9364 (hb. Gr.). Puebla: pine woods, Chinantla, *Liebmann* (type, sketch in hb. Gr.); hills near Amozoc, alt. 2700 m., *Pringle*, no. 9360 (hb. Gr.). Morelos: Popocatepec, alt. 2700 m., *Schaffner*, acc. to Sch. Bip., l. c. Vera Cruz: Mt. Orizaba, alt. 3000 m., *Seaton*, no. 181 (hb. Gr.); Mirador, *Liebmann*, no. 417 (sketch in hb. Gr.). Oaxaca: Sierra de las Cruces, alt. 2560 m., *Schaffner*, acc. to Sch. Bip., l. c.; west slope of Mt. Zempoaltepec, alt. 2325 to 2420 m., *Nelson*, no. 567 (hb. Gr.).

The material at hand shows conclusively the inconstancy of the characters upon which *H. tolucanum* was founded.

8. **H. Lemmoni**, Gray. Habit, stature, and foliage as in the preceding; inflorescence thyrsoidal; peduncles tomentulose, without dark hairs — Proc. Am. Acad. xix. 70 (1883), & Syn. Fl. i. pt. 2, 430. — Arizona: Bear Spring, Cave Cañon, *Lemmon*, no. 2803 (hb. Gr.); Santa Rita Mountains, *Pringle* (hb. Gr.). Doubtfully distinct from the preceding.

9. **H. Rosei**, n. sp. Perennial, about 5 dm. tall, branched from below the middle; stem purple, densely spreading-hirsute with long grayish slightly deflexed hairs: leaves about 5, chiefly cauline, 5 to 6 cm. long, about half as broad, ovate-oblong, obtuse, undulate and (often obsoletely) cuspidate-denticulate, sparsely villous, somewhat paler beneath: panicle lax, the lower branches long and naked from the axils of ovate-oblong obtuse foliaceous bracts; pedicels filiform, pale-tomentulose, hispidulous; heads about fifty; involucre narrowly campanulate, 8 to 9 mm. long, nigrescent, externally subappressed-pubescent: achenes purplish black, slender, slightly tapering upwards, 4 to 4.5 mm. long; pappus white, becoming sordid. — Tepic: in the Sierra Madre between Santa Gertrudis and Santa Teresa, 8 August, 1897, *Dr. J. N. Rose*, no. 2080, in part (hb. U. S. Nat. Mus., tracing and fragments in hb. Gr.). Readily distinguished from the related species by its short and broad leaves.

+ + + Achenes 2 to 3 mm. long.

++ Stems decidedly leafy; leaves large (8 to 12 cm. long).

10. **H. Rusbyi**, Greene. Stem erect, 3–4-leaved, 6 dm. high, covered to the middle with spreading villous pubescence, nearly glabrous above; leaves lance-oblong, nearly entire, villous upon both surfaces, acute, the cauline sessile by broad amplexicaul base: branches of the loosely corymbose inflorescence minutely flocculent-tomentose, neither they nor the pale bracts (12 to 14 in number) of the narrow cylindric involucre hispid: heads about 23-flowered. — New Mexico: Mogollan Mountains, *Rusby* (hb. Gr.). Chihuahua: about 64 km. from Guadalupe y Calvo, alt. 2450 m., *Nelson*, no. 4816 (hb. U. S. Nat. Mus.). Mt. Mohinora, *Nelson*, no. 4897 (hb. U. S. Nat. Mus.).

11. **H. Wrightii**, n. comb. Stem tall and stout, fistulose, 4 to 9 dm. high, very hirsute to the summit, 5–9-leaved; caudex stout, comose: basal leaves oblanceolate, narrowed to a long petiolar base, glaucescent, obsoletely glandular-denticulate, villous-hirsute upon both surfaces: cauline leaves lance-oblong, attenuate: panicle many-headed; the peduncles

somewhat rigid, hispidulous; involucre campanulate, in fruit nearly 1 cm. broad, the scales tomentulose and covered with long spreading tawny or darker colored glandular setae: corollas yellow: achenes columnar, nearly black, striate, glabrous, 2 to 2.5 mm. long; pappus bright white or slightly tawny. — *H. Rusbyi*, var. *Wrightii*, Gray, Proc. Am. Acad. xix. 68 (1883), & Syn. Fl. i. pt. 2, 428. *Crepis ambigua*, Gray, Pl. Wright. i. 129 (1852), not of Pl. Fendl. *H. mexicanum*, Gray, Proc. Am. Acad. xix. 70 (1883), in part, not Less. *H. mexicanum*, var. *niveopappum*, Gray in Wats. Proc. Am. Acad. xviii. 110 (1883). — WESTERN TEXAS: between the Limpio and Rio Grande rivers, *Wright* (hb. Gr.). COAHUILA: Sierra Madre south of Saltillo, *Palmer*, no. 757 (coll. of 1880), hb. Gr., doubtful. CHIHUAHUA: hills near Chihuahua, *Pringle*, nos. 592, 771 (both in hb. Gr.). LOWER CALIFORNIA: Saucito, *Brandegee* (hb. Gr.). SAN LUIS POTOSI: in shady places near the city, *Schaffner*, nos. 287, 288 (both in hb. Gr.); *Parry & Palmer*, nos. 552 553 (in both hb. Gr.). DURANGO: among hills, *Palmer*, no. 653 (coll. of 1896), hb. Gr.

12. **H. abscissum**, LESS. Stem 5 to 9 dm. high, hirsute, especially below the middle: basal leaves oblanceolate, obtuse and mucronate or acute, slightly paler beneath, including the long gradually narrowed petiolar portion 17 cm. in length, hairy upon both surfaces, rarely subglabrous, cuspidate-denticulate to subsinuate-dentate; cauline leaves about 4, thin, oblong-lanceolate, the lower attenuate at the base, the upper somewhat cordate-clasping: inflorescence at first thyrsoidal but at length open and 3 to 4 dm. long; heads small, about 26-flowered, slender-peduncled: achenes 2 mm. long; pappus sordid. — Linnaea, v. 132 (1830); Fries, Symbol. Hierac. 148 (where the achenes are incorrectly described as elongated), & Epicris. Hierac. 150 (excluding the indefinite references to plants of the southern United States); Gray, Proc. Am. Acad. xix. 70, & Syn. Fl. i. pt. 2, 430; Hemsl. Biol. Cent.-Am. Bot. ii. 259. *H. strigosum*, Don, Trans. Linn. Soc. xvi. 175 (1830), ex char.; Arv.-Touv. Spicileg. Hierac. 7. *H. Schaffneri*, Sch. Bip. acc. to Arv. Touv. l. c. *H. thyrsoideum spectabile*, Fries, Symbol. Hierac. 141 (1848). *H. Friesii*, Sch. Bip. Bonplandia, ix. 174, 326 (1861), not Hart. *H. Schultzii*, Fries, Epicris. Hierac. 150 (1862). — NUEVO LEON: on the Sierra Madre, *Pringle*, nos. 2205, 2838 (both in hb. Gr.). SAN LUIS POTOSI: grassy hillsides, Las Canvas, *Pringle*, no. 5077 (hb. Gr.). TERRITORY OF TEPIC: between Dolores and Santa Gertrudis, *Rose* (hb. U. S. Nat. Mus.). HIDALGO: Real del Monte, *Coulter* (hb. Gr.). STATE OF MEXICO: near Tacubaya, *Schaffner*,

no. 66 (hb. Gr.); pine woods near Eslava, alt. 2420 m., *Pringle*, no. 9363 (hb. Gr.). MICHOACAN: hills, Patzcuaro, *Pringle*, no. 5270 (hb. Gr.). MORELOS: mountain woods above Cuernavaca, alt. 2260 m., *Pringle*, no. 8053 (hb. Gr.). VERA CRUZ: near Orizaba, *Bourgeau*, no. 2800 (hb. Gr.); Volcano Tuxtla, alt. 600 to 1450 m., *Nelson*, no. 462 (hb. Gr.); Orizaba, *Botteri*, no. 144 (hb. Gr.). PUEBLA: Mount Popocatepetl, alt. 3000 m., *Schaffner*, no. 68 (hb. Gr.). OAXACA: Sierra de San Felipe, alt. 6000 m., *Conzatti & González*, no. 695 (hb. Gr.); Sierra de San Felipe del Agua, alt. 695 m., *Conzatti & González*, no. 401, in part (hb. Gr.); Oaxaca, *Conzatti & González*, alt. 525 m., no. 1004 (hb. Gr.). CHIAPAS: *Ghiesbreght*, nos. 20, 574 (in both hb. Gr.); near San Cristobal, alt. 2100 to 2660 m., *Nelson*, no. 3225 (hb. U. S. Nat. Mus.). GUATEMALA: Department of Santa Rosa, alt. 750 m., *Heyde & Lux*, no. 3784 (of J. D. Smith's distrib.).

13. **H. anthurum**, FRIES. Very tall; stem-leaves linear, the lower attenuate into a petiole, the upper sessile: inflorescence becoming 6 dm. long; involucres very hirsute with dark glandular hairs; pappus reddish. — Symbol. Hierac. 153 (1848), & Epicris. Hierac. 157; Hemsl. Biol. Cent.-Am. Bot. ii. 259. — OAXACA: Sierra de Zempoaltepec, *Liebmann*. Not seen; characters compiled.

++ ++ Stem-leaves 0 to 3, rarely over 7 cm. long.

14. **H. comatum**, FRIES. Leafy and sericeous at the base: stem leafy and strongly hirsute near the base, 4 to 6 dm. high, subglabrous above: leaves oblong-lanceolate, hirsute upon both surfaces, thickish, much paler and glaucous beneath; the lower cauline and radical obtuse or rounded, cuspidate-denticulate, the upper lance-linear, attenuate: heads loosely paniculate, small; peduncles spreading, filiform, canescent-tomentulose, finely glandular-pubescent. — Symbol. Hierac. 148 (1848), including var. *irregulare*, a much branched narrow-leaved form, & Epicris. Hierac. 152; Hemsl. Biol. Cent.-Am. Bot. ii. 259. — PUEBLA: in pine woods at Chinantla, alt. 2100 to 2400 m., *Liebmann* (sketch in hb. Gr.); near Calchicomula, *Rose & Hay*, no. 5806 (hb. U. S. Nat. Mus.). OAXACA: Sierra de San Felipe, grassy wooded slopes, alt. 2400 m., *Pringle*, no. 4701 (hb. Gr.), dry ridges under pines, alt. 2560 to 3000 m., *Pringle*, no. 4708 (hb. Gr.), alt. 2860 to 3300 m., *Nelson*, no. 1077 (hb. U. S. Nat. Mus.); Sierra de San Felipe del Agua, alt. 2300 m., *Conzatti & González*, no. 401, in part (hb. Gr.); west slope of Mount Zempoaltepec, alt. 2300 to 2410 m., *Nelson*, no. 593 (hb. Gr.).

15. **H. oaxacanum**, n. sp. Caudex hairy at the oblique summit,

elongated, sending off tough simple roots; stem simple, 2 to 5 dm. high, flexuous, covered throughout with long spreading hairs: basal leaves 2 to 3, oblong-lanceolate, rounded and apiculate at the apex, narrowed at the base, 3 to 5 cm. long, 10 to 16 mm. broad, hirsute upon both surfaces, paler beneath, glandular-denticulate: petioles nearly as long; cauline leaves about 3, somewhat remote, oblong or oblanceolate, obtusish to acute, denticulate, somewhat narrowed toward the subclasping base: heads about 3; peduncles densely tomentose and bristly with glandular hairs; involucral scales dark colored, attenuate, glandular-pubescent, in fruit 1 cm. long: corollas apparently white or very light yellow: achenes reddish black, columnar, 3 mm. long; pappus in dried specimens sordid white. — State of Mexico: valley of Mexico, *Bourgeau*, no. 384 (hb. Gr.), *Schaffner*, no. 309 (hb. Gr.). Oaxaca: Sierra de San Felipe, alt. 3000 m., *Pringle*, no. 4715 (hb. Gr.), type. Distinguished from the preceding by its fewer larger heads, and by the stem, which is setose to the summit with hairs 2 mm. long.

16. **H. mexicanum**, Less. Stems 1 to several, slender, 2 to 5 dm. high, flexuous, sometimes branched, more or less hairy toward the base, glabrous (to the unassisted vision) above or very finely tomentulose, never setose; caudex woolly-tufted at its summit: leaves essentially confined to a basal rosette, thickish, oblong-lanceolate, obtuse to acute, including the narrowed petiolar base 3 to 7 cm. long, 1 to 1.5 cm. broad, more or less villous-pubescent upon both surfaces, much paler beneath, glandular-denticulate; cauline leaves none or 1 or 2, much reduced, oblong-linear, acute: heads 3 to 10 to (rarely) 26, narrow, about 20-flowered; involucral scales dark colored, narrow, from smoothish to tomentulose and minutely bristly-glandular: achenes 2 to 3 mm. long. — Linnaea, v. 133 (1830); Fries, Symbol. Hierac. 139, & Epicris. Hierac. 149; Gray, Proc. Am. Acad. xix. 70, in part; Hemsl. Biol. Cent.-Am. Bot. ii. 260. ? *H. Lagopus*, Don, Trans. Linn. Soc. xvi. 176 (1830); Fries, Epicris. Hierac. 143. *H. niveopappum*, Fries, Symbol. Hierac. 139 (1848). *H. thyrsoideum plebejum*, Fries, Symbol. Hierac. 142 (1848). *H. praemorsiforme*, Sch. Bip. Bonplandia, ix. 174, 327 (1861). *H. multicaule*, Schaffner acc. to Sch. Bip. Bonplandia, ix. 173 (1861); Fries, Epicris. Hierac. 149 (1862). *H. orizabaeum*, Arv.-Touv. Spicileg. Hierac. 17 (1881), ex char. *H. intybiformia*, Arv.-Touv. Ann. Conserv. Genèv. 95 (1897). — Jalisco: between Huejuquilla and Mesquitec, *Rose* (hb. U. S. Nat. Mus.). State of Mexico: San Angel, alt. 2410 m., *Schaffner* (hb. Gr.); Sierra de las Cruces, alt. 2600 m., *Schaffner*, *Pringle*, no. 5322 (hb. Gr.); dry pine woods Nevada de

Toluca, alt. 2900 m., *Pringle*, no. 4284 (hb. Gr.). PUEBLA: Popocatepetl, *Schaffner*, no. 67 (hb. Gr.). VERA CRUZ: Mt. Orizaba, *Schiede*, no. 285 (hb. Gr.), type; alt. 3950 m., *Liebmann* (sketch in hb. Gr.); alt. 3050 m., *Galeotti*, no. 2173 (hb. Gr.); *Seaton*, no. 182 (hb. Gr.); *Rose & Hay*, nos. 5750, 5751 (hb. U. S. Nat. Mus.). GUERRERO: between Tixtla and Chilpancingo, alt. 1820 to 2140 m., *Nelson* (hb. U. S. Nat. Mus.); top of Sierra Madre near Chilpancingo, alt. 2750 to 3180 m., *Nelson*, no. 2192 (hb. U. S. Nat. Mus.). OAXACA: summit of Mt. Zempoaltepec, alt. 3475 m., *Nelson*, no. 634 (hb. U. S. Nat. Mus.).

17. H. irasuense, BENTH. Near *H. mexicanum* but with a stem less tall and seldom branched, covered throughout by a dark bristly glandular pubescence as well as by a fine short and sordid tomentum: leaves longer, 3 to 13 cm. in length, 1 to 2 cm. broad, subentire or conspicuously glandular-dentate: heads 10 to 15, crowded. — Benth. in Oerst. Vidensk. Meddel. 1852, p. 113; Fries, Epicris. Hierac. 144. Referred by Hemsl. l. c. 259 to *H. lagopus*, Don. — COSTA RICA: on the Volcano Irazu, alt. 2925 m., *Oersted*; also at Laguna del Reventado, alt. 2300 m., 1 January, 1901, *H. Pittier*, no. 14,074 (hb. Gr.).

* * Pubescence on both surfaces of the leaves long, truly woolly, of matted or felted hairs.

18. H. Pringlei, GRAY. Stem 34 to 68 cm. high, woolly toward the base: leaves chiefly basal, rosulate and decumbent upon the ground, obovate to oblanceolate, 7 to 10 cm. long, 20 to 35 mm. wide, obscurely cuspidate-denticulate: heads 12 to 18, in an open corymb; involucre in fruit 1 cm. long, involved in close wool through which short light-colored bristles protrude: achenes columnar, reddish brown, 3 mm. long; pappus essentially white. — Proc. Am. Acad. xix. 69 (1883), & Syn. Fl. i. pt. 2, 429; Arv.-Touv. Ann. Conserv. Genèv. i. 96. — SOUTHERN ARIZONA: Santa Rita Mountains, 6 May, 1881, *Pringle*, no. 314 (hb. Gr.), 2 June, 1884, *Pringle* (hb. Gr.), *Lemmon*, no. 369 (hb. Gr.). To be expected in Mexico.

19. H. jaliscense, n. sp. Root of strong dark brown fibres; stem subflexuous, 48 to 72 cm. high, simple to the corymbosely branched inflorescence, glabrate above, lanate toward the base, the hairs long, white, widely spreading or somewhat reflexed, matted: basal leaves 2 to 5, oblanceolate, 5 to 15 cm. long, 1.5 to 3 cm. wide, erect, cuspidate-denticulate, rounded or obtuse, often apiculate at the apex; cauline leaves 1 to 3, lanceolate to linear: heads about 10; peduncles bristly with pale stipitate glands; involucre 8 to 10 mm. long; inner scales narrowly linear, attenuate, glandular, bristly chiefly near the midrib, scarcely at all woolly: achenes columnar, striate, reddish brown, 3 mm. long; pappus slightly tawny. — JALISCO: near Guadalajara, gravelly banks, *Pringle*, no. 5133 (hb. Gr.), hillsides, *Pringle*, no. 2990 (hb. Gr.), dry gravelly banks, *Pringle*, no. 4461 (hb. Gr.). TERRITORY OF TEPIC: between Dolores and Santa Gertrudis, *Rose*, no. 2044 (hb. U. S. Nat. Mus.). STATE OF MEXICO: Sierra de Ajusco, alt. 2440 m., *Pringle*, no. 7211 (hb. Gr.).

Var. Ghiesbreghtii, n. var. Pedicels and involucral scales covered with close wool (tawny in dried specimens) as well as stipitate glands: habit, etc., of the typical form. — CHIAPAS: growing in the plains, cool region, *Ghiesbreght*, no. 573 (hb. Gr.).

DOUBTFUL SPECIES.

H. STUPOSUM Fries. Leafy at base, the summit of the caudex woolly; stem 1 to 1.2 dm. high, almost naked, branched, slightly scabrous: radical leaves densely rosulate, petiolate, lanceolate, repand-dentate; the cauline reduced: inflorescence thyrsoidal, clothed with white floccose pubescence; involucre soft-villous: achenes narrowed downward; pappus white. — Vet. Acad. Förh. 1856, p. 146, & Epicris. Hierac. 148; Hemsl. Biol. Cent.-Am. Bot. ii. 260. — MEXICO: *Ehrenberg*.

H. FRIGIDUM, Wedd. Chloris, And. i. 225 (1855); Klatt in Engl. Jahrb. viii. 52. — Klatt, l. c., ascribes this South American species to Guatemala, a report based upon Lehmann's no. 1526, which is not now in the Klatt Herbarium. No specimen from north of Panama has been seen by the writers, which corresponds with the South American plant.

IV. — SYNOPSIS OF THE MEXICAN AND CENTRAL AMERICAN SPECIES OF ALNUS.

By M. L. FERNALD.

SEVERAL Mexican and Central American Alders of very diverse characters have been generally called *A. acuminata* and *A. jorullensis*. This disposition of the forms has been most unsatisfactory, and the following synopsis is offered as a partial solution of the difficulties presented by them.

* Mature strobiles 2 cm. or more long (rarely shorter in *A. oblongifolia* and *A. glabrata*).

+- Petioles pubescent.

++ Leaves ovate to obovate, rounded at base.

A. acuminata, HBK. Bark of branches ashy brown, smooth: buds pubescent: leaves short-acuminate, finely and doubly serrate-dentate, 0.5 to 1.5 dm. long, 0.4 to 1 dm. broad, subcoriaceous, sparingly pilose on the midrib above, more abundantly so on the midrib and nerves beneath; petiole about 1 to 1.5 cm. long: peduncles stout, glabrous or minutely puberulent: staminate aments becoming 1 dm. or more long; the ashy-brown strobiles 3 or 4, sessile or short-pedicelled, ovoid-oblong, 2.2 to 2.6 cm. long, about 1.5 cm. thick: nutlet lanceolate, 4 mm. long, 1.75 mm. broad, bordered by an equally broad coriaceous wing. — Nov. Gen. & Sp. ii. 20; Regel in DC. Prodr. xvi, pt. 2, 183. Described from the Andes of Peru, this species has been made to include practically all the forms of South America, Mexico, California and New Mexico. From the original description, however, the species appears to be a well marked plant represented in the Gray Herbarium from Guatemala and Bolivia. GUATEMALA, San Lucas, Dept. Zacatepequez, alt. 1700 m., April, 1890 (*J. D. Smith*, no. 2188): BOLIVIA (*Bang*, no. 1893).

++ ++ Leaves oblong-lanceolate or elliptic, subcuneate at base.

A. oblongifolia, TORR. Bark of branches reddish brown, smooth; of the branchlets puberulous or glabrate: buds glabrous: leaves acute, sharply double-serrate, 0.5 to 1.5 cm. long, 3 to 6.5 cm. broad, more or less glandular-dotted, coriaceous, pilose or glabrate above, paler and more or less pilose on the prominent veins beneath; petioles slender, 1 to 2.5 cm. long: peduncles puberulent or glabrate: staminate aments becoming 1 dm. long; the brown or slightly ferrugineous strobiles 3 to 5, pedicelled, sub-cylindric, when mature about 2 cm. long, 1 cm. thick: nutlet obovate, 3 mm. long, 2 mm. broad, with a narrow submembranaceous wing. — Bot. Mex. Bound. 204. *A. serrulata*, γ *oblongifolia*, Regel, l. c. 188. *A. acuminata*, Sargent, Silva, ix. 79, t. 457, in part, not HBK. — NEW MEXICO, banks of the Mimbres and near Santa Barbara, tree 9.23 m. high (*Charles Wright*, no. 1864): CALIFORNIA, San Bernardino, "tree 80 feet high, 2–3 feet in diameter," 1876 (*C. C. Parry & J. G. Lemmon*). SONORA, Huchuerachi, alt. 1230 m., Dec., 1890 (*C. V. Hartman*, no. 322, *F. E. Lloyd*, no. 464).

+ + Petioles glabrous.

++ Leaves pilose on the nerves beneath.

A. arguta, SPACH. Bark of the branches and branchlets brown, glabrous: buds short-stalked, glabrous: leaves ovate or obovate, acute or bluntish, rounded or somewhat narrowed at base, finely but sharply double-serrate, 0.4 to 1 dm. long, 3 to 7.5 cm. broad, sparingly glandular-dotted, but glabrate above, paler beneath and pilose especially along the nerves; petiole slender, 1 to 2.5 cm. long: peduncles slender, glabrate; staminate aments 4 to 7 in terminal panicles (becoming about 1 dm. long): the dark reddish-brown strobiles 3 or 4, sessile, sub-cylindric, when mature 2 to 3 cm. long, about 1 cm. thick: the cuneate obovate or oblong nutlet 2.5 cm. long, 1.5 mm. broad, somewhat broader than the coriaceous wing. — Ann. Sci. Nat. ser. 2, xv. 205. *A. acuminata, a genuina*, Hemsl. Biol. Cent.-Am. Bot. iii. 165, not Regel. *A. jorullensis*, var. *castaneaefolia*, Hemsl. l. c. 166, not Regel. *Betula arguta*, Schlecht. Linnaea, vii. 139. — VERA CRUZ, Mt. Orizaba, April 10, 1867 (*Bilimek*, no. 404), without date (*Botteri*, no. 857); alt. 3400 m., Aug., 1891 (*H. E. Seaton*, no. 231). Originally described from VERA CRUZ and TAMAULIPAS.

++ ++ Leaves glabrous, or at most only slightly pubescent when young.

A. glabrata, n. sp. Bark of the branches brown, glabrous: buds short-stalked, sparingly puberulent or glabrous: leaves oblong-lanceolate or elliptic, long-acuminate, narrowed or slightly rounded at base, coarsely and doubly serrate-dentate especially above, paler and dull beneath, the yellowish or brownish nerves prominent; petiole to 1 to 3.5 cm. long: peduncles thickish, glutinous; staminate aments 3 or 4, becoming 7 or 8 cm. long; the dark brown or blackish strobiles oblong or cylindric, sessile or short-stalked, in maturity 1.8 to 2.8 cm. long, 9.5 to 14 mm. thick: nutlets oblong to oblong-lanceolate, 3 to 5 mm. long, 1.5 to 2 mm. broad, with slightly narrower coriaceous wings. — *A. acuminata*, Sargent, l. c., in part, not HBK. — GUANAJUATO, Mt. San Nicholas, April, 1882, and Santa Rosa, April, 1901 (*A. Dugès*): QUERETARO, between Mexico and Queretaro, November, 1827 (*Berlandier*, no. 1294): HIDALGO, Dublan, December 4, 1902 (*C. G. Pringle*, no. 11,179): MEXICO, Valley of Mexico, June 1, 1865 66 (*Bourgeau*, no. 244), Dec. 9, 1892 (*C. G. Pringle*, no. 4361); near Tizapan, alt. 2300 m., Jan. 30, 1899 (*C. G. Pringle*, no. 8022): OAXACA, Valley of Oaxaca, alt. 1570 to 1786 m., Sept. 8, 1894 (*E. W. Nelson*, no. 1254); near Huajuapam, alt. 1725 to 2000 m., Nov. 16, 1894 (*E. W. Nelson*, no. 1976); Coyula de Cuyamecalco, alt. 2155 m., Apr. 24, 1895 (*L. C. Smith*, no. 495). Said to be a large tree.

* * Mature strobiles less than 2 cm. long (exceptional plants of *A. oblongifolia* and *A. glabrata* may be looked for here).

+ Mature leaves ferrugineous-velutinous beneath.

A. ferruginea, HBK. Branches brown, the younger parts more or less pubescent: buds very pubescent: leaves ovate, acuminate at tip, rounded or narrowed at base, coarsely and doubly serrate, 0.5 to 1.5 dm. long, 3.5 to 8 cm. broad, dull green and minutely pilose above; petiole thickish, 1 to 2 cm. long: peduncles villous or glabrate, the staminate aments becoming 1.5 dm. long: the dark brown short-oblong strobiles subsessile or short-stalked, in maturity 1.5 to 1.9 cm. long, about 1 cm. thick: nutlets cuneate-obovate, 3 mm. long, 2 mm. wide, with narrower coriaceous wings. — HBK. l. c. 21. *A. acuminata, δ ferruginea*, Regel, l. c. 184; Hemsl. l. c. 165. — Described from Colombia. Extending into Central America and Southern Mexico. CHIAPAS (*Ghiesbreght*, no. 160): GUATEMALA, Coban, Alta Verapaz, alt. 1488 m., Feb. 1886 (*H. von Tuerckheim* in Exsicc. J. D. Smith, no. 351).

++ Mature leaves merely pilose on the nerves beneath.

A. jorullensis, HBK. Branches reddish brown, smooth, the younger parts puberulent or glabrate: buds pruinose: leaves oblong or oblong-obovate, thick, mostly 7 to 13 cm. long, 2.5 to 5.5 cm. wide, rounded or acute at tip, subcuneate or rounded at the subentire base, crenate-serrate toward the tip, glaucous and sublucid above, brownish beneath and closely pruinose with glandular or waxy atoms, the prominent nerves more or less pilose; petioles thickish, 3 to 10 mm. long, pubescent or glabrate: peduncles thick, pruinose or glabrate; staminate aments 3 or 4, becoming 6 cm. long; the reddish-brown strobiles 3 or 4, sessile or subsessile, oblong-ovoid, 1 to 1.8 cm. long, barely 1 cm. thick: nutlets cuneate-obovate to suborbicular, 2.5 mm. long, 2 to 2.5 mm. broad, with narrow membranaceous wings. — HBK. l. c. 20; Regel, l. c.; Hemsl. l. c. 166, in part. *A. acuminata*, Sargent, l. c. in part, not HBK. Sierra Madre of Central and West-Central Mexico. Described from Volcan de Jorullo, Michoacan. MICHOACAN, mountains near Patzcuaro, Dec. 21, 1891 (*C. G. Pringle*, no. 5057); Sierra Madre, without locality (*Seemann*): MEXICO, foothill of Mt. Ixtaccihautl, Jan. 5, 1899 (*C. C. Deam*, no. 130).

Var. **exigua**, n. var. Leaves short-oblong, 2 to 4 cm. long, 1 to

2.4 cm. broad, the nerves conspicuously pilose beneath; petioles 2 to 4 mm. long, pilose: strobiles globose or short-oblong, about 1 cm. long. — GUANAJUATO, mountains of Santa Rosa, April, 1901 (*A. Dugès.*)

V. — DIAGNOSES AND SYNONYMY OF MEXICAN AND CENTRAL AMERICAN SPERMATOPHYTES.

By J. M. GREENMAN.

Echeandia Pringlei, n. sp. Roots fascicled, tuberiform; the tuberous portion 1 to 2 cm. long, 5 to 7 mm. in diameter: leaves chiefly radical, 1 to 4 dm. long, 2 mm. broad, about 9-nerved, glabrous on both surfaces, more or less involute, hirtellous-ciliolate-margined, firm in texture, yellowish in the dried state, sometimes falcate; the fibrous portion of the bases of the older leaves persistent around the base of the scapiform stem: scape simple or branched, 2.5 to 4 dm. high, bearing 1 to 3 leaf-like bracts: floral bracts lance-ovate, acute, scarious: flowers two in the axil of each bract; pedicels 5 to 10 mm. long, jointed below the middle: perianth white; divisions lanceolate, about 1 cm. long, 1.5 to 2.5 mm. broad, acute, 3-nerved: stamens nearly or quite as long as the perianth-divisions; anthers united into a tube 6.5 to 8 mm. in length: capsule oblong, 6 mm. long, transversely nerved. — MEXICO. State of Jalisco: Etzatlan, collection of 1903, *C. G. Pringle*, no. 11,852 (hb. Gr.).

Sisyrinchium macrophyllum, n. sp. Erect, 4 to 6 dm. high, glabrous throughout: roots long, fleshy, almost tuberous: stems narrowly winged, leafless to the first fork, branched above into a several-headed inflorescence: leaves lance-attenuate, 5 to 6 dm. long, 1 to 5 cm. broad, acute, prominently nerved, narrowly subcartilaginous-margined: bracts of stem and inflorescence lance-acuminate, 8 cm. or less long: peduncles in fruiting specimens 5 to 8 cm. in length; spathes 2–5-flowered; valves lanceolate, 3 to 3.5 cm. long, acute, the outer slightly shorter than the inner: capsules oblong, triangular, 1.2 to 2 cm. long, one half as broad, glabrous: seeds subspherical, 2 mm. in diameter. — MEXICO. State of San Luis Potosi: Alvarez, 28 September to 3 October, 1902, *Dr. Edward Palmer*, no. 205 (hb. Gr. and hb. U. S. Nat. Mus.).

Phoradendron Palmeri, n. sp. Pubescent throughout; younger parts subtomentose: stem and branches terete: leaves narrowly oblong or oblong-spatulate, 1 to 3.5 cm. long, 3 to 8 mm. broad, obtuse, entire, narrowed below into a subpetiolate base: spikes of fertile flowers solitary, sessile or short-pedunculate, 2–3-jointed, in anthesis 3 to 6 mm. long, 7–13-flowered: flowers 3-merous, fruiting spikes 1.5 to 2 cm. long, on peduncles 2 to 6 mm. in length: berry oblong, 3 to 4 mm. long, hirtellous-puberulent. — MEXICO. State of San Luis Potosi: Alvarez, 5 to 10 September, 1902, *Dr. Edward Palmer*, no. 119 (hb. Gr. and hb. U. S. Nat. Mus.).

In general appearance this species resembles *Phoradendron brachystachyum*, D. Oliv., but differs in the denser tomentum, the longer and pedunculate fruiting spikes, and finally by the more oblong pubescent fruit.

Echites Smithii, n. sp. Stem terete, closely and evenly puberulent: leaves opposite, short-petiolate, oblong-lanceolate, 3 to 6 cm. long, 1 to 2.5 cm. broad, acute, entire, subcordate at the base, dark green and glabrous or slightly hirtellous-puberulent above especially near the margins, paler and glabrous beneath, bearing 2 small glands on the upper surface of the blade at its junction with the petiole; midrib and the 5 to 7 pairs of lateral nerves rather prominent beneath; petioles 2 to 4 mm. long, puberulent: inflorescence usually supra-axillary, racemose, few (1–3)-flowered, shorter than the leaves; peduncles 5 mm. or less in length; bracts lance-linear, 4 to 5 mm. long, acute, often purplish; pedicels 5 to 8 mm. long and as well as the peduncles puberulent: calyx deeply 5-parted, persistent; lobes lanceolate, acute, 4 to 5 mm. long, green or slightly purplish-tinged: glands 2 to 4, opposite each sepal: corolla salverform, yellow or yellowish white, glabrous; tube 10 to 12 mm. high; lobes oblique, about 8 mm. in length: stamens included; anthers blunt, 2.5 mm. long: immature follicles pubescent: disk deeply 5-lobed. — MEXICO. State of Oaxaca: Salomá, altitude 1981 m., 9 August, 1895, *Albert L. Smith*, no. 672 (hb. Gr.); Huancilla-Nochixtlan (Nochistlan), alt. 2000 m., June, 1901, *C. Conzatti & V. González*, no. 1198 (hb. Gr.).

MANDEVILLA OAXACANA, Hemsley, Biol. Cent.-Am. Bot. ii. 316. To this species should be referred specimens collected by Mr. C. G. Pringle on dry ledges of foothills above the city of Oaxaca, Mexico, at an altitude of 1830 m., 29 May, 1895, no. 4662 (hb. Gr.). This number of Mr. Pringle's *exsiccatae* was erroneously determined as *Echites secundiflora*, A. DC., under which name it was distributed. Hereto may also be referred specimens secured by Señor C. Conzatti on hills of the San Felipe del Agua, Oaxaca, altitude 1800 m., 1 September, 1895, no. 578 (hb. Gr.).

The species is apparently common about the City of Oaxaca, and is well characterized by the tubular-funnelform corolla. In all other characters it agrees well with the genus *Echites*.

Gonolobus calcicola, n. sp. Stems twining, ligneous at the base, hirtellous-puberulent, the younger parts also hirsute with spreading or somewhat reflexed hairs: leaves petiolate, ovate, 2 to 6 cm. long, 1 to 3 cm. broad, acute or acuminate, entire, hirtellous-pubescent above, granulose beneath and often scattered-hirsute especially on the prominent midrib and nerves, revolute-margined; petioles 10 to 22 mm. long, hirtellous-pubescent: inflorescence in axillary congested several-flowered cymes; peduncles shorter than the petioles: flowers medium-size: calyx deeply 5-parted; divisions lance-elliptic, 6 to 7 mm. long, 2 to 3 mm. broad, acute, ciliate, externally hirsutish pubescent: corolla dark brownish-purple, about 2 cm. in diameter, externally minutely puberulent, upper or inner surface corrugated-verrucose, bearing a few scattered whitish hairs: crown 5-lobed; lobes fleshy, unequally papillose or somewhat warty: fruit not seen. — MEXICO. State of Morelos: limestone hills, Jojutla, near Cuernavaca, 30 August, 1902, *C. G. Pringle*, no. 11,367 (hb. Gr.).

In general habit *G. calcicola* suggests *G. congestus*, Decne., but the smaller narrower leaves, fewer larger flowers, and the lobed crown serve to separate readily the two species.

NAMA UNDULATUM, HBK., var. MACRANTHUM, Choisy, Hydrol. 18, t. 2, f. 1; DC. Prodr. x. 182; Gray, Proc. Am. Acad. x. 330; Hemsl. Biol. Cent.-Am. Bot. ii. 364. *N. Berlandieri*, Gray, l. c. viii. 282. Specimens belonging to this interesting variety have been secured by Mr. E. W. Nelson at Montemorelos, State of Nuevo Leon, no. 6697. Although the variety has been generally understood as having merely sessile and amplexicaul leaves, yet the specimens now at hand, from Mr. Nelson, show very distinctly that the leaves are not only sessile and amplexicaul, but more or less decurrent on the stem. Moreover a careful examination of Berlandier's nos. 699, 2116 (hb. Gr.) show some of the leaves to be decurrent. The plant is a well-marked variety, and further material, especially complete specimens, may prove it to be specifically distinct.

Cynoglossum Pringlei, n. sp. Stems leafy to the inflorescence, striate, hirsute-pubescent: stem leaves sessile, semiamplexicaul, oblong-lanceolate, 3 to 12 cm. long, 1 to 4 cm. broad, acute, entire, dark green above, paler beneath, hirsute-pubescent on both surfaces: inflorescence compound, many-flowered, more or less bracteate: pedicels 0.5 to 2 cm. long, recurved at least in the later stages, appressed-pubescent: calyx

5 parted, glabrous except at the very base; lobes 3 to 5 mm. long, subrotund, imbricated: style 1.5 to 2 mm. long, persistent: corolla not seen: nutlets broadly ovate or subrotund, dorso-ventrally compressed, 10 to 12 mm. long, nearly or quite as broad, glochidiate-hispid on the upper or outer surface, smooth beneath; scar triangular-ovate. — Mexico. State of Mexico: Fultenango Cañon, altitude 2500 m., 9 October, 1902, *C. G. Pringle*, no. 11,350 (hb. Gr.): "Morelia (Michuacán) et de Moro Leon (Guanajuato)," *A. Dugès* (hb. Gr., collection of 1893).

Lithospermum Nelsonii, n. sp. An herbaceous perennial: stems erect from a ligneous base, 2.5 to 4.5 dm. high, sparingly branched above, pubescent with spreading hirsute hairs intermixed with a subappressed puberulence: lower stem-leaves oblanceolate, 3 to 7 cm. long, 8 to 18 mm. broad, obtuse or submucronate-acute, narrowed below into a winged sub-petiolate base, hirsute-hispid on both surfaces, more or less 3-nerved; upper stem-leaves sessile, oblong-lanceolate: flowers on short pedicels, disposed in the axils of the uppermost leaves forming a leafy-bracteate scorpioid raceme: calyx deeply 5-parted; divisions linear, about one half as long as the tube of the corolla, acute, hirsute-pubescent: corolla salverform, white or pale yellow; tube 13 to 14 mm. long, externally appressed puberulent, within glabrous below, papillose-puberulent above, bearing in the throat opposite the lobes 5 conspicuous folds; lobes subrotund, 4 to 5 mm. long and broad: stamens inserted high on the corolla-tube, included: style persistent: nutlets white, ovate, 3 mm. long, acute, slightly crested on the back toward the summit. — Mexico: State of Nuevo Leon: Cerro la Silla, near Monterey, 20 March, 1902, *E. W. Nelson*, no. 6682 (hb. Gr., and hb. U. S. Nat. Mus.).

Capraria mexicana, Moric. in DC. Prodr. x. 429. Specimens collected by Mr. E. W. Nelson at Soto la Mariana, State of Tamaulipas, no. 6643 (hb. Gr., and U. S. Nat. Mus.), agree in all details with the original description of the above species. Although *C. mexicana* has been considered by some authors to be identical with the widely distributed species *C. biflora*, L., yet on account of its glabrous character throughout, the uniformly narrow-lanceolate acuminate leaves, the long anthers, and the more acute corolla-lobes, it seems to the writer well worthy of specific rank. In addition to Mr. Nelson's no. 6643, the following specimens may be referred to *C. mexicana*, Moric: en route from San Luis Potosi to Tampico, December, 1878, to February, 1879, *Dr. E. W. Palmer*, no. 1125 (hb. Gr.); Wartenberg, near Tantoyuca, prov. Huasteca, State of Vera Cruz, *L. C. Ervendberg*, no. 184 (hb. Gr.).

STEMODIA SCHOTTII, Holzinger, Cont. U. S. Nat. Herb. i. 286. Imperfect specimens collected by Dr. Edward Palmer between the Frio and the Nueces Rivers on the road to Lerios, Texas, and distributed under no. 1053 (hb. Gr.), were referred by Dr. Watson, Proc. Am. Acad. xviii. 130, to *Stemodia durantifolia*, Swartz; but a careful examination of Dr. Palmer's specimen in the light of additional material shows very clearly that the plant should be referred to the above well characterized species, *Stemodia Schottii*, Holzinger. Strong and vigorous specimens of this species were secured by Mr. E. W. Nelson between San Fernando and Jimenez, State of Tamaulipas, Mexico, 26–27 February, 1902, no. 6619 (hb. Gr., and hb. U. S. Nat. Mus.). These two collections extend the geographical range of *S. Schottii* from Rio Grande, where it was first found through the Valley of the Rio Grande nearly to the Gulf of Mexico.

Dyschoriste Pringlei, n. sp. Stems several, 1 to 2 dm. in length, erect or ascending from a ligneous perennial base, densely hirsute-pubescent or subtomentose: leaves lance-elliptic to slightly obovate, 1.5 to 4 cm. long, 0.5 to 1.6 cm. broad, obtuse or acute, entire, narrowed below to a subpetiolate base, sparingly hirsute-pubescent on both surfaces: flowers crowded in the axils of the upper leaves forming a subcapitate leafy inflorescence: calyx 13 to 14 mm. long, densely pubescent with white flaccid-hirsute hairs, divided to somewhat below the middle; divisions lance-attenuate: corolla tubular-campanulate, 3 to 4 cm. long, externally pubescent, more or less purplish-maculate at least in the dried state: stamens adnate to the corolla for about one half its length; anthers rather conspicuously calcarate: ovary glabrous; style pubescent: mature capsule not seen. — MEXICO. State of Jalisco: deep cañons near Guadalajara, 1 July, 1889, *C. G. Pringle*, no. 2907 (hb. Gr.), distributed as "*Calophanes capitatus*, Oerst."; Rio Blanco, near Guadalajara, 22 July, 1902, *C. G. Pringle*, no. 11,313 (hb. Gr.).

Justicia (Dianthera) Canbyi, n. sp. Stems 3 dm. or more high, sparingly branched, subterete, slightly furrowed, hirsute-pubescent in lines from node to node or not infrequently over essentially the entire surface: leaves petiolate, ovate, 1.5 to 5 cm. long, 1 to 3 cm. broad, acute, entire, revolute-margined, cuneate to subcordate at the base, dark green above, pale beneath, usually glabrous on both surfaces except for hirsute hairs along the midrib, occasionally with a few scattered hairs over either surface; petioles 5 mm. or less in length, ciliate: flowers on axillary or terminal spikes more or less paniculately disposed; bracts and bracteoles small, subulate, shorter than the calyx: calyx 3 mm.

long, deeply 5-parted, puberulent; the posterior segment much smaller than the other segments: corolla, 2-lipped, purple; tube slender, cylindrical, 22 mm. long, externally pubescent; upper or posterior lip suberect, entire, acute; lower or anterior lip deeply 3-lobed, spreading: capsule 12 mm. long, pubescent: seeds verrucose. — MEXICO. State of Tamaulipas: Soto la Marina, 2 March, 1902. *E. W. Nelson*, no. 6650 (hb. Gr., and hb. U. S. Nat. Mus.). State of Nuevo Leon: near Monterey, 19 March, 1900, *William M. Canby*, no. 187 (hb. Gr.). State of Hidalgo: Zimapan, *Th. Coulter*, no. 1213 (hb. Gr.). This number of the Coulter collection was referred by Hemsley, Biol. Cent.-Am. Bot. ii. 516, to *Siphonoglossa pilosella*, Torr., from which species it is certainly distinct. From the shape of the corolla the species here described might very well pass as a *Siphonoglossa*, but the character of the inflorescence and the absence of a spur or any appendage on the lower anther-cell place it rather with *Justicia (Dianthera)*. In general appearance *Justicia (Dianthera) Canbyi* is not unlike certain *Rhinacanthi* of the Old World.

RANDIA OBCORDATA, Watson, Proc. Am. Acad. xxiv. 53. The only recorded station for this species is the original locality, namely, on high mesas near Guaymas, State of Sonora, Mexico. Its occurrence at Manzanillo, State of Colima, where excellent fruiting specimens were secured by Dr. Edward Palmer, 2 to 18 March, 1891, no. 801 (hb. Gr., and hb. U. S. Nat. Mus.) indicates that the species has a rather extended distribution along the west coast of Mexico.

CRUSEA SUBULATA, Gray, var. **leiocarpa**, n. var. Carpels 2 mm. long, glabrous: habit and other characters of the species. — MEXICO. State of San Luis Potosi: Alvarez, 28 September to 3 October, 1902, *Dr. Edward Palmer*, no. 147 (hb. Gr., and hb. U. S. Nat. Mus.).

Stevia vernicosa, n. sp. Shrub, 1 m. or less high: stem branched; branchlets terete, often verrucose, glabrous or somewhat pubescent towards the inflorescence, reddish: leaves opposite, lance-elliptic to oblanceolate, 1.5 to 4.5 cm. long, 4 to 8 mm. broad, obtuse, rather closely serrulate or crenate-serrulate, narrowed below to a subpetiolate base, glabrous, glandular-punctate and glutinous: heads about 12 mm. high, 5-flowered, disposed in dense cymules terminating the stem and branches: involucre 6 to 8 mm. high; bracts of the involucre 5, lanceolate, acute or obtuse, glutinous: pappus of 1 to 5 setae nearly or quite equalling the corolla-tube, and of 5 minute scales: corollas white: mature achenes 4 to 4.5 mm. long, puberulent. — MEXICO. State of Morelos: on dry ledges, Sierra de Tepoxtlan, altitude 2310 m., 15 March, 1899, *C. G.*

Pringle, no. 7698 (hb. Gr.), distributed as "*Stevia salicifolia,* Cav." State of Mexico: Mt. Ixtaccihautl, altitude 3350 to 3650 m., 1903, *C. A. Purpus,* no. 185 (hb. Gr. and hb. Brandegee). Mr. Pringle's specimen above cited was referred to *S. salicifolia,* Cav. to which species *S. vernicosa* is evidently related, but from which it differs in having smaller closely instead of remotely serrulate leaves; moreover, the leaves are not attenuate at both ends, and the lateral veins are barely visible instead of conspicuous as in typical *S. salicifolia.*

Carphochaete Schaffneri, n. sp. An herbaceous perennial: stems usually several from a ligneous base, 3 to 4.5 dm. high, glabrous or puberulent: leaves opposite, sessile, subamplexicaul, linear-lanceolate, 1 to 4 cm. long, 1 to 3.5 mm. broad, acute, entire, glabrous, glandular-punctate: inflorescence loosely corymbose: heads 2.5 to 3 cm. long, 4-flowered: involucre 1.3 to 1.5 cm. high; bracts of the involucre lanceolate and acute to lanceolate more or less abruptly narrowed at the apex and submucronate-acute, externally as well as the peduncles glandular-hirtellous: pappus of 6 to 8 elongated linear-lanceolate 1-nerved paleaceous aristae and 2 to 5 short oblong obtuse pales: corollas 1.5 to 1.7 cm. long, white with a purplish throat, glabrous: achenes 10 to 12 mm. long, minutely hirtellous. — MEXICO. State of San Luis Potosi: Valley of San Luis Potosi, September, 1876, *Dr. J. G. Schaffner,* no. 241 (hb. Gr.), distributed as "An *Stevia triflora,* DC."; region of San Luis Potosi, altitude 1850 to 2465 m., *Parry & Palmer,* no. 329 (hb. Gr.). The number here cited from Parry & Palmer's collection was referred by Hemsley, Biol. Cent.-Am. Bot. ii. 108, to *Carphochaete Grahami,* A. Gr. from which species *C. Schaffneri* is easily distinguished by the narrower acute leaves and the more lanceolate glandular-hirtellous unciliated involucral bracts.

Brickellia megalodonta, n. sp. Stem terete, green or purplish, pubescent: leaves opposite, petiolate, triangular-ovate to triangular-lanceolate, 3 to 7 cm. long, 1.5 to 5 cm. broad, acuminate, acute, unequally and coarse dentate, subcordate to somewhat cuneate at the base, 3-nerved, olive-green and sparingly pubescent above, paler and pubescent on the nerves beneath: inflorescence a terminal somewhat leafy paniculate cyme; peduncles 2.5 cm. or less in length, pilose-pubescent and with glandular hairs intermixed: heads 12 to 13 mm. high, about 15-flowered: involucral bracts imbricated, 4-5-seriate, lance-acuminate, gradually narrowed to the tip, conspicuously nerved, pale-green, often tinged with purple; the outer bracts shorter than the inner: pappus silvery white, slightly shorter than the yellowish-green corolla: mature achenes 4 mm. long, pubescent. — MEXICO. State of Jalisco: Guadalajara. 22 September, 1903, *E. W. D. Holway*, no. 5022 (hb. Gr.).

In general appearance this species resembles *B. Coulteri*, and *B. brachiata*, Gray. From the former it differs in having constantly larger leaves and longer petioles, in the pubescence of peduncles, etc.; from the latter species it differs conspicuously in the lance-attenuate involucral bracts, which are gradually narrowed to the tip, not abruptly contracted at the apex and mucronate.

Barroetea sessilifolia, n. sp. Annual: stem erect, about 8 dm. high, usually much-branched, terete, hirsutish-pubescent with spreading or subappressed hairs; branches arcuate-ascending: leaves opposite, sessile, ovate, 1.5 to 5 cm. long, 1 to 4 cm. broad, subulate-dentate, hirsute-pubescent on both surfaces, especially on the midrib and veins beneath, 3-nerved from the base: inflorescence a loose panicle: heads about 1 cm. high: involucre 4–5-seriate; bracts of the involucre lanceolate, subulate-acuminate, glabrous, conspicuously nerved, pale-green, scarious-margined: mature achenes 3 to 3.5 mm. long, dorso-ventrally compressed, the outer surface 3–4-nerved, the inner 1–2-nerved, dull brown, slightly pubescent on the nerves. — MEXICO. State of Morelos: limestone hills near Yantepec, altitude 1220 m., 21 October, 1902, *C. G. Pringle*, no. 9865 (hb. Gr.), distributed as "*Brickellia diffusa*, Gray." State of Guerrero: vicinity of Acapulco, October, 1894 to March, 1895, *Dr. Edward Palmer*, no. 625 (hb. Gr.), originally determined as "*Barroetea subuligera*, Gray."

BARROETEA SUBULIGERA, Gray, var. **latisquama**, n. var. Habit and foliage of the species: heads 30–35-flowered; involucral bracts lanceolate to lance-oblong, mostly abruptly contracted at the apex and terminated by a mucro, more or less tinged with purple: achenes 3 mm. long, ciliate, sparingly pubescent on both surfaces. — MEXICO. State of Jalisco: on dry rocky mountains near Etzatlan, 27 October, 1903, *C. G. Pringle*, no. 8773 (hb. Gr.).

Gutierrezia argyrocarpa, n. sp. Suffruticose: stem much-branched, striate, hirtellous-puberulent: leaves linear to linear-spatulate, 1 to 1.5 cm. long, 1 to 2.5 mm. broad, obtusish, entire, glabrous on both surfaces: heads numerous, 4 to 5 mm. high, sessile or pedunculate, terminating the branches in cymose inflorescences: involucre campanulate, 3-seriate; bracts of the involucre narrowly oblong, appressed, herbaceous-tipped, the outer shorter: receptacle convex, not elongated: ray-flowers mostly 6; pappus of the ray-flowers similar to that of the flowers of the disk but shorter; rays pale yellow: disk-flowers about 12: achenes densely appressed-sericeous; pappus of 12 or more unequal somewhat lacerated scales nearly or quite as long as the achene. — MEXICO. State of Hidalgo: on chalk bluffs of Tula, altitude 2075 m., 20 September, 1902, *C. G. Pringle*, no. 11,361 (hb. Gr.).

The habit and general appearance of *G. argyrocarpa* is like *G. Berlandieri*, Gray, from which, however, it differs by the longer pappus, fewer ray-flowers, and by the more densely pubescent achenes. From *G. eriocarpa*, Gray, it differs in having fewer flowers in the head, a merely convex not high conical receptacle, and by the strongly appressed pubescence of the achene.

Erigeron irazuense, n. sp. Suffrutescent: stems erect or ascending, 1 to 2 dm. high, pilose-pubescent, purplish: leaves spatulate to oblanceolate, 1 to 3.5 cm. long, 3 to 8 mm. broad, apiculate-acute, sparingly mucronate-dentate in the upper half, gradually narrowed below the middle to a winged subpetiolar base, pilose-hirsute on both surfaces: heads few, 10 to 12 mm. high, terminating the stem and branches on 3 cm. or less long pubescent peduncles: involucre 2–3-seriate, pubescent; bracts of the involucre lance-attenuate, equalling or slightly exceeding the numerous flowers of the disk, more or less purplish; the outer bracts somewhat shorter than the inner: ray-flowers 3-seriate; rays narrow, white or roseate: achenes pubescent. — COSTA RICA. "Endroits humides," La Playita, altitude, 3300 m., Volcan Irazu, 31 January, 1900, *H. Pittier*, no. 14,075 (hb. Gr., and hb. Physico-Geogr. Cost. Ri.).

DIPLOSTEPHIUM SCHULTZII, Wedd. Chloris Andina, i. 204. *D. rupestre*, Klatt, Bull. Soc. Bot. Belg. xxxi. pt. 1, 197, reprint, p. 15, not Wedd. The late Professor Klatt referred Pittier's no. 178, collected on the summit of Mount Irazu, Costa Rica, 30 March, 1888, to *Diplostephium rupestre*, Wedd. A careful study of the specimen under this number in the Gray Herbarium shows that it is identical rather with *D. Schultzii*, Wedd. To this species is also to be referred Pittier's no. 14,072 (hb. Gr., and hb. Inst. Phisico.-Geogr. Cost. Ri.). As far as known to the writer *D. Schultzii* has not been hitherto recorded from North America.

Baccharis glandulosa, n. sp. Glandular-pubescent throughout: stems caespitose from a perennial root, erect, 0.5 to 1 m. high, striate, reddish-brown: leaves alternate, sessile, amplexicaul, lanceolate, 5 to 8 cm. long, 1 to 1.5 cm. broad, acute, mucronate-denticulate, dark green above, paler beneath: inflorescence a terminal leafy paniculate cyme: heads of the pistillate plant about 1 cm. high with many (100–120) pistillate flowers surrounding 8 to 15 hermaphrodite flowers in the centre of the head; involucre campanulate; bracts of the involucre unequal, 4–5-seriate, linear-attenuate, more or less purplish, the outer shorter than the inner; pappus white, about 5 mm. in length: achenes slightly puberulent: heads of the staminate plant also about 1 cm. high, many (60-70)-flowered; involucral characters similar to those of the pistillate plant. — MEXICO. Federal District: Serrania de Ajusco, altitude, 2885 m., 7 December, 1903, *C. G. Pringle*, no. 8782 (hb. Gr.).

Baccharis oaxacana, n. sp. An herbaceous perennial, often stoloniferous, freely rooting at the suffruticose base: stems erect or ascending, 2 to 5 dm. in height, sparingly branched above, terete, more or less glandular-pubescent: leaves alternate, sessile, oblong-obovate, 2 to 7.5 cm. long, 1 to 3 cm. broad, mucronate, shallowly dentate or mucronate-denticulate, narrowed below the middle to a broadly winged subpetiolate auriculate-amplexicaul base, glandular-hirtellous, paler and somewhat villous-pubescent beneath especially on the midrib and lateral nerves: inflorescence a terminal corymbose cyme; peduncles short-stipitate-glandular: heads of the pistillate plant about 1 cm. high, many (70–75)-flowered; involucre campanulate, glandular-puberulent to essentially glabrous; bracts of the involucre unequal, linear-lanceolate, shorter than the flowers, deeply tinged with purple; flowers either all pistillate or sometimes with a few (2–4) perfect (?) flowers in the centre of the head; pappus white or slightly tawny, 5 to 6 mm. long; achenes glabrous or inconspicuously puberulent: heads of the staminate plant about 8 mm. high, 60–70-flowered, the outer cycle not infrequently consisting of 5 pistillate flowers; involucral bracts sub-4-seriate, narrowly lanceolate, acute, purplish and slightly glandular-puberulent. — MEXICO. State of Oaxaca: Sierra de San Felipe, altitude 2895 m., 13 December, 1895, *C. G. Pringle*, no. 6257 (hb. Gr.), distributed as *Baccharis hieraciifolia*, Hemsl.; in the same locality, 3 August, 1894, *C. G. Pringle*, no. 5669 (hb. Gr.).

This species is most nearly related to *B. hieraciifolia*, Hemsl., to which Mr. Pringle's specimens were referred by the writer. A more thorough study in the light of additional material shows very clearly that *B. oaxacana* differs constantly from *B. hieraciifolia* in having the leaves sessile oblong-obovate and auriculate-amplexicaul, not ovate-lanceolate and acuminate at both ends. *B. oaxacana*, as well as the preceding species, *B. glandulosa*, Greenm., may be regarded as somewhat intermediate between *Conyza* and *Baccharis*, as they are not distinctly dioecious. The sexes are, however, sufficiently differentiated to warrant both being placed definitely in the genus *Baccharis*.

Desmanthodium fruticosum, n. sp. Shrub, 1 to 1.5 m. in height: branches terete, striate, reddish-brown: leaves opposite, ovate, 4 to 8 cm.

long, 2 to 4.5 cm. broad, acuminate, acute, shallowly dentate, narrowed below into a subpetiolate base, 3-nerved, dark green and essentially glabrous above, paler and crisp-pubescent on the conspicuously anastomosing veins and somewhat ferruginous-pubescent on the midrib and nerves beneath: inflorescence corymbose, and as well as the younger branches ferruginous-pubescent in decussating lines: heads in glomerules 5 to 6 mm. high and nearly or quite as broad, disposed in a terminal crowded corymb; bracts of the glomerule and of the involucre brownish, glabrous or nearly so: achenes oblanceolate, about 2 mm. long, black and glabrous. — MEXICO. State of Jalisco: Zapotlan, 9 October, 1903, *E. W. D. Holway*, no. 5137 (hb. Gr.).

CLIBADIUM PITTIERI, Greenm., forma **phrixium**, n. form. Stem pubescent with spreading hirsute-hispid hairs: foliar characters and details of the head the same as the species. — COSTA RICA. "Confluent du Puerto Viejo et du Sarapiqui," January, 1893, *P. Biolley*, no. 7399 (hb. Gr., and hb. Inst. Physico.-Geogr. Cost. Ri.); "Buissons à Tuis," altitude 650 m., December, 1897, *Ad. Tonduz*, no. 11,479 (hb. Gr., and hb. Inst. Physico.-Geogr. Cost. Ri.).

Rumfordia aragonensis, n. sp. Stem erect, striate, glabrous or slightly pubescent: leaves opposite, detoid-ovate to ovate-lanceolate, 7 to 22 cm. long, 2.5 to 11 cm. broad, acuminate, acute, mucronate-denticulate, abruptly narrowed below the middle into a winged petiole continuous about the stem, thin or membranous, 3-nerved, sparingly puberulent on both surfaces: inflorescence a terminal corymbose cyme: heads about 12 mm. high, radiate: involucre 2-seriate; outer bracts of the involucre foliaceous, ovate and acute to ovate-lanceolate and acuminate-acute, glabrous or essentially so; inner bracts of the involucre lanceolate, somewhat navicular, stipitate-glandular, more or less closely surrounding the glabrous ray-achenes: ray-flowers sub-biseriate, fertile; rays small, linear-oblong, the expanded portion about 5 mm. long, 1 mm. broad, obtuse or rarely toothed, usually 2-nerved: disk-flowers 20 to 30: achenes obovate, glabrous, one half shorter than the pales of the receptacle. — COSTA RICA. "Broussailles près d'Aragon," Turrialba, altitude 620 m., 2 January, 1899, *H. Pittier*, no. 13,246. In general aspect *R. aragonensis* resembles *R. polymnioides*, Greenm., but is readily distinguished from it by the absence of pilose hairs on the stem and in the inflorescence, the smaller fewer-flowered heads, the shorter pales of the receptacle, and the narrowed fewer-nerved barely dentate rays.

Sclerocarpus sessilifolius, n. sp. Stems ascending, 3 dm. or more long, sparingly branched, striate, hirsute-hispidulous: leaves mostly opposite, sessile or essentially so, ovate to ovate-lanceolate, 2 to 5 cm. long, 1 to 2 cm. broad, acute, remotely few-dentate, dark green and hirsute-hispid above, paler and almost soft-hirsute beneath, cuneate to subcordate at the base, 3-nerved, often reflexed: heads large, 1.5 to 2 cm. high, including the rays 4 to 5 cm. in diameter, terminating the stem on naked 8–15 cm. long peduncles: involucre biseriate; bracts of the involucre unequal, foliaceous, lance-linear to narrowly oblanceolate, 6 to 12 mm. long, hirsute-pubescent: ray-flowers about 15, conspicuous, neutral; rays lemon-yellow, 1.5 to 2 cm. long, one half as broad: disk-flowers numerous: corollas deeply 5-toothed; teeth pubescent on the upper or inner surface with long black hairs: pappus reduced to an inconspicuous crown, often bearing one or two larger squamellae: achenes glabrous. — MEXICO. State of Sinaloa: foothills of the Sierra Madre, between Rosario and Colomas, 13 July, 1897, *Dr. J. N. Rose*, no. 1629 (hb. Gr., and hb. U. S. Nat. Mus.); near Colomas, 20 July, 1897, *Dr. J. N. Rose*, no. 3224 (hb. Gr., and hb. U. S. Nat. Mus.).

Aspilia Rosei, n. sp. Suffruticose, hirsute-hispid throughout: stems as seen 1.5 to 2.5 dm. high, branched: leaves opposite, linear to lanceolate, 1 to 5 cm. long, 2 to 10 mm. broad, acute, entire or inconspicuously denticulate, strongly revolute-margined, sessile or narrowed below into a short petiole, appressed-tuberculate-hispid above, rather sparingly hirsute-pubescent and prominently nerved beneath: heads about 1 cm. high, 1.5 to 2 cm. broad including the rays, terminating the stem and branches: peduncles 0.5 to 4 cm. in length: involucre 2-seriate; outer bracts of the involucre foliaceous, lanceolate or slightly spatulate, 8 to 10 mm. long, acute or acutish, externally hirsute-pubescent, equalling or more often slightly exceeding the inner oblong obtuse strongly ciliate involucral bracts: ray-flowers 6 to 8, neutral; rays pale yellow: disk-flowers about 20: pappus of both ray- and disk-flowers a lacerated crown of united scales, often with a short awn on the inner angle of the disk-achenes. — MEXICO. Territory of Tepic: in the Sierra Madre, between Santa Gertrudis and Santa Teresa, 8 August, 1897, *Dr. J. N. Rose*, no. 2111 (hb. Gr., and hb. U. S. Nat. Mus.). State of Durango: 13 August, 1897, *Dr. J. N. Rose*, no. 2245 (hb. Gr., and hb. U. S. Nat. Mus.).

Aspilia purpurea, n. sp. An herbaceous perennial, hirsute-pubescent throughout with appressed or somewhat spreading hairs: stems several, erect, or ascending from a woody base, about 3 dm. in length, branched above: leaves opposite, sessile or nearly so, elliptilanceolate to oblong-obovate, 2 to 5 cm. long, 0.5 to 2 cm. broad, acute, dentate to subentire, green above, slightly paler beneath: heads 10 to 12 mm. high, including the rays 3 to 4 cm. in diameter, terminating the stem and branches on long mostly naked peduncles: involucre narrowly campanulate, 2–3-seriate; the outer involucral bracts herbaceous, lanceolate, acute, usually exceeding the inner; the inner bracts of the involucre oblong, ciliate, often purplish: ray-flowers commonly 6, neutral; rays, including the short tube, 12 to 18 mm. long, 5 to 9 mm. broad, dark purple: disk-flowers about 20; pappus lacerate-coroniform with an occasional short awn on the inner angle of the achene, persistent: mature achenes slightly obovate, 3 to 4 mm. long, hirsute-pubescent with upwardly subappressed hairs. — MEXICO. State of Chiapas: Valley of Jiquipilas, altitude 650 to 850 m., 16 to 18 August, 1895, *E. W. Nelson*, no. 2924 (hb. Gr., and hb. U. S. Nat. Mus.).

Aspilia xylopoda, n. sp. An herbaceous perennial: stems several from a stout ligneous base, erect, 4 dm. or more high, simple below, branched above, hirsute-hispid with spreading hairs intermixed with a hirtellous puberulence, green and striate, purplish above: leaves opposite, lanceolate to lance-oblong, 3 to 8 cm. long, 0.8 to 2.5 cm. broad, acute, entire or remotely denticulate, narrowed at the base into a short petiole, tuberculate-hispid above, hirsute-pubescent beneath: heads 10 to 12 mm. high, including the rays 1.5 to 2 cm. in diameter: involucre 2-seriate; outer bracts of the involucre herbaceous, lanceolate, 12 mm. in length, acute, pubescent on both surfaces; inner involucral bracts oblong, slightly shorter than the outer, obtusish, externally pubescent, long-ciliate, purplish: ray-flowers 8, neutral; rays purple: disk-flowers 20 to 30: pappus a short lacerated crown of united scales: immature achenes pubescent. — MEXICO. State of Durango: 13 August, 1897, *Dr. J. N. Rose*, no. 2260 (hb. Gr., and hb. U. S. Nat. Mus.); also same locality and date, *Dr. J. N. Rose*, no. 2245 in part (hb. Gr., and hb. U. S. Nat. Mus.).

The species here proposed is related to *Aspilia purpurea*, Greenman, from which, however, it is readily separated by the longer leaves, and the spreading pubescence on the stem.

Viguiera morelensis, n. sp. Stem erect, branched above, upwardly appressed hirsute-hispid, also finely puberulent, reddish-brown: leaves alternate, lanceolate, 3 to 7 cm. long, 1 to 2.5 cm. broad, acuminate, acute, dentate to subentire, revolute-margined, narrowed below into a subpetiolate base, 3-nerved, appressed-hirsute and somewhat rugose above, hirsute-pubescent beneath especially on the conspicuously reticulated veins: heads 10 to 12 mm. high, about 3 cm. in diameter, terminating the stem and branches in a paniculate cyme: involucre campanulate, 3–4-seriate; bracts of the involucre lanceolate, acute, herbaceous-tipped and more or less squarrose, externally hispid-pubescent, ciliate: ray-flowers orange-yellow, 12 to 14: disk-flowers numerous; pappus of two caducous aristae with minute intermediate scales: achenes somewhat compressed laterally, oblique, 2 to 2.5 mm. long, black, glabrous. — MEXICO. State of Morelos: near Cuernavaca, 12 November, 1902, *C. G. Pringle*, no. 11,295 (hb. Gr.).

The rather strongly compressed achenes of the species here described suggests at first a generic affinity with *Encelia*, but the whole habit of the plant is that of a *Viguiera*; and, moreover, its specific relationship seems to be with *V. Palmeri*, Gray, and *V. urticiformis*, Hemsl. From both of these species *V. morelensis* differs in having alternate leaves, and in having somewhat squarrose but not attenuated involucral bracts.

Perymenium (?) calvum, n. sp. An herbaceous perennial: stems erect from a ligneous base, slender, about 1.5 dm. high, upwardly strigillose, purplish: leaves opposite, sessile, broadly ovate to ovate-lanceolate, 1.5 to 2.5 cm. long, 0.8 to 1.6 cm. broad, mucronate-acute, shallowly dentate, appressed-hispidulous on both surfaces, slightly paler beneath: peduncles long and slender, 6 to 9 cm. in length, appressed-hispidulous: heads about 1 cm. high, radiate: involucre 2-seriate; bracts of the involucre oblong-lanceolate, 8 to 9 mm. long, somewhat acuminate, acute, strigillose: ray-flowers 5 to 7, fertile; rays yellow: disk-flowers several: achenes of both ray- and disk-flowers bald, glabrous, those of the disk laterally compressed and sub-4-angled. — MEXICO. State of Sinaloa: in the foothills of the Sierra Madre, near Colomas, 14 July, 1897, *Dr. J. N. Rose*, no. 3225 (hb. Gr., and hb. U. S. Nat. Mus.). Notwithstanding the lack of pappus the plant here described is placed for the present at least in the genus *Perymenium*. In general habit it resembles *P. tenellum*, Gray.

Helianthella iostephanoides, n. sp. An herbaceous perennial: stems 5 to 7 dm. high, erect, simple below, branched above, striate, reddish-brown, hirsute-pubescent with spreading or subappressed usually jointed hairs: leaves chiefly basal, oblong to lance-oblong, 1 to 1.5 cm. long, 2 to 4 cm. broad, obtuse or acute, remotely denticulate, slightly sinuate, narrowed below into a winged petiole, sparingly hirsute-pubescent on both surfaces, subtrinerviate from near the base; stem-leaves few, remote and smaller: heads few, 1.5 cm. high, including the rays 3.5 to 4 cm. in diameter, disposed on long bractless pubescent peduncles 8 to 18 cm. in length: involucral bracts 2-seriate, lanceolate, 12 to 15 mm. long, 3 to 5 mm. broad, acute, densely hirsute-pubescent: ray-flowers about 13; rays yellow, 1.5 to 2 cm. long, 7 to 10 mm. broad, 3-dentate: disk-flowers numerous: pappus of two aristae with intermediate lacerated and more or less united squamellae: achenes laterally compressed, slightly pubescent, ciliate along the inner margin. — MEXICO. State of Zacatecas: in the Sierra Madre, 18 August, 1897, *Dr. J. N. Rose*, no. 2391 (hb. Gr., and hb. U. S. Nat. Mus.).

Zexmenia Rosei, n. sp. An herbaceous perennial: roots tuberiform: stem erect, 3 to 4 dm. high from a ligneous base, sparingly branched below, simple above, densely pubescent with spreading villous hairs with which is intermixed a fine puberulence: leaves confined chiefly to the lower part of the stem, opposite, short-petiolate, oblong-lanceolate to somewhat oblanceolate, 6 to 18 cm. long, 2 to 5.5 cm. broad, acute or obtuse, rather sparingly dentate, dark green and tuberculate-hispid above, paler and prominently reticulate-nerved and densely villous-pubescent beneath especially on the anastomosing veins: inflorescence a few-headed close terminal cyme, subtended by 2 to 4 well developed leaves or foliaceous bracts: heads about 12 mm. high, radiate: involucre campanulate; bracts of the involucre 2-3-seriate, the outer foliaceous, oblong or oblanceolate, obtuse, villous-pubescent, the inner thinner, stramineous below, purplish and ciliate towards the apex: ray-flowers fertile; rays including the tubular portion 12 to 15 mm. long, 4 to 6 mm. broad, deep purple: disk-flowers numerous: pappus of both ray- and disk-flowers much-reduced, consisting of a crown of minute scales: achenes of both ray- and disk-flowers narrowly winged, glabrous. — MEXICO. Territory of Tepic: between Pedro Paulo and San Blacisto, 4 August, 1897, *Dr. J. N. Rose*, no. 1987 (hb. Gr., and hb. U. S. Nat. Mus.).

OTOPAPPUS EPALEACEUS, Hemsl., var. (?) **Pringlei**, n. var. Habit and foliar characters of the species: stem above and the ultimate branches pubescent with spreading or reflexed subvillous hairs: heads exclusive of the rays 12 to 16 mm. in diameter; inner involucral bracts lanceolate, acutish; rays short, pale-yellow; pappus of 2 unequal awns with intermediate scales which are 0.5 to 0.75 mm. in length: mature achenes 4 mm. long, glabrous, longitudinally 3-ribbed on both surfaces. — MEXICO. State of Morelos: near Cuernavaca, altitude 1585 m., 12 September, 1896, *C. G. Pringle*, no. 6521 (hb. Gr.), distributed as *Otopappus acuminatus*, Watson. State of Guerrero: Iguala, 3 November, 1902, *E. W. D. Holway*, no. 5313 (hb. Gr.).

Verbesina (Platypteris) Tonduzii, n. sp. Stem brownish, striate, glabrous or sparingly pubescent, narrowly winged: leaves opposite, simple, ovate, 5 to 14 cm. long, 2.5 to 6 cm. broad, acuminate, sharply and unequally sinuate-dentate, abruptly contracted below the middle and then gradually narrowed to the insertion of the petiole, hispid-pubescent on both surfaces or merely hispidulous beneath: heads relatively few, on 3 to 8 cm. long peduncles, discoid, 12 to 15 mm. high, 1.5 to 2.5 cm. in diameter, many-flowered: involucre campanulate; bracts of the involucre 3–4-seriate, lanceolate, obtuse or obtusish, rather thick, densely puberulent: corollas in the dried state somewhat reddish: mature achenes broadly winged, 5 to 7 mm. long, nearly or quite as broad, glabrous or somewhat verrucose. — *V. Fraseri*, Klatt, Bull. Soc. Bot. Belg. xxxi. pt. 1, 205 (reprint 23) in part, not Hemsl. *V. crocata*, Rob. & Greenm. Proc. Am. Acad. xxxiv. 537 in part, not Less. — COSTA RICA. "Dans les buissons à las Vueltas, Tucurrique," altitude 635 m., November, 1898. *Tonduz*, no. 12,765 (hb. Gr.); "haie à Turrialba," altitude 200 m., 6 May, 1891, *Pittier*, no. 4136 (hb. Gr.): "broussailles à Buenos Aires," January, 1892, *Pittier*, no. 4905 (hb. Gr.). Most nearly related to *V. crocata*, Less. with which species it has been confused, but from which it is readily separated by the fewer seriated involucre, shorter and more obtuse bracts of the involucre, simple and undivided leaves, and narrowly winged petiole gradually narrowed to the base.

GARCILLASSA RIVULARIS, Poepp. Nov. Gen. & Sp. iii. 46, t. 251. With this species the writer identifies the following: COSTA RICA. "Bois de la vallée du Rio Tuis," altitude 600 m., November, 1893, *Ad. Tonduz*, no. 8096 (hb. Gr., and hb. Inst. Physico.-Geogr. Cost. Ri.); "broussailles à Tuis," altitude 650 m., November, 1897, *Ad. Tonduz*, no. 11,482 (hb. Gr., and hb. Inst. Physico.-Geogr. Cost. Ri.). Mr. Tonduz's specimens agree in every detail with the original description and illustration above cited. The genus is, as far as known, monotypic, and does not seem to have been hitherto recorded as occurring north of the Isthmus of Panama.

Coreopsis cuneifolia, n. sp. Suffruticose: stem covered with a grayish bark, di- or trichotomously branched; branchlets at first upwardly subappressed-pubescent, later glabrate except at the nodes: leaves opposite, simple at least as far as seen, cuneate or oblanceolate-cuneate, 1.5 to 3.5 cm. long, 6 to 12 mm. broad, acute, dentate in the upper half with usually 7 to 9 spreading mucronate teeth, cuneate and entire below, narrowed into a subpetiolate base, glabrous on both surfaces or sparingly pubescent beneath, glandular-punctate: heads pedunculate, about 1 cm. high, 1.5 to 2 cm. broad, terminating the stem and branches and either solitary or in few-headed paniculate-cymes: outer involucral bracts few, herbaceous, slightly spreading; inner bracts of the involucre oblong, 6 mm. long, 2 to 2.5 mm. broad, obtuse: ray-flowers few, five or six, fertile; rays with a short sparingly pubescent tube and a broad-oblong expanded portion, 6 mm. long and nearly as broad: disk-flowers about 12; achenes, as well as those of the ray-flowers, oblong, strongly obcompressed, destitute of pappus, glabrous. — MEXICO. State of Durango: 16 August, 1897, *Dr. J. N. Rose*, no. 2344 (hb. Gr., and hb. U. S. Nat. Mus.). State of Jalisco: on dry rocky mountains near Etzatlan, 2 October, 1903, *C. G. Pringle*, no. 8781 (hb. Gr.).

In habit *Coreopsis cuneifolia* suggests *C. petrophiloides*, Rob. & Greenm., from which it is amply distinct in the character of the foliage, the pubescent branches, and in the glabrous epappose achenes. In the technical characters of the head *C. cuneifolia* is more nearly related to *C. mexicana*, Hemsl. (*Electra mexicana*, DC.), but is readily separated by the cuneate or oblanceolate-cuneate leaves, by the pubescent branchlets, etc.

LEPTOSYNE ARIZONICA, Gray, var. **filiformis**, n. var. Glabrous throughout: leaves 3–5-parted; segments filiform, 8 cm. or less in length. — MEXICO. State of Sinaloa: Sierra de Choix, 80 km. N. E. of Choix, 15 October, 1898, *E. A. Goldman*, no. 258 (hb. Gr., and hb. U. S. Nat. Mus.). Aside from the filiform attenuated leaf-segments no important characters are observed separating the variety from typical representatives of the species.

LEPTOSYNE PINNATA, Rob., var. **integrifolia**, n. var. Leaves simple, undivided, entire or crenulate: pappus evident, coroniform, often of several short unequal more or less lacerated scales. — MEXICO. State of Durango: near el Salto, 12 July, 1898, *E. W. Nelson*, no. 4580. Mr. Nelson's plant at first glance seems very different from the type-specimen of *Leptosyne pinnata*; but aside from the characters enumerated, namely, the entirety of the leaves and the fairly well developed pappus, there are no further differences.

BIDENS LUDENS, Gray, Proc. Am. Acad. xxi. 390. This species was founded on Pringle's no. 293 collected on hills and plains near the City of Chihuahua, Mexico, in October of 1885. The printed label accompanying this plant in the Gray Herbarium bears the name "*Leptosyne Arizonicus*, Gray." The unfortunate confusion of names probably resulted either in reporting the collection or in printing the labels. The plant, however, is unquestionably a *Bidens*, and the correction of the label may now be definitely made.

Perityle Rosei, n. sp. Stems erect, 1 to 2 dm. high from a ligneous base, more or less striate, puberulent: leaves opposite, or the uppermost alternate, petiolate, ovate-triangular, frequently somewhat halberd-shaped, 1 to 3 cm. long, 0.5 to 2 cm. broad, acute or rounded at the apex, entire or subentire, subtruncate at the base and decurrent on the petiole, puberulent on both surfaces: heads about 6 mm. high, on peduncles 3 cm. or less in length, many-flowered, radiate: involucre of about 24 lanceolate acute bracts, and as well as the peduncles pubescent: ray-flowers about 13; rays white: disk-flowers 50 to 60: pappus of both both ray- and disk-flowers of two slender subequal setae, which are nearly or quite as long as the achene, and of intermediate more or less united erose scales: mature achenes about 2.5 mm. long, subcartilaginous-margined and ciliated, glabrous on both surfaces. — MEXICO. State of Jalisco: in the Sierre Madre, west of Bolaños, 15–17 September, 1897, *Dr. J. N. Rose*, no. 2947 (hb. Gr., and hb. U. S. Nat. Mus.).

GALEANA HASTATA, La Llave. Although this species was well described by La Llave in 1824, nevertheless by subsequent authors it seems to have been little understood, and somewhat traditionally treated as an herbaceous monotype of doubtful affinity. Kunth in 1820 referred specimens secured by Humboldt and Bonplaud at Valladolid, Mexico, to the Linnaean genus *Unxia*. Lessing in 1832, recognizing the discrepancy between the Mexican annual and typical *Unxia*, apparently unaware of the already published *Galeana*, founded a new genus, *Chlamysperma*, on the Humboldt and Bonpland plant, and established the binomial *Chlamysperma pratense*. Later, in 1873, Bentham and Hooker referred the *Chlamysperma pratense*, Less. to *Villanova*, a genus with pinnatisect leaves and different technical characters of the head; and at the same time retained *Galeana* of La Llave as a distinct genus. Hemsley in the Biologia Centrali-Americana, and Hoffmann in Engler and Prantl's Natürlichen Pflanzenfamilien follow the course of Bentham and Hooker. There are in the Gray Herbarium several specimens, which were identified by Dr. Gray with *Galeana hastata*, and which too correspond in every detail with the careful characterization of La Llave. These specimens, together with a part of the original plant on which Lessing based his new genus *Chlamysperma*, as well as a considerable suite of recently collected material, have been examined by the writer, and there can be no doubt but that they all represent one and the same species, as in every essential morphological character there is exact correspondence. The synonomy and citation of specimens are as follows:

GALEANA HASTATA, La Llave in La Llav. & Lex. Nov. Veg. Descr. i. 12 (1824); DC. Prodr. vii. pt. 1, 257; Benth. & Hook. Gen. Pl. ii. 406; Hemsl. Biol. Cent.-Am. Bot. ii. 215. *Unxia pratensis*, HBK. Nov. Gen. & Sp. iv. 279, t. 401 (1820). *Chlamysperma pratense*, Less. Syn. Comp. 256 (1832). *C. arenarioides*, Hook. & Arn. Bot. Beech. Voy. 300, t. 64 (1840). *Villanova pratensis*, Benth. & Hook. l. c. 405 (1873); Hemsl. l. c. 213; Hoffm. in Engler & Prantl, Nat. Pflanzenf. iv. Ab. 5, 259. — MEXICO. San José del Coral, *La Llave*. Capula and Valladolid, *Humboldt & Bonpland*. State of Vera Cruz: Cordillera of Vera Cruz, altitude 915 m., *Galeotti*, no. 2198 (hb. Gr.). State of Jalisco: without locality, *Beechey*; near Guadalajara, July, 1886, *Palmer*, no. 263 (hb. Gr.); barranca near Guadalajara, 1 November, 1888, *Pringle*, no. 2177 (hb. Gr.); hills near Guadalajara, 8 October, 1889, *Pringle*, no. 2330 (hb. Gr.); between Colotlan and Bolaños, 7 to 9 September, 1897, *Dr. J. N. Rose*, no. 3673 (hb. Gr., and hb. U. S. Nat. Mus.). State of Colima: Volcan de Colima, altitude 1220 m., 1840, *Galeotti*, no. 2449 (hb. Gr.); Manzanillo, 1863, *Xantus*, no. 330 (hb. Gr.). State of Morelos: Cuernavaca, 3 January, 1900, *Chas. C. Deam*, no. 8 (hb. Gr.); Cuernavaca, 6 July, 1900, *Chas. C. Deam*, no. 55 (hb. Gr.). State of Chiapas: Valley of Jiquipilas, altitude 600 to 850 m., 16 to 18 August, 1895, *E. W. Nelson*, no. 2931 (hb. Gr., and hb. U. S. Nat. Mus.). GUATEMALA. Department of Santa Rosa: Santa Rosa, altitude 915 m., July, 1892, *Heyde & Lux*, no. 3364 (hb. Gr.), exsiccatae of John Donnell Smith.

HYMENOTHRIX GLANDULOSA, Watson, var. **Nelsonii**, n. var. With habit and general characters of the species but much less glandular-pubescent: bracts of the involucre often purplish-tinged on the margins: pappus usually of 12 narrowly oblong-obovate or slightly spatulate scales about 1.5 mm. in length, fully twice longer than in the species, obtuse; midnerve not excurrent. — MEXICO. State of Chihuahua: in the Sierra Madre near Colonia Garcia, 1–20 August, 1899, *E. W. Nelson*, no. 6210 (hb. Gr., and hb. U. S. Nat. Mus.).

BAHIA GLANDULOSA, Greenm., Proc. Am. Acad. xxxix. 116 (1903). To this species is to be referred specimens secured by Dr. J. N. Rose near Plateado, State of Zacatecas, Mexico, 3 September, 1897, no. 2772 (hb. Gr., and hb. U. S. Nat. Mus.). This collection extends the geographical range of the species considerably southward and eastward from the hitherto recorded localities.

Tagetes Hartwegii, n. sp. An erect herbaceous perennial, glabrous throughout: stem 8 to 9 dm. high from a ligneous base, striate, purplish: leaves bipinnately parted, 2 to 8 cm. long, 1.5 to 6.5 cm. broad; segments linear-lanceolate, acute, subequally serrate-dentate, often setigerous, bearing a single row of glands on either half of the blade at the base of the teeth: heads about 1.5 cm. high, including the rays 2.5 to 3.5 cm. in diameter, terminating the stem and branches on long slender 0.5 to 1 dm. long peduncles: involucre subcampanulate, 10 to 12 mm. high, 6 to 8 mm. in diameter, 8-dentate; teeth obtuse, slightly pubescent: ray-flowers 8; pappus of 1 to 5 narrow unequal paleaceous awns not exceeding the involucre; tube about 5 mm. in length, pubescent; rays subobovate to oblong-cuneate, 1.2 to 1.5 cm. long, 7 to 11 mm. broad, emarginate, deep orange-yellow: disk-flowers 35 to 45; pappus much-reduced or quite obsolete; corollas deeply 5-toothed with the tube pubescent below, glabrous above, and the teeth pubescent on the inside especially along the margins: achenes 6 mm. long, glabrous. — *T. peduncularis*, Benth., Pl. Hartw. 17 (1839), not Lag. — MEXICO. State of Jalisco: in pine woods, Bolaños, 1837, *Hartweg*, no. 118 (hb. Gr.); Sierra Madre, west of Bolaños, 16 September, 1897, *Dr. J. N. Rose*, no. 3722 (hb. Gr., and hb. U. S. Nat. Mus.). Hartweg's number above cited was referred doubtfully by Mr. Bentham to *Tagetes peduncularis*, Lag.; but the excellent specimens secured by Dr. Rose, which are identical with the Hartweg plant in the Gray Herbarium, show very clearly that the two species have but a superficial resemblance. The single row of submarginal leaf-glands, the 8-dentate involucre, the numerous flowers, and finally the usually epappiferous character of the disk-flowers render *T. Hartwegii* an easily recognizable species, readily distinguished from *T. peduncularis*, Lag., with which it has been confused.

Tagetes jaliscensis, n. sp. An erect glabrous annual 3 to 4.5 dm. in height: stem simple and terete below, branched and striate or angulate-striate above, green or somewhat purplish: leaves opposite below, alternate above, 3 to 10 cm. long, 1.5 to 4 cm. broad, deeply pinnatisect with 9 to 15 narrowly lanceolate acute subincised-dentate segments and with 4 to 8 smaller setiferous segments at the base; the punctate glands more or less scattered in the leaf-lamina: heads corymbosely disposed on 2 to 4 cm. long upwardly thickened peduncles: involucre slightly fusiform, 1.5 to 1.7 cm. high: ray-flowers 5; tube 8 to 9 mm. long, inconspicuously puberulent; rays cuneate-oblong, 5 to 6 mm. long, and nearly or quite as broad, emarginate, deep orange: disk-flowers about 12; corollas tubular, rather deeply 5-lobed; lobes pubescent and, as well as the upper portion of the tube, colored like the rays: pappus of both ray- and disk-flowers of 2 anterior lance-attenuate paleaceous aristae 8 to 10 mm. in length, and of 3 posterior more or less coalescent pales less than one-half the length of the outer or anterior ones, at maturity grayish purple: mature achenes 8 to 9 mm. long, puberulent. — MEXICO. State of Jalisco: on dry rocky mountains above Etzatlan, altitude 610 to 915 m., 2 October, 1903, *C. G. Pringle*, no. 8768 (hb. Gr.).

HYMENATHERUM AUREUM, Gray, Proc. Am. Acad. xix. 42 (1883); Syn. Fl. i. pt. 2, 359. The following specimens are identified with this species: MEXICO. State of Chihuahua: Colonia Diaz, 20–21 September, 1899, *E. W. Nelson*, no. 6446 (hb. Gr., and hb. U. S. Nat. Mus.). Although *H. aureum* is well known from Texas to Colorado, it seems not to have been hitherto recorded from Mexico.

Porophyllum Holwayanum, n. sp. Shrub 1 to 1.5 m. high, glabrous throughout: stem covered with a brownish cortex: leaves alternate, petiolate, elliptic-oblong, 1 5 to 5 cm. long, 7 to 22 mm. broad, rounded and submucronate at the apex, crenate-margined, narrowed at the base into a slender 5 to 12 mm. long petiole; glands chiefly marginal, from 5 to 12 on each half of the blade, and a single gland terminating the midnerve: heads numerous, about 2 cm. long, nodding on slender 1 to 1.5 cm. long peduncles, disposed in a terminal leafy panicle: involucre cylindrical, 16 to 17 mm. long, green or slightly purplish; bracts of the involucre 5, narrowly oblong, obtuse: flowers usually 14, exceeding the .involucre; corolla with a slender tube about equalling the pappus, and a narrowly campanulate 5-toothed limb, yellowish-green or not infrequently slightly purplish-tinged at the base of the limb: anthers purplish: achenes puberulent, about 1 cm. in length. — MEXICO. State of Jalisco: Sayula, 8 October, 1903, *E. W. D. Holway*, no. 5130 (hb. Gr.); near Chapala, 5 October, 1903, *Rose & Painter*, no. 7652 (hb. Gray, and hb. U. S. Nat. Mus.). A species nearly related to *P. nutans*, Rob. & Greenm., but differing in the uniformly larger leaves, longer involucre and more numerous marginal leaf-glands.

Pectis (§ **Pectothrix**) **Barberi**, n. sp. Annual, scapose, simple or branched from the base: leaves rosulate, oblanceolate, 1 to 2 cm. long, 2 to 5 mm. broad, obtuse or subsetiferous-mucronate, setiferous-ciliate at the narrowed base, otherwise entire, glabrous on both surfaces, paler beneath: glands of the leaves marginal: scape 0.5 to 1.5 dm. high, slender, minutely bracteate, bearing a single terminal head: heads 8 to 10 mm. high, including the rays 1 to 1.5 cm. in diameter: involucre narrowly campanulate, 5 to 6 mm. high, glabrous and purplish; bracts of the involucre usually 13, lanceolate or narrowly oblong, obtuse, scarious-margined, obtusely keeled toward the base: ray-flowers about 13; pappus of 2 slender setae; tube 1.5 mm long, puberulent; rays oblong, 5 to 6 mm. long, yellow; style-branches greenish: disk-flowers 40 to 45; pappus of 5 slender subequal setae and numerous shorter unequal setae, often purplish: mature achenes 3 mm. long, inconspicuously puberulent. — MEXICO. State of Chihuahua: in the Sierra Madre near Colonia Garcia, altitude 2285 m., 19 August, 1899, *Townsend & Barber*, no. 269 (hb. Gr.), distributed "*Pectis longipes*, Gray"; between Colonia Garcia and Pratt's Ranch, below Pacheco, 22 to 24 August, 1899, *E. W. Nelson*, no. 6269 (hb. Gr., and hb. U. S. Nat. Mus.).

P. longipes, Gray, to which the Townsend & Barber plant was referred is distinctly a perennial; its stems are ascending or erect, usually branched and more or less leafy; the leaves, moreover, are linear or linear-lanceolate. *P. Barberi* on the other hand is clearly an annual, with scapoid habit and rosulate oblanceolate leaves; it is one of the most clear cut species of the genus.

Pectis (§ Eupectis) propetes, n. sp. Low annual, usually prostrate and much-branched: stems puberulent in lines, purplish: leaves narrowly oblong to oblanceolate, 1 to 3.5 cm. long, 2 to 5 mm. broad, obtuse or mucronate-acute, entire, setiferous-ciliate toward the base, inconspicuously hirtellous-margined, pale green and glabrous on both surfaces: glands of the leaves scattered: heads 8 to 10 mm. high, on puberulent peduncles 4.5 cm. or less in length: involucre subcylindrical, 6 to 7.5 mm. high, glabrous; bracts of the involucre 5, oblong, obtuse or truncate at the apex, scarious-margined, obtusely keeled toward the base, green or slightly purplish: ray-flowers 5, conspicuous; pappus commonly of 2 lanceolate pales; rays including the tube 8 to 9 mm. long, yellow, in the early stages slightly tinged with purple on the under or outer surface: disk-flowers 20 to 25; pappus of 4 or 5 lance-acuminate pales about one-half as long as the subbilabiate corollas: achenes about 4 mm. long, pubescent. — MEXICO. State of Zacatecas: near San Juan Capistrano, 19 August, 1897, *Dr. J. N. Rose*, no. 2436 (hb. Gr., and hb. U. S. Nat. Mus.). In general aspect *Pectis propetes* resembles *P. prostrata*, but it is readily distinguished from it by the elongated peduncles and the conspicuous ray-flowers.

Pectis (§ Pectothrix) puberula, n. sp. A slender dichotomously branched annual, 1 to 2 dm. in height: stems puberulent, often purplish: leaves linear or linear-lanceolate, 1 to 3 cm. long, 4 mm. or less broad, acute, bearing 1 to 4 pairs of setae near the base, otherwise entire, glabrous on both surfaces, slightly hirtellous on the margins and on the midrib beneath, provided with a row of submarginal glands on either half of the blade, more or less revolute-margined: peduncles slender, filiform, 4 cm. or less in length: heads small, about 6 mm. high: involucre 4 mm. high, glabrous; bracts of the involucre 8, linear, blunt, obtusely keeled: ray-flowers 8; pappus of 2 slender setae and a small intermediate anterior scale; rays including the tubular portion 4 mm. long: disk-flowers about 15; pappus of numerous slender unequal setae: mature achenes of both ray- and disk-flowers about 2.5 mm. in length, puberulent. — MEXICO. State of Sinaloa: Lodiego, 9 to 15 October, 1891, *Dr. Edward Palmer*, no. 1605 (hb. Gr., and hb. U. S. Nat. Mus.). In a natural sequence this species is to be placed near *P. papposa*, Gray, and *P. Palmeri*, Watson. From the former it is readily and easily separated by having the heads disposed on long slender peduncles instead of subfastigiate; from the latter it differs in the smaller fewer-flowered heads, shorter involucre and more slender peduncles. *P. puberula* differs from both *P. papposa* and *P. Palmeri*, in having the stem and branches puberulent.

PECTIS SINALOENSIS, Fernald, var. **lancifolia**, n. var. Leaves sessile, lanceolate, 2.5 cm. or less long, 6 mm. or less broad, usually hirtellous-puberulent on the upper surface and on the midrib beneath, bearing 1 to 6 pairs of setae toward the base: peduncles 11 cm. or less in length: rays golden-yellow or before expansion slightly copper-colored externally, somewhat larger than in the type of the species. — MEXICO. Territory of Tepic: between Concepcion and Acaponeta, 29 July, 1897, *Dr. J. N. Rose*, no. 1893 (hb. Gr., and hb. U. S. Nat. Mus.).

Artemisia Pringlei, n. sp. Annual, or biennial (?), with a single erect stem, or sometimes several-stemmed from a common base, 4 to 8 dm. high, more or less white-tomentose throughout: lower leaves bi-tripinnately parted, 2 to 6 cm. long, 3 cm. or less broad, white-tomentose on both surfaces, later as well as the stem below somewhat glabrate; segments linear, obtusish, revolute-margined; upper leaves similar, becoming less parted and finally reduced to simple lance-attenuate entire bracts in the inflorescence: the terminal racemiform panicles 0.5 to 2.5 dm. in length, 1 to 2.5 cm. in diameter: heads in anthesis 3 to 4 mm. high, equally broad, nodding: involucre campanulate, subtriseriate, externally white-tomentose; bracts of the involucre oblong to oblong-ovate, obtuse or rounded at the apex, scarious-margined: flowers 30 to 35, about one-half of which, namely the outer ones, are pistillate, the others hermaphrodite: corollas short-stipitate-glandular: achenes epappose. — MEXICO. State of Chihuahua: plains near the City of Chihuahua, 11 September, 1885, *C. G. Pringle*, no. 625 (hb. Gr.), distributed as "*Artemisia Ludoviciana*, Nutt., *var.?*"; near Lake Santa Maria, 7 September, 1899, *E. W. Nelson*, no. 6387 (hb. Gr., and hb. U. S. Nat. Mus.). This species is amply distinct from *A. Ludoviciana* both in the character of the foliage and the inflorescence.

CACALIA PELTATA, HBK., var. **Coulteri**, n. var. Lower leaves peltate, 7–10-lobed with rounded or subcircular sinuses: inflorescence 8 to 15 cm. in diameter, densely pubescent with jointed villous hairs: heads 12–16-flowered. — MEXICO. State of Vera Cruz: Real del Monte, *Th. Coulter*, no. 420 (hb. Gr.); Mt. Orizaba, August, 1891, *Henry E. Seaton*, without number (hb. Gr.). State of Mexico: Nevado de Toluca, 15 October, 1903, *Rose & Painter*, no. 7939 (hb. Gr., and hb. U. S. Nat. Mus.).

CIRSIUM OCHROCENTRUM, Gray, var. **durangense**, n. var. Leaves pinnatisect; divisions lance-linear, spine-tipped, at first arachnoid-tomentulose above but soon glabrate revealing the dark green surface and yellowish midvein, densely and permanently white-tomentose beneath. — MEXICO. State of Durango: about the City of Durango, 1 August, 1898, *E. W. Nelson*, no. 4633 (hb. Gr., and hb. U. S. Nat. Mus.).

Onoseris silvatica, n. sp. An herbaceous perennial about 1 m. high: leaves alternate, chiefly basal, runcinate-pinnatifid, short-acuminate, acute, unequally sinuate-dentate, arachnoid-tomentulose above, densely white-tomentose beneath; terminal segment large, triangular-sagittate, 1 to 3 dm. long, nearly or quite as broad, palmately nerved, cordate; lateral segments smaller, divaricate or somewhat reflexed, often reduced to a more or less continuous wing on the rhachis or petiole: inflorescence a naked loose paniculate cyme: peduncle or scape striate-angled, floccose-tomentose: heads 2 to 2.5 cm. high, 40–45-flowered: involucre subcampanulate, 2 cm. high, tomentulose; bracts of the involucre 4-seriate, the outer shorter: outer flowers of the head bilabiate, producing well-developed dark-red or purplish rays: inner or disk-flowers deeply and subequally 5-toothed: achenes appressed - sericeous - pubescent. — *O. paniculata*, Klatt, Bull. Soc. Bot. Belg. xxxi. pt. 1, 214, not [Moc. & Sessé] DC. — COSTA RICA. "Forêts des collines de Nicoya," January, 1900, *Ad. Tonduz*, no. 13,597 (hb. Gr.); "forêt de l'Alto del Rodeo," altitude 1100 m., 28 December, 1889, *Pittier*, no. 1622 (hb. Gr.); "tailles du Rodeo del Pacaca," altitude 900 m., 1 2 January, 1891, *Pittier*, no. 3312 (hb. Gr.). From *O. Isotypus*, Benth. & Hook. f. (*O. paniculata*, [Moc. & Sessé] DC.) this species is readily distinguished by having larger more numerously flowered heads, longer involucral bracts, longer pappus and appressed-pubescent achenes.

Crepis Barberi, n. sp. Perennial: stems scapoid, 5 to 7 dm. high, erect, simple below, branched above, glabrous or essentially so: leaves chiefly rosulate, narrowly oblanceolate, 6 to 15 cm. long, 1 to 2 cm. broad, subruncinate-pinnatifid, bearing lateral ascending to reflexed narrow elongated teeth, obtuse or obtusish, glabrous on both surfaces: heads 15 to 18 mm. high, including the rays about 3 cm. in diameter: involucre at the base, as well as the peduncles, more or less tomentulose; bracts of the involucre lanceolate, obtuse, blackish or dark green, scarious-margined: flowers numerous; rays yellow: achenes columnar, about 10-ribbed, glabrous. — MEXICO. State of Chihuahua: in the Sierra Madre, near Colonia Garcia, altitude 2285 m., *Townsend & Barber*, no. 206 (hb. Gr.); Sierra Madre, *E. W. Nelson*, no. 6107 (hb. Gr., and hb. U. S. Nat. Mus.). This species is most nearly allied to *C. runcinata*, Torr. & Gray.

VI. SOME NEW SPECIES OF MEXICAN AND NICARAGUAN DICOTYLEDONS.

By M. L. FERNALD.

Chlorophora mollis. Tree; the branches covered with greenish gray rough bark bearing many pale lenticels; young branchlets cinereous with soft fine pubescence: leaves ovate, subcordate at base, caudate-acuminate at tip, 3 to 7.5 cm. long, 2 to 4.5 cm. broad, finely crenate-serrate near the middle, and pilose on both surfaces, dull green above, cinereous beneath, the nerves strongly oblique (making an angle of 30° with the midrib); the slender cinereous petiole 6 to 8 mm. long: pistillate heads canescent-puberulent, 4.5 to 6 mm. in diameter, on slender canescent peduncles 2 or 3 mm. long. — OAXACA, Tomellin Cañon, June 23, 1899 (*Rose & Hough*, no. 4672). Differing markedly from the well known *C. tinctoria* in its soft cinereous pubescence, the very oblique nerves of the leaves and the smaller heads of pistillate flowers. Staminate aments not seen.

Cordia (Laxiflorae) chiapensis. Shrub with brown bark, pubescent with appressed tuberculate stiff hairs: leaves elliptic-lanceolate to ovate-lanceolate, 3 to 8 cm. long, 1 to 3.3 cm. broad, subentire or crenulate, or coarsely crenate-serrate toward the usually blunt-acuminate apex, entire below and cuneate to a short (2 to 4 mm. long) petiole, scabrous above, more or less pilose-hispid beneath on the nerves: cymes subaxillary or terminal: peduncles 2 to 5 cm. long, pilose-hispid with mostly appressed hairs; primary branches mostly 3, more or less forked: calyx hemispherical, appressed-pubescent, becoming 4 mm. high, cleft half-way to base into deltoid awn-tipped lobes: corolla 6 or 7 mm. long, about equalled by the stamens: fruit ovate-oblong, 6 mm. long. — CHIAPAS, table-land about Ocuilapa, alt. 1045 to 1170 m., Aug. 21, 1895 (*E. W. Nelson*, no. 3003).

Salvia (Micranthae) tehuacana. Simple or branching annual 3 to 5 dm. high; the stems and petioles setulose with crisp spreading white hairs; the middle internodes 1 to 1.5 dm. long, the lower and upper much shorter: leaves broad-ovate, blunt, rounded at base, the primary ones 3 to 5 cm. long, coarsely short-dentate, sparingly pilose-setulose or glabrate above, glabrous beneath or with occasional hairs on the midrib; lower petioles equalling or exceeding the blades, the upper much shorter: racemes stiff, becoming 1 or 2 dm. long; rachis densely glandular-puberulent; the verticels 2–6-flowered, all becoming remote, the lowest finally 4 cm. apart; bracts caducous, ovate, 1 cm. long, strongly ciliate, the scarious body hardly equalling the subulate herbaceous awn: calyx campanulate, setulose, sparingly glandular, in anthesis 5 or 6 mm., in fruit 8 mm. long; the lobes ovate, blunt: corolla 1 cm. long, the slender white tube exserted; the smooth blue lip much exceeding the galea. — PUEBLA, thin soil of calcareous hills near Tehuacan, Aug. 1901 (*C. G. Pringle*, no. 8587). Nearest related apparently to *S. permixta*, Briquet (no. 15 of the writer's synopsis). Differing from that in its setulose stem, long awn-tipped bracts, and blunt calyx lobes. Superficially resembling *S. podadena*, Briquet (*S. ageratifolia*, Fernald, Proc. Am. Acad. xxxvi. 499), but quickly distinguished by the glandless hairs of the stem, the rounded not cordate more shallowly toothed leaves, the glandular-puberulent rachis and the blunt calyx-lobes.

Salvia (Vulgares) molina. Herbaceous, branching, nearly 1 m. high, the stem closely fine-puberulent, especially on the deeply sulcate decussating bands: leaves firm, 0.5 to 1 dm. long, rhombic-ovate, attenuate-acuminate, cuneate at base to a slender petiole 1 to 2 cm. long, coarsely but bluntly serrate, green above and minutely pubescent with short appressed hairs, cinereous beneath and closely fine-puberulent: primary racemes becoming 4 dm. long, the secondary ones much shorter; verticels 10–40-flowered, becoming subequally remote: bracts ovate-oblong, acuminate, pulverulent, caducous: pedicels 3 to 5 mm. long, puberulent: calyx tubular-campanulate, puberulent, with flaring lips and rounded sinuses, in anthesis 6 or 7 mm. long, the lower lobes subulate-acuminate, the upper lobe acuminate: corolla 11 or 12 mm. long, the pale tube exserted, the blue lip hardly equalling the pilose galea: style pilose. — MEXICO, by streams, Zamora, alt. 1540 m., May 23, 1901, and July 25, 1902 (*C. G. Pringle*, nos. 8504, 11,124). Related to *S. jaliscana*, Briquet, and *S. fluviatilis*, Fernald (nos. 75 & 79 of the writer's Synopsis). From the former distinguished by its firmer more finely toothed leaves, its finer puberulence, its much more numerous flowers in more remote verticels, and the rounder, not acute, sinus of the calyx. From the latter species it is distinguished by the closely puberulent lower surface of the more prominently toothed and attenuate-acuminate leaves and its more numerous flowers in more remote verticels.

Salvia (Candicantes) fasciculata. Shrub with pale brown bark; the branches erect and rather rigid, the young parts granular-pruinose: leaves narrowly linear, strongly revolute, canescent beneath, glabrous and dark green above, 1 to 2.5 cm. long, approximate on the short branches and often with close axillary fascicles of shorter leaves: racemes terminating the branches and short branchlets, 1 to 3 cm. long; rachis puberulent, enlarged below the subremote (3 to 5 mm. apart) 2-flowered verticels: bracts persistent, ovate, 0.5 to 2 mm. long; pedicels 3 mm. long: calyx tubular-campanulate, in anthesis 4 mm. long, minutely pubescent with white stellate hairs, in fruit larger and glabrate; the tube thrice exceeding the ovate teeth; upper lip tridentate: corolla 8 mm. long, purplish, pilose, the galea about equalling the lip: stamens and style exserted, glabrous. — OAXACA, mountains above Oaxaca, alt. 2250 m., December, 1900 (*C. Conzatti & V. González*, no. 1151). Closely related to *S. Coulteri*, Fernald (no. 91 of the writer's Synopsis), from which it differs among other characters in its narrow leaves dark green and glabrous above, puberulent rachis, few flowers and glabrate calyx.

Salvia (Scorodoniae) pannosa. Branches very slender and minutely pilose-puberulent; the internodes mostly 4 or 5 cm. long: leaves from linear-oblong to elongate-deltoid, the primary ones 3 to 5 cm. long, 0.5 to 1.8 cm. broad at the truncate or barely subcordate base, the tip bluntish, the margin closely fine-crenate, sometimes irregularly so at base, dark green and rugulose above, white beneath with close felt-like indument of minute simple hairs; petioles almost filiform, 1 to 1.5 cm. long: raceme simple, terminal, becoming 1.5 dm. long: rachis glandular-puberulent: verticels 4–8-flowered, all becoming remote, the lowest 3.5 cm. apart; pedicels slender, 2 or 3 mm. long, glandular-puberulent:

bracts ovate, aristate, 3 mm. long, caducous: calyx cylindric, bluish, glandular-puberulent, the tube five times exceeding the ovate acuminate lobes, in anthesis 6 mm. long: corolla 1.5 cm. long, the pale tube long-exserted; the deep-blue lip and the pilose galea subequal: style pilose. — PUEBLA, limestone hills near Tehuacan, alt. 1720 m., Aug. 23, 1901 (*C. G. Pringle*, no. 8593). Related to *S. alamosana*, Rose (no. 108 of the writer's Synopsis), but differing in its less branching habit, closer finer pubescence, slender petioles, more deltoid leaves and fewer flowers.

Salvia (Fulgentes) ancistrocarpha. Coarse erect herb; the stems closely glandular-puberulent above, and hispidulous especially upon the angles with short whitish flat trichomes: leaves ovate, cordate with closed sinus or subtruncate, acutish, closely crenate, scabrellous above, hispidulous on the nerves beneath, the primary ones 8 to 10 cm. long, 5 to 6 cm. broad at base: racemes elongate, the primary one becoming 4 dm. long: verticels 4–9-flowered, the lowest 2 or 3 cm. apart: bracts round-ovate, scarcely veiny, coarsely ciliate, tipped by a hook-like slender awn 4 or 5 mm. long, caducous: pedicels 2 or 3, becoming 6 mm. long: calyx tubular-campanulate, in anthesis 13 to 15 mm. long, in fruit pulverulent, hirtellus on the veins, the teeth acuminate: corolla pure red, 3 to 3.5 cm. long, the slightly pilose galea shorter than the broad lower lip: style glabrous. — MEXICO, Tultenango Canyon, Oct. 7, 1902 (*C. G. Pringle*, no. 8674). A gorgeous plant closely related to and somewhat suggesting *S. adenophora*, Fernald (no. 152 of the writer's Synopsis). Differing in its herbaceous stem, its harsh pubescence, its longer acuminate-toothed calyx, its larger nearly glabrous more vivid corolla, and in the smoother bracts with their peculiar claw- or hook-like appendages.

Salvia (Tubiflorae) Townsendii. Stems about 1 m. high, minutely pruinose-puberulent: leaves narrowly ovate or ovate-lanceolate, elongate-acuminate, about 1 dm. long, 2.5 to 3 cm. broad, dark green and glabrate above, pale and minutely pilose beneath, crenate, on puberulent petioles 1 to 2 cm. long: racemes 1 to 2.5 dm. long, closely flowered; the verticels 12–20-flowered, the lowest becoming 2 cm. apart, the upper crowded: pedicels 3 to 7 mm. long, ascending: calyx in anthesis 3 to 3.5 mm. long, strongly glandular-dotted, and minutely hispidulous on the nerves, the lobes very short and broad: corolla villous, deep purple-violet, 2.5 cm. long, the galea exceeding the drooping lip: stamens and style exserted, glabrous. — CHIHUAHUA, near Seven Star Mine, Sept. 16, 1899 (*C. H. T. Townsend & C. M. Barber*, no. 426). Nearly related to *S. michoacana*, Fernald (no. 187 of the writer's Synopsis), but distinguished by its closer cinereous puberulence, longer crenate (instead of sharply serrate) shorter-petioled leaves, and deeper-colored flowers.

Acnistus Pringlei. Branches stout, covered with a soft almost corky ashy brown bark: leaves elliptic-lanceolate or narrowly oblong, the principal ones 4 to 7.5 cm. long, slightly puberulent above, tomentulose and becoming glabrate beneath, on slender petioles 0.7 to 1.5 cm. long: flowers in crowded fascicles: pedicels 1 to 1.5 cm. long, slightly thickened above: calyx glabrous, at first campanulate, 2 or 3 mm. high, with ascending blunt lobes, later rotate and larger: corolla tubular-funnelform, straw-colored, 8 mm. long, with spreading or finally recurved bluntish lobes: stamens and style much exserted: fruit pulpy, subglobose, pale, about 7 mm. in diameter. — MEXICO, wet meadows, Valley of Zamora, alt. 1540 m., May 25, 1901, and July 25, 1902 (*C. G. Pringle*, nos. 8509, 11,049). Near *A. Benthami*, Miers, but clearly differing in its much smaller flowers and shorter pedicels.

Castilleja patriotica. Simple or branched at base, 3 to 5.5 dm. high; stems pilose-hirsute throughout or glabrate: leaves scabrous but thin, hispidulous, 3 to 5 cm. long, linear-lanceolate, dilated below the 2 to 6 strongly divergent linear lobes: bracts similar but less cleft, sometimes entire: flowers loosely racemose; pedicels 1 cm. or less long: calyx tubular, 3 to 4 cm. long, pilose-puberulent, red, green, and whitish; corolla 4.5 to 5.25 cm. long, green and whitish, the back softly pubescent; the long-exserted galea 2.75 to 3 cm. long; lower lip green, about 3 mm. long, with lanceolate lobes: fruiting calyx somewhat enlarged: capsule oblong, acuminate, 1.5 cm. long. — CHIHUAHUA, Sierra Madre near Colonia Garcia, alt. 2310 m., July 20, 1899 (*C. H. T. Townsend & C. M. Barber*, no. 156); Cumbre, Aug.–Nov., 1885 (*Edw. Palmer*, no. 363); Mapula Mts., alt. 2200 m., Oct. 30, 1886, and cool slopes of the Sierra Madre, Sept. 16, 1887 (*C. G. Pringle*, nos. 1154, 1350). Related to *C. tenuiflora*, Benth., and *C. affinis*, Hook. & Arn. Differing from both in its much longer galea (distinctly exceeding the tube) and in its thinner divaricately cleft leaves.

Castilleja Pringlei. Dwarf rather tufted perennial, with many slender decumbent pilose stems 3 to 6 cm. high: lowest leaves crowded and bract-like, ovate, 3 to 4 mm. long; the upper 1.5 to 2 cm. long, lanceolate to oblong-lanceolate, pilose, simple or with deeply 3-lobed tips: bracts similar, the narrow lobes usually red-tinged: calyx tubular, 2.5 to 3.5 cm. long, straw-colored below, deep red above, densely pilose except at the puberulent tips, split equally before and behind one fourth to the base, into oblong blunt shortly 2-cleft lobes: corolla slightly exserted, the narrow galea pilose, the somewhat cleft saccate lower lip about 1 mm. long. — HIDALGO, damp alpine meadows at 3080 m. altitude, Sierra de Pachuca, July 21, 1901, and Aug. 22 1902 (*C. G. Pringle*, nos. 9674, 8666, *Rose & Hay*, no. 5581); Mt. Popocatapetl, alt. 3630 m., Aug. 7 and 8, 1901 (*Rose & Hay*, no. 6022). A beautiful plant somewhat resembling *C. Schaffneri*, but with shorter stems, much larger red flowers, and densely pilose calyx.

Utricularia lobata. Scapes 2 to 5 cm. high, bearing 1 or 2 lance-subulate bracts, 1–2-flowered; bladder-bearing branches very short, basal or wanting; the scattered bladders barely 1 mm. long: leaves wanting or solitary and radical, narrowly spatulate, 3-nerved, 1 to 1.3 cm. long: pedicels at most 1 mm. long, subtended by 3 or 4 slightly longer subulate bracts: calyx of 2 elliptic-oblong lobes 3 to 4 mm. long: corolla violet with yellow centre, the upper lip erect, slightly dilated and subtruncate above, 3 or 4 mm. long; lower lip 7 to 9 mm. long, deeply 3-lobed, the middle lobe broad, more or less cut; spur subulate, arcuate, equalling or slightly exceeding the lower lip: fruit not seen. — MEXICO, springy soil, Sierra de las Cruces, alt. 3080 m., Sept. 2, 1901 (*C. G. Pringle*, no. 8607). Related to *U. resupinata*, B. D. Greene, but differing among other characters in its deeply lobed lip. From *U. verapazensis*, Morong, distinguished by its narrower longer leaf, and its much shorter pedicels subtended by subulate, not blunt, bracts.

Pectis erecta. Perennial, with a slender ligneous caudex: stems erect, simple or branching above, pulverulent, 8 to 15 cm. high: leaves linear, 1.5 to 3 cm. long, 1 to 2 mm. broad, pale green, firm, subulate-tipped, mostly ascending, with 2 to 4 irregular lines of pellucid dots beneath, and with 2 to 5 pairs of whitish cilia at base: peduncles filiform, erect, 3.5 to 6 cm. long, with 2 to 5 lance-attenuate bracts: heads 1 to 1.5 cm. broad, several-flowered; involucre turbinate, 5 to 6 mm. high, of 5 oblong or narrowly obovate blunt finely ciliate upwardly flattened bracts: rays oblong, 5 to 7 mm. long, yellow becoming crimson: ray-pappus of 2 or 3 elongate (4 or 5 mm. long) dark upwardly barbed aristiform pales more or less connate at base, with 4 to 8 much shorter unequal whitish attenuate pales: disk-pappus similar but the pales more numerous, the elongate dark ones 6 or 8: achenes slightly setulose. — NICARAGUA, Grenada, Department of Grenada, Feb. 28, 1903 (*C. F. Baker*, no. 2556). In habit resembling *Pectis diffusa*, Hook. & Arn. of the § *Pectothrix*, but with the pappus more nearly of *P. depressa*, Fernald, of the § *Eupectis*.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 152.

THE HEREDITY OF COAT COLOR IN MICE.

By Glover M. Allen.

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I. Introduction.

In the fall of 1900 the writer commenced a series of breeding experiments with a view to studying the inheritance of coat characters in the house mouse (*Mus musculus*) and its various domesticated varieties. This work has been carried on during the four years past under the direction of Professor W. E. Castle, to whose guidance and inspiration is due whatever merit the present contribution may possess. The writer's earlier studies were made on the heredity of complete albinism in its relation to the character pigmentation. It shortly became evident that Mendel's principles of heredity, then being brought to notice by certain botanists, were applicable to the case, and this view has since been fully confirmed by more extended observations. The principles of segregation and resolution of characters among hybrids seem fundamental, and an attempt has been made in this investigation to throw further light on the mechanism of these processes. The writer is aware that the principles enunciated by Mendel, though far-reaching, must not be assumed necessarily to explain all hereditary phenomena, yet they appear to afford the key to much that has previously seemed enigmatical.

During the four years since the independent rediscovery of the Mendelian laws in 1900, by the botanists de Vries (:00, :00^a) and Correns (:00), a considerable literature has sprung up on the subject. The older experiments have been re-examined in the light of the new theory, and other experiments have been instituted to elucidate still further the phenomena of hybridization. At least three recent investigators have experimented with mice, and have published, in part, some of their results. Hence certain of the observations detailed in the present paper are not altogether new, but nevertheless are of value as being corroborative of results independently obtained. The fact that there are several workers in the same field is of further advantage in that an opportunity is offered for the comparison and coördination of results. In the following pages an attempt is made to explain the phenomena observed, according to the principles enunciated by Mendel; yet it is not urged that other explanations are impossible. It is simply a question of what hypothesis best explains the facts.

II. Varieties of Mice and Rats.

The house mouse (*Mus musculus*), as familiarly known in our dwellings and outbuildings, is probably of Eurasian origin, but has been carried by man into all parts of the civilized world. Its color is usually spoken of as gray, the "agouti" of fanciers, and though subject to some slight variation with age and habitat, is nevertheless of a fairly constant type. The pelage of a fully grown specimen taken at Cambridge, Mass., consists dorsally of two sorts of hairs; the one has a very fine black tip with a subapical ring of a light ochraceous color occupying about one-fifth the length of the hair, while the basal portion is dark plumbeous. Scattered among these particolored hairs are others, less numerous but longer and entirely black, so that the combined effect is the "house mouse gray." The centre of the dorsal surface is slightly darker than the sides, while the ventral surfaces are covered (though more thinly) with hairs plumbeous at the base, but with the tips varying in different individuals from a dirty white to a very pale ochraceous tint. The ears, tail, and feet are sparsely covered with short, light gray hairs, and the skin of these parts is dark gray. The ochraceous bands on the particolored hairs of the upper surfaces deepen in tint with the age of the individual, so that old adults often have a rufous tinge on the flanks.

From gray house mice such as these it is supposed that the various domesticated breeds have arisen. Bateson (:03[b]) has published the partial results of an examination by Miss F. M. Durham of the pigments contained in the hair of house mice. These are found to be three: (1) densely opaque black, (2) less opaque brown, and (3) transparent yellow. All three pigments may coexist in the same hair, or certain hairs may contain only black and brown, others only black and yellow. The present writer also has found these three pigments in one or another of the strains of mice with which he has experimented. According as one, two, or all three of these pigments are present in greater or less degree, the mouse is referred by fanciers to a particular color type. Bateson has given a list of the types known to him, enumerating in all thirteen. These, in brief, are as follows:

(1) *Ordinary cinnamon*, or *agouti*, being the usual "wild gray," in which all three pigments are present in the hairs.

(2) *Golden agouti*, a distinctly tawny type, containing the yellow and brown pigments in the hairs, but not the black.

(3) *Sable*, of the agouti type, but with the flanks nearly clear yellow.

(4) *Blue-and-tan*, believed by Bateson to be the same as "sable" but with less of the black pigment.

(5) *Chocolate* or *plum*, containing the brown pigment alone.

(6) *Silver fawn*, a very light chocolate.

(7) *Yellow*, containing yellow only.

(8) *Cream*, a very light, or "diluted" yellow.

(9) *Black*, containing, according to Bateson, black and brown, the former not extending to the tips of the large contour hairs. The same author states that he has not seen a pure black, i. e., one without the brown pigment.

(10) *Blue*, a "dilute" form of black, the pigment granules being less numerous.

(11) *Albino*, in which there is no pigment whatever.

(12) *Black-eyed white*, of which two strains are noted, the eyes in one being a blackish-red.

(13) *Variegated*, in which "irregular, small spots of black or chocolate occur on a white ground."

So far as regards pigment, therefore, the known types of fancy mice are six, — namely, (1) *gray or "agouti,"* including the "*sable*," and *blue-and-tan* breeds, in which all three pigments are present; (2) the "*golden agouti*," in which brown and yellow are the only pigments present; (3) *black*, including "*blue*," in which there are only the black and chocolate pigments; (4) *chocolate*, containing brown pigment only; (5) *yellow* and *cream*, in which only the yellow pigment is present; (6) *albino*, lacking pigment both in the skin and in the eye.

The gray mouse may be looked upon as an individual in which are present the three pigment elements, — black, chocolate, and yellow. By the elimination of one of these elements there are produced the black-chocolate, or the chocolate-yellow combinations; if two elements are eliminated, either chocolate or yellow individuals result. A pure black is to be expected, for if the black pigment may be eliminated and so produce the golden-agouti effect, it would seem that it should also be able to appear alone.

Pied forms of these various types occur, and are generally referred to as *spotted* varieties.

Among rats (*Mus norvegicus* Erxl.) the gray coat bears a considerable superficial resemblance to that of the mouse, although hitherto no examination of its pigments appears to have been made. Moreover, so far as known, the pigment characters of the rat have undergone resolution only to the extent of producing black individuals. Pied and albino varieties are also well known.

The origin of the various breeds of fancy mice is to a considerable extent obscure. In a wild state, melanistic individuals of the house mouse are known to occur, and, according to Davatz ('93), are not uncommon in the valleys of Graubünden, Switzerland. They were first noticed there at about 1860, and were described by Fatio ('69) as a distinct species under the name of *Mus poschiavinus*. They were captured in houses and factories, which they inhabited in common with mice of the usual gray color, and they seemed to be about one-half as numerous as the gray individuals. Albino mice and rats are also not of rare occurrence in a wild state. According to J. W. Loudon ('50) albino mice were not uncommon, particularly in Savoy and some parts of France, a half century ago, and other instances of their appearance here and there might be cited. The question may be raised, however, as to the likelihood of such sporadic cases being due to the introduction of albinos from other sources, and their subsequent escape. Nevertheless it is a matter of common knowledge that albinos may occur *de novo* among many species of mammals, as in deer, bison, squirrels, moles, and others (see Cantoni, '80).

It is thus not difficult to conceive how the albino and the black breeds now domesticated have originated. But the origin of other fancy varieties is not so clear. Thus Pallas (: 78) mentions three varieties : the albino with pink eyes, a black variety, and one gray with white spots. J. B. Fischer ('29) enumerates five different varieties of the house mouse : "totus albus oculis rubicundis," "totus niger," "albo-maculatus," "helvolus," "albus cinereo-maculatus." The first three varieties mentioned probably correspond with those recognized by Pallas ; namely albino, black, and gray-white (perhaps also black-white). The two new varieties in Fischer's list are : (1) "helvolus," supposed by C. Carter Blake (in Fancy Mice) to be the same as isabella (yellow) and (2) "albus cinereo-maculatus," that is, white spotted with gray. Dehne ('55), writing of the house mouse found in Europe, mentions mice of the usual gray color above, but varying in the color of the belly from pure white to yellowish. The albino, he says, is common, though formerly a rarity. He mentions also a light yellow ("erbsen gelb") variety, with *pink* eyes, and a gray-white spotted variety.

The origin of the spotted varieties is unknown. Swinhoe ('70), in writing of the mammals of China and Formosa, states that black-white mice are often kept by the Chinese and that "these are brought from the Straits." The so-called dancing mice are usually attributed to Japan. They are smaller, and of more delicate proportions than the wild house mouse or its tame European varieties, and if actually of Japanese origin, may have been derived from one of the indigenous species closely related to *Mus musculus*. Ordinary dancing mice are black-white, but there occur also individuals which are wholly black or white, or which are spotted fawn-and-white, and have pink eyes (Darbishire, : 02). They are called dancing mice from a habit which they have of turning rapidly in circles of varying diameter, the direction of motion being usually clockwise. This habit is by some attributed to a nervous disorder, and by others to an abnormality of the semicircular canals of the ear. It is termed by the French "mouvement de manége". This breed is believed to have been domesticated by the Japanese for a considerable period, as evidence of which Schlumberger ('94) figures and describes an ancient Japanese netzuke, or charm, carved in wood, and representing with great minuteness an entire family of dancing mice. The father and the mother, as well as four of the young, are spotted black-white, while of the four other young, two are entirely white and two entirely black. The two white young, as well as their black-white mother, have pink eyes; all the others have black eyes. The dancing character in mice of this variety is heritable, and, as pointed out by Castle (:03[d], p. 225), is transmitted quite independently of color characters.

Although the actual origin of the various breeds of mice is thus practically unknown, certain of the writer's experiments seem to throw light upon the method by which they may have arisen. For example, albino mice of unknown ancestry were in his experiments bred to gray house mice, and in the first generation only gray young were obtained, indistinguishable in outward appearance from their gray parent. Subsequently these gray offspring were bred *inter se*, and also back crossed with the albino stock. From both of these crosses there resulted, in addition to animals of the gray type, an occasional black individual, and two such black mice, when mated, produced only black young (or, if they contained albinism recessive, they produced albino as well as black offspring, in the Mendelian ratio, 1 : 3). It may be that in this case a segregation of the pigment elements has taken place in such a way that the yellow element has disappeared from certain individuals, and this segregation has been favored in some way by the cross made. On the other hand, it is not improbable that in this particular case the black character may have been transmitted directly through the albino grandparent without a segration of the compound gray character. This view will be explained in detail further on. The brown, or chocolate, element is present in these black mice, but is not evident to the naked eye except at such times as the regular moult of the pelage takes place, when the new hair is distinctly tinged with brown.

From these black mice *of house-mouse derivation* the writer has been unable, by crossing with the same albino stock, to obtain a further segregation of the pigments, but it would seem plausible to suppose that chocolate and yellow might yet be obtained from this stock by such a method. In the case of a black-white stock, however, further segregation was obtained. The black-whites used were purchased of a dealer by Professor Castle, under the name of Japanese mice, though they were not of the dancing variety. These mice, when bred to the same stock of albinos that was used in the house-mouse crosses, gave both gray young and black young. By interbreeding the gray heterozygotes (or hybrids) there was produced, in addition to gray and black offspring, an occasional golden-agouti or a chocolate individual, these two sorts being about equally numerous. A similar result was obtained by crossing a gray with a black heterozygote, but these same black heterozygotes when bred *inter se* produced only blacks.

It is probable that the various color varieties of mice, as now kept by fanciers, have originated through segregation of elements found in the compound gray character, caused perhaps by crossing with dissimilar varieties. Long domestication and the conditions incident to confinement have very likely been effective in aiding this so-called "breaking up of the type."

It may be of interest to mention here what appears to be a pathological variety of the house mouse, namely, the so-called "rhinoceros mouse." This appears to have first been figured and described by Gaskoin ('56), whose colored plate represents an animal of the size of the common mouse, but entirely devoid of hair, except for a few whiskers. The skin, however, is thrown into a series of transverse corrugations, and extends along the sides as a broad fold running from the anterior to the posterior limbs and loosely enveloping them. This character and the generally wrinkled appearance of the skin give the animal somewhat the appearance of a diminutive rhinoceros. According to Gaskoin, no vestige of hair or hair follicles could be discovered in microscopic sections of the skin. He states that two individuals of this sort, a male and a female, were captured by a laborer about a straw stack which stood in a wood not far from Taplow Paper Mills, England. The female was pregnant and in a week had five young, all of which were of the same "rhinoceros" type. Gaskoin also mentions a similar specimen which was captured in the fireplace of a room, in 1820, and is preserved in the museum of the College of Surgeons, England.

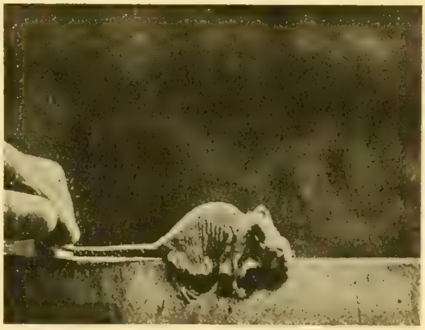

FIGURE 1.

The present writer obtained a mouse exhibiting this same condition

(Figure 1). It was captured in a house in Boston, where it had been seen with a white and a black-white mouse. Unfortunately it died before any young could be obtained from it. Its skin was soft, and it was similar in every way to those figured by Gaskoin, save that the ears had a few hairs on their distal portion, and there were a number of whiskers.

During the course of the writer's experiments, a certain normally haired female gave birth to a litter of young, and these, after acquiring their first coat of hair, became sick and perished one by one, till finally a single individual of the litter remained. This mouse, while still very young, lost all its hair, except for a small amount on the ears, nose, feet, and tail. It soon began to develop the transverse corrugations and the lateral fold of skin, and would doubtless have become a "rhinoceros mouse," had its career not been cut short at this interesting stage. It is clear that this condition is a pathological one, and there are indications that it is heritable, but exact tests of this have as yet proved impossible.

III. Care and Management.

A few words respecting the care and management of mice may not be without value to those who contemplate experimenting with these animals. Tame white mice and rats seem to have little desire to escape from their boxes, and hence cause little trouble. But wild individuals are very timid and at the first alarm are apt to rush wildly for the nearest opening, and escape. For this reason, it was found that a double compartment cage, with separate doors for each compartment, was the best. A small opening at the far end of the partition between the compartments allows the inmates to gain the shelter of the opposite chamber while one side of the cage is being examined or cleaned. As the results of experiments with various sorts of cages, Professors Mark and Castle designed one which has been used in the work at the Harvard Zoölogical Laboratory for the past two years with great satisfaction. As the basis for the cage (Figure 2), a wooden tray is used, of the standard size employed throughout the Museum of Comparative Zoölogy, viz., 27½ inches long, 18 inches wide, and 3½ inches high, outside measurements, made from half-inch stock. At the back of this tray, three upright posts 13 inches high are erected, one at either end and a third at the middle. Cross pieces connect the tops of these posts, and two slanting side pieces run from the tops of the end posts to the front of the tray. The sides as well as a central partition are of wire netting, and the two doors, hinged to the top, are wooden frames covered on the inner side with netting. Racks were constructed, designed to hold ten or twenty of these boxes in five tiers. Thus an economy of space was secured, while the compactness of the system made feeding and cleaning easier. The floors of the cages are kept covered with clean sawdust, and wooden berry baskets, one to each compartment, inverted and filled with cotton, serve for nests.

As for food, bread and milk are used as a staple, care being taken that the mixture should not contain more liquid than the bread will absorb. Oats and bits of vegetable, as well as green grass, afford a welcome variety.

In work of this sort, where it often becomes necessary to test the same mouse with various other individuals, some means of marking the animals is needed, so that a given individual may be identified readily from among similarly colored mice. To this end the writer has used the system employed by Professor Castle in connection with guinea-pigs and rabbits, namely, that of punching small holes in the ears. The left ear may be used for tens, the right for units. A small circular hole at the anterior base, tip, or posterior base, stands for 1, 2, or 3, respectively. A scallop or notch in the same places represents 4, 5, or 6, respectively; while by using two notches, combinations standing for 7, 8, or 9 are readily made. An ear unpunched stands for 0. After the numbers from 1 to 99 have

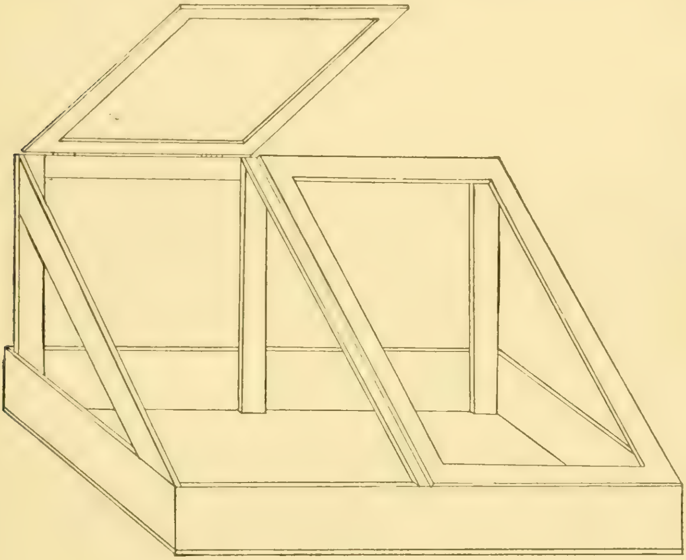

FIGURE 2.

been exhausted for numbering the mice, the individuals of the second hundred may be marked in like manner, care being taken to avoid giving the same number to two mice of the same color. For numbers of the third hundred, a round hole may be punched in the centre of the left ear, and for those of the fourth hundred, a similar hole in the right ear. With care, these marks may be accurately placed, and this with no apparent inconvenience to the animals. The mice may be recorded serially in a blank-book kept for the purpose, and the writer has also found it of advantage to use a separate card for each animal. For this purpose library cards may be used, the animal's pedigree entered on the back, and data as to matings, litters, etc., on the front. The cards may be used in the form of an index, and those of the individuals in any one cage can thus be readily kept together or changed about as occasion requires.

In the experience of the writer, mice are found to thrive and breed best where but a single pair is kept in a cage. In common with certain other mammals, these mice have a strong sense of property rights, and if a strange individual is introduced into a cage, he is apt to be attacked at once by the inmates. If the newcomer is not despatched forthwith, he is usually kept in the farthest corners of the cage, and is not suffered to enter the nests until he has shown himself capable of a good defence, or until the fury of his antagonists has abated. To avoid such encounters, as well as the loss of valuable animals, the writer has found it sufficient simply to introduce two animals, which it is desired to mate, into a cage in which neither has been living. Both recognize at once that they are on strange territory, and after cautiously approaching each other and examining the new home, they settle down peaceably without further ado.

The period of gestation in both mice and rats is very nearly 21 days, and the female is in heat again at the birth of a litter (Sobotta, '95). The young should be weaned at three weeks old, for, if allowed to remain longer with the mother, they become poorly nourished and puny from overcrowding of the pen. Not infrequently the male will devour the young, at the time they are born, if he is not removed. More rarely a female mouse will devour her entire litter.

As the young are born blind and naked, several days must elapse before it is possible to determine their coat color. Albinos can be distinguished at birth, however, since their eyes are unpigmented, while individuals which are to become colored have dark eyes at birth, the pigment showing distinctly through the overlying skin. Pigmented animals having pink eyes could not, of course, be distinguished from complete albinos at this stage, but none of this sort have been obtained in these experiments.

Mice may be mated at the age of three months, and they will continue to breed until a year old or slightly longer. But although they may live to be 15 or 18 months old, they are much less prolific in the latter months of their life. Moreover, in the writer's experience, they rarely breed so often as once in three weeks, although they are capable of so doing.

IV. Mendel's Laws of Heredity.

Mendel's great discovery of the principles in heredity of dominance and segregation of characters is now so generally known as hardly to need extensive notice here. These two important laws were first set forth by Mendel in 1866, though for nearly half a century they passed quite unnoticed. In 1900 the botanists de Vries and Correns, working independently of each other, again brought to light Mendel's principles, and were able to give them further confirmation through their own experiments. The three following years have seen a keen and growing interest in the field thus opened, and already a considerable body of literature has appeared for or against the Mendelian laws. A full exposition of the Mendelian doctrine may be found in the works of Bateson (:**02**) and Castle (:**03**^{b}, :**03**^{d}), and it is from these that the following brief statement is principally drawn.

In his experiments with peas, Mendel established the fact that when two varieties are crossed, whose seeds are respectively yellow and green, the hybrid seeds produced are yellow, that is, the yellow color of cotyledon is dominant, while the green color does not appear in the cross-breds, though it will be represented in their germ cells. If D represent the dominant character and R the recessive character set off against it, we may express Mendel's result by this formula (1): $D \times R = 4D(R)$,* the parenthesis indicating that the enclosed character is present in the germ cells but does not appear in the soma. In such cases of simple dominance, the first filial generation, F_1,† resembles in external appearance the parent possessing the dominant character. That this resemblance does not signify identity in character may be shown by breeding *inter se* the D(R) hybrids obtained from the above cross. Mendel proved that for his peas the hybrids, D(R), produced germ cells D and

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[*] In this equation the symbols at the left of the sign of equality indicate the character of the parents, those at the right the character of their offspring.

[†] This convenient notation, suggested by Bateson (:**02**), will be employed throughout this paper.

R in approximately equal numbers. This result is probably brought about by the expulsion, in maturation, of the D or the R character from the germ containing these two elements. This phenomenon is the so-called principle of segregation. If one hybrid be fertilized by another like hybrid, there will thus be possible, according to the laws of chance, *two* combinations of a D with an R gamete to *one* each of a D with a D and an R with an R, thus:

$$\begin{array}{c} D + R \\ D + R \\ \hline D + 2D(R) + R \end{array}$$

This result may be expressed by formula (2):

$$D(R) \times D(R) = D + 2D(R) + R.$$

Generation F_2 will thus consist of three sorts of individuals: (*a*) individuals D, which are pure dominants and reproduce their kind only; (*b*) individuals D(R), which, though indistinguishable in somatic characters from those of class (*a*), are nevertheless heterozygotes and, like their parents, produce gametes D and R in equal numbers; and (*c*) individuals R, which have the recessive character alone, and when paired with other R individuals produce only R offspring.

For class (*a*), if kept by itself, we may then construct formula (3): $D \times D = 4\ D$; and for class (*c*) formula (4): $R \times R = 4\,R$. The formula for class (*b*) is of course the same as that for their parents, the primary hybrids, viz., formula (2) above. In addition to these crosses, the D(R), or heterozygous, individuals may be back-crossed with the original parent forms. Thus, if the D(R) animal, producing gametes D and R in equal proportions, be bred to one of the pure dominant type, producing gametes D and D, there will be two sorts of combinations possible, one of which will be a combination of D with D, the other a combination of D with R. The chances for the occurrence of each are equal. The result may be expressed by formula (5): $D(R) \times D = 2D(R) + 2D$. In external appearance, the entire progeny of this cross may be similar, but the existence of the two sorts of individuals may be demonstrated by crossing each with the recessive form, R. In the case of the D individuals, such a cross will produce only heterozygotes, D(R), in accordance with formula (1). But the D(R) class, producing equal numbers of D and R gametes, on being crossed with the recessive type, will give in equal proportions the two sorts, D(R) and R individuals. This result we may express by formula (6): $D(R) \times R = 2D(R) + 2R$. Thus in a cross of two heterozygotes, there are obtained, — formula (2), — in addition to a heterozygous class similar to the parents, two pure classes, each reproducing respectively the one or the other parent type. On the other hand, when a heterozygote is back-crossed with either of the pure parental types, — formulae (5) and (6), — no new forms result, but there is simply a reproduction of the two which united in fertilization.

For convenience in reference these formulae of Mendelian inheritance may be brought together as follows:

To the left of the sign of equality is indicated the character of the parents mated; to the right, the classes and numerical proportions of the offspring to be expected.

(1) $D \times R = 4D(R)$
(2) $D(R) \times D(R) = D + 2D(R) + R$
(3) $D \times D = 4D$
(4) $R \times R = 4R$
(5) $D(R) \times D = 2D(R) + 2D$
(6) $D(R) \times R = 2D(R) + 2R$

A similar set of formulae may be deduced for cases in which the dominant character is a mosaic of the two characters which, if occurring separately, are to each other as dominant and recessive. Thus the character of a spotted mouse (pigmented, but with white areas at certain places) is dominant over that of a complete albino, and the pigmented heterozygotes may produce gametes of two sorts, the one bearing the character partial albinism, the other bearing the character complete albinism. The various combinations possible for these two characters may be expressed by the following formulae:

(1*a*) $DR \times R = 4DR(R)$
(2*a*) $DR(R) \times DR(R) = DR + 2DR(R) + R$
(3*a*) $DR \times DR = 4DR$
(4*a*) $R \times R = 4R$
(5*a*) $DR(R) \times DR = 2DR + 2DR(R)$
(6*a*) $DR(R) \times R = 2DR(R) + 2R$

These formulae indicate the results of various matings in cases of simple dominance and segregation. In cases where compound characters become resolved and the elements reunite again in cross-breeding, it is not certain that the relations are quite so simple. Nevertheless, the results so obtained seem to indicate that here, too, segregation does take place in a more or less definite way.

V. The Heredity of Albinism.

A. *Complete Albinism.*

1. *House mice and albinos.* In a preliminary paper (Castle and Allen, :03) Professor Castle and the writer have shown that complete albinism in mice and rats is a recessive character in a Mendelian sense, and our observations corroborate Cuénot's (:02) results along similar lines. The writer bred wild house mice, captured in stables and houses in Cambridge, to a stock of albino mice obtained from a Boston dealer. Nothing could be learned as to the ancestry of these albinos, but when bred *inter se* they had produced only albinos. As a result of this cross of wild gray with albino mice, only pigmented individuals were obtained. By breeding albino males to female house mice, eight litters were secured, there being a total of 28 young. The average number of young produced at a birth was thus 3½, and the commonest number was 3 or 4. There was one litter of 7 and one of 6, while a third consisted of but 2 individuals. By the reciprocal cross, in which the gray house mouse was the male, and the albino the female parent, 38 young were produced in nine litters, an average of 4⅗ to a litter. The sex was ascertained of 46 of these 66 mice, and there were found to be 21 females and 25 males, an excess of about 8 per cent of males. This is in accord with the work of Schultze (:03), who usually obtained a slight excess of males among the offspring of his white (albino) mice.

Several of the cross-breds produced by the matings described were paired. The Mendelian expectation in this case is, by formula (2), that a quarter of the progeny will be albinos, a quarter will be pure dominants, and the remaining half will be, like their parents, heterozygous, pigmented but containing the albino character in a recessive condition. Quantitatively, this expectation was not precisely realized. For of the 69 mice produced, theoretically 17 should have been albinos, while 52 should have been pigmented. There were actually 25 albinos and 44 pigmented individuals, or 36 per cent albinos and 64 per cent pigmented. The excess of albinos is thus 11 per cent, or there are about half again as many albinos as expected.

The distribution of the young by litters in this generation is shown in Table A.

Disregarding the first two litters in which the precise Mendelian ratio of pigmented and albino young is realized, we find that six of the eight remaining lots show an excess of albinos ranging from 0.25 to 4 per litter, or an average of less than two (actually 1.5). Only two litters con-

TABLE A.

	Pigmented.	Albino.	Expected No. Pigmented.	Expected No. Albino.	
Litter 1 “ 2	9	3	9	3	exact agreement.
“ 3	4	. .	3	1	excess of pigmented.
“ 4	5	3	6	2	“ albino.
“ 5	4	2	4.5	1.5	“ “
“ 6	2	3	3.75	1.25	“ “
“ 7 “ 8	8	8	12	4	“ “
“ 9	5	2	5.25	1.75	“ “
“ 10	3	3	4.5	1.5	“ “
“ 11	4	1	3.75	1.25	“ pigmented.
Total	44	25	51.75	17.25	

tained an excess of pigmented individuals, and the excess is very slight, 0.25 in one case, and 1 in the other, an average of less than 1. The entire excess of albinos being 7.75, the average excess is thus less than one for each of the nine litters which show deviation from the Mendelian ratio. To be sure, the totals are not large, and the results may be purely a matter of chance in the union of the gametes, yet the deviation may have some significance which has not yet been fully appreciated. Thinking that the production of an excess of albinos might be an individual peculiarity of the parents, wherein an unequal number of pigment and non-pigment gametes were given off, two of the heterozygous, F_1, males were bred to albinos; but these produced an excess of *pigmented* individuals, there being in 17 litters 35 pigmented mice as against 23 albinos, where — according to formula (1) — equality is expected. Curiously, the excess of albinos in the litters from cross-breds nearly offsets the shortage in the lots obtained by back crossing with the albino parent. Thus in the first case, out of a total of 69 young, we expect 17 to be albinos, and obtain 25. In the second case, 29 albinos are expected, and only 23 occurred. In all, therefore, 48 albinos were produced, where 46 are expected. The deviations from the Mendelian ratios may, therefore, be due simply to the relatively small numbers of young obtained.

Cuénot (: 02) in similar experiments procured a much larger number of mice, and his figures are much more nearly in accord with the theoretical expectation. By breeding together the F_1 heterozygotes, he obtained 198 pigmented mice and 72 albinos, or 26.6 per cent albinos. By back-crossing the heterozygotes with albinos, Cuénot obtained an approximate equality of pigmented and of albino mice, but the precise numbers are not given.

The pigmented animals obtained by interbreeding the F_1 heterozygotes should be of two sorts according to Mendelian principles, the one of pure dominants, the other of heterozygotes; and these two classes should be to each other numerically as 1 : 2. No actual test of this numerical ratio has as yet been made in the case of mice. To make such a test, it is necessary to breed each of the F_2 pigmented individuals to an albino or to a heterozygote (containing the albino character recessive), and a sufficient number of young must be so obtained to determine whether or not the particular individual is producing germ cells having the albino character. It is obvious that the cross with an albino is preferable, in this case, to the cross with an individual known to be heretozygous. For the albino, if bred to an animal containing the albino character recessive, will (by formula 6) produce 50 per cent of albinos, whereas from two heterozygous animals, only half as many are to be expected. The writer has tested 10 of the pigmented mice of generation F_2 by breeding them to albinos. Five of these individuals produced only pigmented young, 8, 11, 9, 5, and 11 respectively. We may therefore conclude that they were in all probability of the class D, or "extracted" (Bateson) dominants. Each of the five other animals of generation F_2 produced both pigmented and albino offspring when bred to albinos. They were, then, of the class D(R). The totals of their litters are 23 pigmented, 26 albino, offspring. It is therefore plain that, qualitatively, the Mendelian expectation is realized, in the case of generation F_2, where the three classes, D, D(R), and R should be present in the proportion 1 : 2 : 1. The fact that the ten mice of classes D and D(R), although selected at random, were 5 of them of class D and 5 of class D(R), instead of being one-third D and two-thirds D(R), may be not without significance in this particular case, wherein there is some slight evidence of irregularity. Yet the result may be purely a chance occurrence, for the numbers are small.

It is of interest to examine the separate litters making the above total of 23 pigmented and 26 albino young (see Table B).

TABLE B.

Litter.	Parents.	Albino.	Pigmented.
1	♀ 242* × albino	3	2
2	" "	3	2
3	" "	3	..
4	♂ 246 × albino	3	3
5	" "	4	4
6	♀ 210 × albino	3	
7	♀ 208 × albino	1	6
8	" "	3	1
9	" "	..	1
10	♀ 212 × albino	3	3
Total		26	23

Only three of the ten litters (Nos. 4, 5, and 10) show the expected equality. Two of these were from the same pair, and as in the case of the third litter, represent all the young obtained from the parents. Evidently each of the two heterozygous parents was producing equal numbers of D and of R gametes. In case of the pair producing the first three litters, there is an excess of 33⅓ per cent of albinos, and one entire litter of three contained no pigmented animals at all. The heterozygous parent, if we may judge from the small number (13) of young produced, furnished more albino than pigment gametes. The opposite seems to have been the case with the pigmented parent of litters 7, 8, and 9, which aggregate 4 albinos, 8 pigmented offspring. Nevertheless the result is doubtless a chance one, since, although the first litter shows a shortage of albinos, the second litter presents an excess. The total result, 26 : 23, is a close approximation to the Mendelian expectation with the slight excess in favor of the albinos. Thus it appears that although, in individual lots, there may be a considerable irregularity in the proportions of the two sorts of gametes produced by the heterozygote, yet in the long run the 1 : 1 ratio is practically realized.

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* Different individuals are distinguished by numerals, as stated on page 69.

Cuénot (: 02) also tested the pigmented mice of generation F_2 which he obtained by breeding *inter se* the heterozygotes of generation F_1. This he did by simply interbreeding these pigmented animals. In this case, according to the Mendelian formulae, three sorts of unions are possible: (1) D × D, (2) D × D(R), and (3) D(R) × D(R). From the first two combinations no albinos will be produced; from the third, there should be produced 25 per cent albinos. Cuénot found these expectations realized, for of a number of the pigmented mice selected at random, about one-half produced only pigmented young (189), and the other half produced the expected 25 per cent of albinos, the actual numbers being 162 pigmented : 57 albinos, or 26 per cent albinos. In case of the mice whose union produced no albinos, either one or both animals of each pair must have been of the D class. If one-half were of the D class and the other half of the D(R) class, the precise Mendelian expectation would be realized. Since at least one-half of all the pigmented mice were certainly known to be D(R) because they produced albinos, it is probable that, in those unions which produced no albinos, one at least of the two parents mated was a pure D, the other being probably in some cases heterozygous, in other cases pure.

The two tables, C and D, show the total numbers of pigmented and of

TABLE C.

	Parents both D(R).	Pigmented Young.	Albinos.	Expected No. of Albinos.
1	♂ gr. 48, 49, 56 × ♀ gr. 50, 51, 52	44	25	17.25
2	♂ gr. (bl.-wh. × alb.) × ♀ gr. (house m. × alb.) .	19	7	6.50
3	♂ gr. 143 × ♀ gr. 229	4	2	1.50
4	“ “ “	2	3	1.25
5	“ “ “	5	. .	1.25
6	“ “ “	1	. .	.25
7	♂ bl. 246 × ♀ bl. 242	5	2	1.75
8	“ “ “	. .	2	.50
9	“ “ “	1	. .	.25
10	♀ bl. 457 × ♀ bl. 456	. .	1	.25
		81	42	30.75

TABLE D.

	Parents: one D(R), one R.	Pigmented Young.	Albinos.	Expected No of Albinos.	
1	♂ gr. 51 × ♀ alb.	4	. .	2	
2	“ “	. .	3	1.5	
3	“ “	1	2	1.5	
4	“ “	2	. .	1	
5	“ “	1	. .	.5	
6	“ “	2	1	1.5	
7	“ “	4	2	3	
8	“ “	1	1	1	
9	“ “	. .	1	.5	
10	“ “	4	. .	2	
11	“ “	1	. .	.5	20 p. : 10 alb.
12	♂ gr. 56 × ♀ alb.	1	1	1	
13	“ “	2	2	2	
14	“ “	1	1	1	
15	“ “	1	1	1	
16	“ “	3	1	2	
17	“ “	3	1	2	11 p. : 7 alb.
18	♂ alb. 145 × ♀ gr. 52	2	5	3.5	
19	“ × ♀ gr. 50	2	1	1.5	
20	♂ alb. 237 × ♀ gr. 210	1	3	2	
21	♂ alb. 235 × ♀ gr. 208	6	1	3.5	
22	“ “	1	3	2	
23	“ “	1	. .	.5	8 p. : 4 alb.
24	♂ bl. 246 × ♀ alb. 254	3	3	3	
25	“ “	4	4	4	7 p. : 7 alb.
26	♂ gr. 376 × ♀ alb. 255	4	2	3	
27	“ “	1	3	2	5 p. : 5 alb.
28	♂ alb. 246 a × ♀ bl. 242	2	3	2.5	
29	“ “	2	3	2.5	
30	“ “	. .	3	1.5	4 p. : 9 alb.
31	♂ alb. 277 × ♀ gr. 212	3	3	3	
32	“ “	3	1	2	6 p. : 4 alb.
33	♂ alb. 246 a × ♀ bl. 377	3	2	2.5	
34	“ “	4	2	3	7 p. : 4 alb.
35	♂ alb. 412 × ♀ bl. 377	3	3	3	
36	♂ gr. 376 × ♀ alb. 441	3	. .	1.5	
37	♂ alb. 428 × ♀ ♀ 427 a, 429	5	2	3.5	
		84	64	74	

albino young obtained by the writer from all litters in which the pigmented parent or parents were of house-mouse and albino ancestry, and include the lots previously mentioned as well as several not elsewhere noted.

Table C shows that in a total of 123 young there is an excess of about 11 albinos (9 per cent) over the expected number, and this preponderance of the recessive character is seen to be especially marked in generation F_2 (from ♂♂ gr. 48, 49, 56, mated with ♀♀ gr. 50, 51, 52), where it is about 11 per cent. The additional lots, however, reduce this excess to 8 per cent in the total, and it would doubtless be found that larger numbers would show still closer approximation to the theoretical result, as indeed the experiments of Cuénot (: 02) show.

Table D is of more particular interest, since it gives the results of back crossing with an albino a pigmented animal one of whose parents was an albino. Consequently, by Mendelian formula (6), the pigmented mouse is assumed to produce in the long run equal numbers of gametes bearing the pigment and the albino characters respectively. It is evident that if such an animal be bred to an albino, all the gametes it produces that bear the albino character will unite with those of a similar sort furnished by the albino mate, and *albino* young will result. Further, all the gametes it produces that bear the pigment character will also unite with those of the albino, but pigmented young will be produced, each of which will contain the albino character recessive. It is thus possible to determine how many gametes of each sort are produced by the heterozygote at each union, or, at least, how many of them unite with those of a second animal to form new individuals.

In the case of the heterozygous male, No. 51, thirty young were obtained in eleven litters by white females. Instead of the anticipated equality of pigmented and albino mice, the former class stands to the latter as 2 : 1. Most of the litters are small, three consisting each of but a single individual, and but one of these is an albino. Two litters consist each of two young, and in the one case both are pigmented, while in the other there is one pigmented and one albino young. This last litter is the only one of the eleven in which the expected equality of the two classes is realized. There are three litters of three young each. One of these is of albinos only. The other two taken together give an equality of the two classes, the excess in the one exactly counterbalancing the shortage in the other. Two litters, each of four young, contain no albinos at all, while the single litter of six contains four pigmented to two albino young, a ratio again of 2 : 1.

Male heterozygote, No. 51, gave six litters by albinos, and in four of these the exact Mendelian proportion, 1 : 1, is maintained with a total of ten young. In the remaining two litters three pigmented young are obtained to one albino. The total excess of pigmented over albino mice in this case is thus but two in a total of 18 young. A third heterozygote, ♀ gray 208, produced three litters, in two of which there was an excess of pigmented young, while in the third there was an excess of albinos, a total of eight pigmented to four albino young. Two other animals produced each two lots of young; in the case of one there is Mendelian equality in each litter, while in the case of the other, a shortage of albinos in one litter is exactly offset by an excess in the second litter. The remaining cases show a similar state of affairs, for a single large or small litter may contain equal numbers of pigmented and of albino young, while subsequent lots show an excess of the one or the other sort in varying degrees, amounting in some cases to a total exclusion of the pigmented or of the albino class, and this too in litters of as many as three or four young. The most extreme case obtained was one in which there was but one albino in a litter of seven mice, the expectation being equality of albino and pigmented individuals.

It is evident from this table that there is a considerable variation in the relative numbers of gametes of each class brought by the heterozygote to each union. Frequently there is an exact numerical equality of the two classes in separate litters, as is theoretically expected on Mendel's assumption that a heterozygote produces gametes of the two sorts in equal numbers. In other cases the gametes of such an individual produced at one time, or at least all which unite with those of another animal to produce young, may be entirely of the one class or the other, while between these two extremes are found various degrees of variation in the relative proportions of the two sorts. This variation appears to be quite independent of sex. The numerous deviations from the 1 : 1 ratio tend to counterbalance one another, often to a remarkable degree, so that as the numbers obtained become larger, the approximation to the theoretical ratio becomes constantly closer, and *in the long run* we may say that the heterozygote *does* produce equal numbers of gametes of the two classes considered.

In order to show that Mendelian ratios are due simply to the chance combinations of gametes, the following experiment, suggested by Professor Castle, was tried. Equal measured quantities of small white beans were placed in two bags of the same shape and size. One-half the beans in each bag were marked with a spot of red ink, and the other half were left unmarked, so that the former stood for gametes having the character pigmentation, and the latter for gametes having the character albinism. The contents of each bag, when thoroughly mixed, were supposed to represent an indefinite number of the two sorts of gametes produced in equal proportions by a heterozygote. If, now, a single bean be taken at random from each bag, three combinations are possible: (1) both beans may be spotted, representing a pure D individual; (2) one may be spotted, the others unspotted, representing a heterozygous, D(R), individual; or (3) both beans may be unmarked, representing an albino, R. For comparison the same number of drawings was made as there are individuals represented in Table G (page 84) of the progeny of heterozygotes from spotted and albino parents. The total number of these young was 214, of which 159 were pigmented and 55 were albinos, so that the two classes are practically as 3 : 1. Three series of drawings were made. In the first and second of these, the number of beans in each bag was indefinite, but in the third series, the beans representing the gametes of the female parent were made only twice as many as the number of young actually occurring in any given litter of the table and, of course, represented equal numbers of the two sorts in each case. The totals for each of the three series of drawings agreed very closely with the numbers actually obtained with the mice, the agreement being almost perfect in the case of the third drawing, in which the conditions of the experiment were made to correspond most closely with those in the mice. In a small number of consecutive drawings there was sometimes a considerable excess of one class or another over its expected ratio, but these discrepancies tended to balance one another in the long run, just as observed with the mice. Table E gives a summary of these results.

TABLE E.

	Pigmented, D D(R)	Albino, R
Result with the mice (Table G)	159	55
First drawing, number of gametes indefinite	59 + 110 = 169	45
Second " " "	48 + 113 = 161	53
Third drawing, gametes of female double the number of offspring	53 + 107 = 160	54
Expectation	53.5 + 107 = 160.5	53.5

This table shows that the chance unions of the two sorts of gametes, when produced in equal proportions by both parents, give the three possible classes, D, D(R), and R in approximately the proportions 1 : 2 : 1.

A similar series of drawings was made for Table II (page 87), which represents the offspring of heterozygous and albino parents. In this case, where an equality of the two classes is expected, one bag contained only unmarked beans representing gametes of the albino parent. The following table (F) gives the results obtained:

TABLE F.

	Pigmented, D(R).	Albino, R.
Result with the mice (Table II)	69	69
First drawing, number of gametes indefinite	58	80
Second " " "	72	66
Third drawing, gametes of female double the number of offspring	74	64
Average for three drawings	68	70
Mendelian expectation	69	69

2. *Spotted mice and albinos.* In a former paper it has been shown (Castle and Allen, :03) that the piebald mouse, having areas of pigmented and non-pigmented integument, is a distinct type, or "mutation," and the gametes formed by such an animal are in the nature of a mosaic of the two alternative characters. Such mosaic gametes act in heredity as unit characters, and the pigment-albino mosaic is dominant over gametes representing complete albinism. Crosses in which one of the parents is a spotted animal and the other a complete albino may therefore be legitimately considered in investigating the behavior of the unpigmented condition toward a character dominant over it.

In the experiments of the writer, a number of young were obtained by breeding together pure mosaics and albinos. The result was in every case complete dominance of the pigmented condition; in most cases the recessive element of the mosaic disappeared altogether from the soma. The behavior of the mosaic condition will be taken up in greater detail farther on. The important point in the present connection is that the albino character contained in one of the conjugating gametes becomes recessive towards the mosaic character, so that from the union just de-

TABLE G.

No.	Parents of Spotted Stock.	Pigmented Young.	Albinos.	Expected No. of Albinos.
1	♂ gr. 31 a × ♀ gr. 45	2	. .	.50
2	“ “	3	1	1
3	“ “	. .	1	.25
4	“ × ♀ gr. 33	2	1	.75
5	“ “	. .	1	.25
6	“ “	1	3	1
7	“ “	3	2	1.25
8	♂ gr. 82, 84 × ♀ gr. 81, 88	2	. .	.50
9	“ “	1	. .	.25
10	♂ gr. 31 a × ♀ gr. 33	2	. .	.50
11	“ “	2	2	1
12	♂ gr. 82, 84 × gr. ♀ 81, 86	3	1	1
13	“ “	3	. .	.75
14	“ “	2	1	.75
15	“ “	1	. .	.25
16	“ “	2	. .	.50
17	“ “	1	2	.75
18	“ “	2	. .	.50
19	“ “	1	. .	.25
20	“ “	3	1	1
21	♂ bl. 99 × ♀ bl. 94, 98, 101	4	5	2.25
22	♂ gr. 1 × ♀ gr. 3	1	2	.75
23	♂ gr. 1 × ♀ gr. 68	3	1	1
24	“ “	2	1	.75
25	♂ gr. 4 × ♀ gr. 64	3	1	1
26	“ “	3	1	1
27	“ “	6	. .	1.50
28	♂ gr. — × ♀ gr. 87	5	4	2.25
29	“ “	5	. .	1.25
30	♂ gr. 40 × ♀ gr. 38	2	2	1
31	“ “	1	2	.75
32	“ “	4	1	1.25
33	♂ bl. 103 × ♀ bl. 106	5	1	1.50
34	“ “	5	1	1.50
35	♂ gr. 82 × ♀ gr. 81	2	1	.75
36	“ × ♀ gr. 81, 86, 114	3	. .	.75

TABLE G (*continued*).

No.	Parents of Spotted Stock.	Pigmented Young.	Albinos.	Expected No. of Albinos.
37	♂ bl. 99 × ♀ bl. 98	2	..	.50
38	♂ bl. 342 × ♂ bl. 344	5	1	1.50
39	" "	4	2	1.50
40	" "	3	1	1
41	♂ ch.-wh. 402 × ♀ bl.-wh. 403	4	1	1.25
42	♂ gr. 369–374 × ♀ gr. 375	2	..	.50
43	♂ gr. 409 × ♀ gr. 411	5	2	1.75
44	♂ bl. 321 × ♀ bl. 316	3	..	.75
45	♂ ch. 392 × ♀ ch. 393	3	2	1.25
46	♂ gr. 374 × ♀ gr. 418, 419, 421	4	..	1
47	♂ gr. 369, 371, 374 × ♀ gr. 375	3	..	.75
48	♂ gr. 409 × ♀ gr. 411	5	3	2
49	♂ gr. 374 × ♀ gr. 418, 419, 421	7	1	2
50	♀ bl. 92 × ♀ gr. 91, 93, 97	3	..	.75
51	" "	1	..	.25
52	" "	4	1	1.25
53	" "	2	1	.75
54	" "	4	1	1.25
55	" "	2	..	.50
56	" "	3	..	.75
		159	55	53.5

scribed no albino offspring result. The nature of the resulting offspring is discussed under a separate heading.

In these experiments two black-white males were bred to a number of albino females, and all of the young (forty-three in number) were pigmented. A third black-white mouse, a half-brother to the two just mentioned, was also bred to the same stock of females, and produced pigmented as well as albino young. He, therefore, had the albino character recessive, and was what we have called a hybrid mosaic. All the *pigmented* mice obtained from these three animals bred to albinos are, by formulae (1) and (6), hybrid in respect to the mosaic and the albino characters, and should form gametes one-half of which have the dominant (mosaic) character only, and one-half the albino character only. When these pigmented hybrids are intercrossed, the Mendelian expectation is (formula 2) that 25 per cent of the young will be albinos. The following table shows by litters the numbers and sorts of young obtained from all the crosses in which the two parents were hybrids between the black-white stock and albinos.

This table shows that out of a total of 214 mice obtained there are actually 55 albinos, where 53.5 are expected. The experimental result is thus closely in accord with the theoretical.

As in the case of the heterozygotes of house-mouse and albino parentage, the relative numbers of pigmented and albino young *in the individual litters* are rarely those demanded by the Mendelian formula. In fact, the precise theoretical ratio obtains in but 7 out of the 56 litters (Nos. 2, 12, 20, 23, 25, 26, 40), where in each case there were three pigmented and one albino young. In two litters (Nos. 25, 26) from ♂ gr. 4 × ♀ gr. 64 this precise proportion was maintained, but a third lot of six young (No. 27) by the same parents consisted wholly of pigmented mice. Cases in which all the young at a birth are albinos are practically wanting in these experiments, being limited to two litters (Nos. 3, 5), where but a single albino represented the entire progeny. Among the 56 litters, however, there is occasionally an excess of albinos over pigmented young, the most notable case, perhaps, being one (No. 6) in which the expected ratio was precisely reversed. There were in this litter 3 albinos to 1 pigmented individual. In all but 9 of the 56 lots of young, there was a greater or less preponderance of pigmented mice, which is a result of the *nature* expected. Two of these 9 litters (Nos. 11, 30) showed an equality of the two classes, the total number of young in each of the two cases being four. In one large litter (No. 28) of 9 young, four were albinos; in another (No. 49) of 8 young, but one was an albino. In the latter case, however, the excess of pigmented mice is but one. It is obvious that in cases where the litter consists of an odd number of individuals, it is impossible in that particular case to obtain a 3 : 1 ratio of the two classes. Hence in a lot of 5 young the nearest approach to the Mendelian ratio is 4 pigmented to 1 albino. In this case there are but .25 too few albino individuals. If two of the five young are albinos, the excess is still represented by less than an integer. Hence the irregularity in distribution of the two characters is a thing to be expected in small litters. In a long series of litters, however, the discrepancies noticed in individual lots tend to counterbalance one another, so that the final result shows a close approximation to the 3 : 1 ratio. This is seen also in cases where a number of young are obtained from *the same pair*. For example, ♂ bl. 342 × ♀ bl. 344 produced two litters, the one (No. 38) of 5 pigmented to 1 albino,

TABLE H.

No.	Parents.	Pigmented Young.	Albinos.	Expected No. of Albinos.	Ratio, Pigmented : Albino.
1	♂ bl.-wh. 1 · 4 a × ♀ wh.	. .	2	1	18 : 25
2	“ “	3	1	2	
3	“ “	2	2	2	
4	“ “	1	6	3.5	
5	“ “	2	5	3.5	
6	“ “	3	2	2.5	
7	“ “	3	4	3.5	
8	“ “	2	2	2	
9	“ “	2	1	1.5	
10	♂ bl. 35 × ♀ wh. 34	4	1	2.5	
11	♂ bl. 60 × ♀ wh. 22	1	2	1.5	
12	♂ bl. — × ♀ wh. —	4	5	4.5	8 : 7
13	♂ gr. — × ♀ wh. —	4	2	3	
14	♂ wh. 229 × ♀ gr. 105	. .	1	.5	2 : 3
15	“ “	2	2	2	
16	♂ bl.-wh. 240 × ♀ wh. 217	1	2	1.5	
17	♂ gold.-ag. 314 × ♀ wh. —	3	4	3.5	7 : 5
18	“ “	4	1	2.5	
19	♂ bl.-wh. 227 × ♀ wh. 336	3	3	3	
20	♂ bl.-wh. 265 × ♀ wh. —	1	1	1	
21	♂ gold.-ag. 307 × ♀ wh. 141	2	2	2	
22	♂ gr. 40 × ♀ wh. 278	3	2	2.5	5 : 6
23	“ “	1	1	1	
24	“ “	1	3	2	
25	♂ bl.-wh. 265 × ♀ wh. 121	1	1	1	
26	♂ wh. 256 × ♀ gr. 45, 64, 68	1	1	1	
27	♂ gr. 406, 408 × ♀ wh. 281	1	. .	.5	
28	♂ ch.-wh. 362 × ♀ wh. 366	3	2	2.5	
29	♂ wh. 413 × ♀ bl. 416	4	2	3	
30	♂ ch. 274 × ♀ wh. 366	1	1	1	
31	♂ gr. 449 × ♀ wh. 447	3	3	3	
32	♂ ch.-wh. 362 × ♀ wh. 366	3	2	2.5	
		69	69	69.0	

the other (No. 39) of 4 pigmented to 2 albino mice. Neither litter shows the ideal proportion, but taken together the total result is 9 pigmented to 3 albino mice, the exact Mendelian ratio. Other cases show less close approximation, though the totals for several pairs come within a very small number of the 3 : 1 expectation.

It is clear, then, that these heterozygotes from the spotted stock crossed with albinos tend to produce the two sorts of gametes in equal numbers, so that when intercrossed the ratio of pigmented to albino young is approximately 3 : 1.

Table II gives in condensed form the result of a number of crosses in which an albino was bred to a heterozygote of spotted and albino parentage. In all these cases, the expected result is an equality of the albino and the pigmented classes, by the Mendelian formula (6). In 11 out of 32 litters this expectation is precisely fulfilled. Sixteen of the litters contain an uneven number of young, so that exact equality of the two classes is not possible in the separate lots, though their totals aggregate 38 pigmented to 40 albino mice. The totals for both classes show a very close approximation to the 1 : 1 ratio which the Mendelian law requires. As seen in the other tables, the separate litters of young frequently show a considerable deviation from the 1 : 1 ratio of the two classes. In only two cases (Nos. 1, 14), where the numbers were very small (2 and 1 respectively), were there no pigmented individuals. In ten litters there was an excess of pigmented young, while in ten others the albinos were in excess. The excess in the latter group, however, was usually small, ranging from 0.5 to 1.5 individuals, save in one case (No. 4), a litter of 1 pigmented and 6 albino young, where the latter were 2.5 too many. The discrepancies tend to counterbalance each other as the totals grow larger, so that in the entire lot of 138 individuals there is actual equality. We may, then, conclude that heterozygotes, whether male or female, arising from a cross between a pied and an albino stock, tend to produce in equal numbers gametes representing the pigmented and unpigmented conditions.

An account of the writer's experiments on the heredity of complete albinism having been given in what precedes, a review is now added of experiments along similar lines recorded by others.

Crampe (**'77**) gives the partial results of certain crosses with rats, and in a later, more elaborate paper (**'85**) gives a summary of his crosses for a period of some ten years. This latter paper, though containing an immense accumulation of data, is unfortunately so obscure in many places as to cause considerable uncertainty as to what sorts of animals were used in the various crosses. It seems clear that in many cases three stocks, a wild, a pied, and an albino, were interbred, whereas in other cases only two of these stocks were kept together. After a long study of Crampe's tables, the writer has been able to extract from them a number of cases in which it is evident that both parents concerned were pied or mosaic animals having the albino character recessive, as evidenced by the presence of albinos among their young. Crampe seems to have kept some of his rats together in "families," by which term he refers to certain lots. Others seem to have been single pairs. The following Table (I) shows in condensed form the results of these crosses. Other crosses, by which albinos were produced, are considered under a separate head, as the families appear to have had three characters among them, instead of the two here treated.

TABLE I.

No.	Color of Parents.	Gray-white Young.	Black-white Young.	Albinos.	Expected No of Albinos.	Excess of Albinos = +. Shortage of Albinos = —.
1	♂ bl.-wh. × ♀ bl.-wh.	..	..	2	.5	+1.5
2	" "	..	4	2	1.5	+ .5
3	" "	..	12	1	3.25	−2.25
4	" "	..	6	2	2	±
5	" "	..	6	1	1.75	− .75
6	" "	..	17	6	5.75	+ .25
7	♂ gr.-wh. × ♀ gr.-wh.	19	8	5	8	−3
8	" "	16	5	12	8.25	+3.75
9	" "	8	9	4	5.25	−1.25
10	" "	8	4	6	4.5	+1.5
11	" "	24	5	7	9	−2
12	" "	13	..	3	4	−1
		88	76	51	53.75	−2.75

In these crosses, it is plain that the numbers of young given for the respective pairs represent in some cases the totals of successive litters, while in others they indicate single litters. In each case, however, the theoretical number of albinos is very nearly realized, and although there is exact agreement in but one case (No. 4), the shortage or excess in any one lot was not over 3.75, and this (excess) occurred in but a single case (No. 8). Moreover, as previously pointed out, an exact 3 : 1 ratio of pigmented to albino mice is rarely to be expected experimentally, and even then only in lots where the number of young is 4 or a multiple of 4. Hence, it is only " in the long run " that the theoretical ratio may be expected to obtain closely. The totals in this case are 164 pigmented to 51 albino rats (or 23.5 per cent albinos), so that the latter class is but 2.75 individuals, or 1.5 per cent, short of the expected total. The irregularities in respect to the numbers of albinos are very evenly divided among the eleven lots in which the 3 : 1 ratio was not realized. For in five lots (Nos. 1, 2, 6, 8, 10), there was an excess of albinos ranging from 0.25 to 3.75 individuals, with a total of 7.5, as against six lots (Nos. 3, 5, 7, 9, 11, 12), showing a shortage in the number of albinos, ranging from 0.75 to 3 individuals, or a total of 10.25 individuals. Crampe's rats therefore show that complete albinism is inherited among them precisely after the Mendelian principles, though with small numbers the exact Mendelian ratio may not be closely approximated. No extended series of crosses in which the ratio of pigmented to albino young is expected to be as 1 : 1 is given by Crampe.

In two noteworthy communications von Guaita ('98, : 00) gives his results in breeding black-white dancing mice to albinos of an inbred stock. His experiments were performed manifestly without reference to the Mendelian principles, but nevertheless when the results are examined, they afford additional proof in support of those principles.

Castle (: 03^d, p. 226) has already shown that as regards the numbers of pigmented and albino young obtained by von Guaita there is almost perfect accord with the total expected on the Mendelian hypothesis that the two characters undergo segregation. The following table (J) is a slight expansion of the one that Castle has constructed, and shows the numbers of albinos expected and those actually obtained, as well as the numbers of pigmented mice. In some cases, the total number of young obtained from a single pair of mice is not sufficient to show conclusively whether or not either of the parents had the albino character recessive, and for these cases an allowance has to be made ; yet their effect on the total result is insignificant. The Mendelian formula applicable to each case is indicated by the figures in parenthesis in the proper column.

As regards the inheritance of complete albinism, it is seen that from the various crosses, producing in all 309 mice, a total of 77 albinos occurred, where the expectation was that there should be from 75 to 79.

TABLE J.

(Based on Observations of von Guaita.)

Serial No.	Generation.	Pair.	Gray Young.	Black Young.	Gray-white Young.	Black-white Young.	Albinos.	Expected No of Albinos.	Expected Per cent of Albinos.	Mendelian formula applicable.
1	F_1	..	28	..	..	..	..	0	0	(1)
2	F_2	1	5	..	3	1	4	3.25	25	(2)
3	"	2	7	2	4	..	6	4.75	25	(2)
4	"	3	2	..	2	..	..	1	25	(2)
5	"	4	2	2	..	..	4	2	25	(2)
6	F_3	1	..	..	..	..	4	4	100	(4)
7	"	2	13	..	3	..	..	0	..	(1)
8	"	3	..	..	2	..	..	0 or .5	0 or 25	(3, 2, 5)
9	"	4	2	4	..	1	..	0	0	(1)
10	"	5	1	..	..	..	1	1	50	(6)
11	F_4	1	..	..	16	..	..	0	0	(3, 5)
12	"	2	..	..	..	..	..	..	..	eaten.
13	"	3	..	1	..	2	2	1.25	25	(2)
14	"	4	..	10	..	15	7	8	25	(2)
15	"	5	1	1	4	7	..	0	0	(3, 5)
16	"	6	..	..	35	..	9	11	25	(2)
17	"	7	2	..	..	..	1	.75	25	(2)
18	F_5	1	1	..	21	..	10	8	25	(2)
19	"	2	..	..	..	..	6	6	100	(4)
20	"	3	..	7	..	16	20	21.5	50	(6)
21	"	4	..	1	..	1	..	0 or .5	0 or 25	(3, 2, 5)
22	"	5	1	1	1	..	..	0 or .5	0 or 25	(3, 2, 5)
23	F_6	1	..	..	4	..	..	0 or 2	0 or 50	(1, 6)
24	"	2	..	..	..	..	..	..	..	eaten.
25	"	3	..	..	..	..	3	3	100	(4)
			65	29	95	43	77	75–79		

This range of 4 individuals in the theoretical total is due to the fact that in three, or possibly four, cases the litters were so small that they did not show conclusively whether or not the parents were producing gametes having the albino character. For it is obvious that where both parents are pigmented, the expectation is but one albino in four young, provided both parents have the albino character recessive, hence where less than four young are obtained, and no albinos are among them, it cannot be told, *from this test alone,* whether one or both of the parents produce the two sorts of gametes. For if *one* parent produces only gametes having the pigment character, no albino young will result even though the other parent produces the two sorts of gametes (formula 5). The three cases of this sort among von Guaita's crosses (Nos. 8, 21, 22) necessitate making a possible allowance of from .5 to 1.5 individuals in the theoretical total. Another litter (No. 23) for which allowance may be made is one the parents of which were a spotted and an albino mouse. Had the former contained the albino character in a recessive condition, the theoretical result would have been 50 per cent of pigmented and 50 per cent of albino individuals. To be sure, all of the four young resulting from the cross are pigmented, yet in the tables previously given one sees that it is possible to obtain in a single litter, from a cross of this sort, four pigmented individuals with *no* albinos. Hence, if the greatest possible allowance be made, the actual result is but 2 individuals short of the theoretical, while the most favorable allowance would give an exact agreement.

It may be well to explain in brief the methods by which the above conclusions as to the expected proportions of albinos were reached, and also to analyze the results still further. Generation P (the original parents crossed — Bateson) consisted entirely of crosses in which the one parent (the spotted mouse) possessed only the dominant character and the other (the albino mouse) only the recessive character. In all cases, therefore, by formula (1), heterozygotes are produced, and no albinos. The second generation (F_2) was obtained by interbreeding these heterozygotes (F_1). Assuming that segregation of pigment and albino characters takes place in accordance with Mendel's law, it is expected (by formula 2) that 25 per cent of the resulting offspring will be albinos. The fours pairs of heterozygotes (Nos. 2–5) did actually produce albinos except in one case (No. 4), where but four young were born. In a total of 44 individuals of this generation there were actually 14 albinos, an excess of 3 over the theoretical number. Generation F_3 consists of the young of five pairs selected from the preceding generation. The first of these pairs was of two albinos, and only albino young resulted.

The second, fourth, and fifth pairs consisted each of a pigmented and an albino mouse. The only circumstance under which albinos might result from these three pairs is that the spotted parent should have the albino character recessive, when 50 per cent of the resulting offspring should be albinos. Two of these pairs produced, in a total of 16 and 7 young respectively, no albino young, hence it is safe to assume that the pigmented parents were in each case pure, as regards the albino character, and the result falls under formula (1). In the case of pair 5, however, the pigmented parent evidently did contain albinism in a recessive condition, and the litter of *two* young shows the Mendelian equality demanded by formula (6). The third pair consisted of two spotted mice. There are here three possibilities. (1) Both parents may contain albinism recessive, in which case 25 per cent of the young produced should be albinos; (2) only one parent or (3) neither parent may contain that character, so that *no* albinos could occur (by formulae 5 and 3). Since but two offspring were obtained, it cannot be certainly told which expectation is the correct one, for at least four young are theoretically necessary in order that the albino class may be actually present in the proportion of 1 albino to 3 pigmented mice. Generation F_4 was produced by seven pairs, though only six of these contributed to the record given. In each of the six cases, the two parents were pigmented animals, so that (as in the third pair of F_3) three possible unions are represented. In case albinos are produced by any pigmented pair, it is evident, on Mendelian principles, that both parents are producing gametes with the albino character as well as those with the pigment character. Four of the six pairs which form this generation are of this sort, hence, both parents being heterozygotes, 25 per cent of their offspring are expected to be albinos (formula 2). These four pairs produced 84 young, of which 19 were albinos, that is, the albino class shows a shortage of but 2 individuals, or about 2 per cent. In case of the two remaining pairs, only pigmented young were obtained, 16 and 13 respectively. Had all the four parents contained albinism recessive, there would have been expected 4 and 3 albinos respectively among the offspring of the two pairs. Since no albinos occurred among the totals, it is safe to assume that at least one parent of each pair was pure with respect to the character albinism. Possibly both parents in each case were pure in this respect, but the result is the same in so far as the production of albinos is concerned, for none are expected in either event.

Generation F_5 results from the interbreeding of five pairs of mice from generation F_4. Both animals of the first, fourth, and fifth pairs were pigmented, hence in order that the albino class shall be represented in their progeny it is necessary that both mice of each of the three pairs shall be heterozygotes, having the albino character recessive. This condition evidently obtained in case of pair 1, for out of a total of 32 young, 10 were albinos, where but 8 were expected. Neither of the two other pairs produced albinos, but their totals are too small to show conclusively the nature of the parents, the one pair having but two, the other three young. Pair 2 of this generation consisted of two albinos, and as expected, all their young were of that class. Pair 3 consisted of a spotted and an albino mouse. The former evidently had the albino character recessive, for the result of the cross with the albino mate was to produce 23 pigmented and 20 albino mice. This result is less than two individuals short of the expected equality of the albino and the pigmented classes.

Finally, for generation F_6 there are three litters, one of which was destroyed before its character could be determined. Of the remaining two litters, one consists of 3 albinos from albino parents, the expected result. The other pair is of a spotted and an albino mouse. In this case no albinos are expected provided the spotted parent is pure as regards the albino character. As a matter of fact, none of the four young obtained was an albino. Nevertheless it is within the limits of possibility, as the writer's experiments have shown, to obtain no albinos in a litter of four, even when 50 per cent of the young are expected to be of this sort. Hence it is necessary to make allowance for the possibility that the spotted parent may have been a heterozygote containing the albino character recessive.

The crosses made by von Guaita, as far as they throw light on the heredity of complete albinism, may be summed up as follows: (1) In all cases wherein pure pigmented (spotted) animals were mated with albinos the pigmented character of the former proved dominant over total albinism. (2) Unions of two albinos resulted in the production of albino offspring only. (3) In cases where both parents were proved to be heterozygous with respect to the two characters of coat color, 43 albinos to 117 pigmented mice were produced, the albinos being thus 3 individuals, or less than 2 per cent, in excess of the theoretical number. If the three indeterminable cases be added, the number of albinos is but one individual less than that expected. (4) In cases where it is expected that the two classes shall be as 1 : 1, this ratio is very closely approximated, the totals being 24 pigmented : 20 albino mice. It is clear, then, that von Guaita's mice behaved according to the Mendelian expectation, and as far as known to the writer, his experiments are the earliest pub-

TABLE K.

(Based on observations of Parsons and Copeman.)

No. of Exp.	Parents.	Black.	Black-white.	Brown.	Brown-white.	Albinos.	Expected No. of Albinos.	
26	♂ bl.-wh. × ♀ br.-wh.	..	..	..	1	1	.5	+ .5
27	" "	..	2	..	2	2	1.5	+ .5
32	♂ bl.-wh. × ♀ bl.	5	..	..	..	2	1.75	+ .25
33	" "	7	1	..	..	..	2	−2
34	" "	2	2	1	..	..	1.25	−1.25
37	♂ br.-wh. × ♀ br.	2	..	1	..	3	1.5	+1.5
38	" "	..	..	3	2	3	2	+1
42	♂ bl.-wh. × ♀ bl.-wh.	..	7	..	..	..	1.75	−1.75
43	" "	..	4	..	..	2	1.5	+ .5
44	♂ br.-wh. × ♀ bl.-wh.	..	7	..	..	1	2	−1
		16	23	5	5	14	15.75	

lished from which it is possible to obtain statistical proof of the validity of Mendel's laws among mice.

In three recent papers, Darbishire (:02, :03, :03^a) has summarized certain of his results in crossing spotted "fawn-white" Japanese waltzing mice with albino mice, and these observations, too, confirm the law of segregation, in that the crosses show complete albinism to be inherited in the proper Mendelian proportions. In the first cross of the two stocks, only heterozygotes resulted in accordance with formula (1). In his second paper (:03) the results of intercrossing these heterozygotes are given, and it is stated that in a total of 66 young, 13 are albinos, where 16.5 are expected; 225 young are obtained by back-crossing heterozygotes with albinos, and of these 119 are albinos, the expectation being 112.5. The total number of albinos is thus 132, where the expectation is 129. Hence a fairly close agreement is shown. This author's final report is awaited with interest, and is expected to shed further light on questions of Mendelian inheritance.

Finally, Bateson (:03^b), in a discussion of color heredity among rats and mice, adduces 50 matings of mice, contributed by Mr. F. J. Parsons in conjunction with Dr. S. M. Copeman. In Table K are summarized the cases in which both parents were heterozygotes and produced both pigmented and albino young.

These results are quite similar to those previously reviewed. In a total of 63 mice, the albinos appear in practically their calculated proportion, being 14 in number instead of 15.75, which is as close an agreement as might reasonably be demanded. In six of the ten litters, the albinos are in excess, the total excess being 4.25 individuals, yet this is more than offset by the shortage in the four remaining litters, amounting to 6 individuals. In one case, no albinos occurred in a litter of 8, and again none in a lot of 7 young. In other cases the two classes, pigmented and albino, occurred in equal numbers in separate litters, though in none did the albinos outnumber the pigmented mice. In 21 cases, albino mice were bred to heterozygous individuals, so that 50 per cent of the offspring are expected to be albinos. The following table (L) shows the results of these matings.

As pointed out by Bateson, the expected equality of the pigmented and the albino classes is here realized. One black-white male bred to several albino females produced in all 27 pigmented to 31 albino young; and a brown-white male under similar circumstances produced 11 pigmented to 19 albino mice. In these two cases the 1 : 1 ratio is rather closely approximated. On the other hand, four heterozygous females, bred to albino males, give 16 pigmented and 7 albino young, which Bateson believes can hardly be a fortuitous departure from the expected equality. Yet the departure is actually but 4 or 5 individuals, and may be entirely a matter of chance. For even the black-white male, previously mentioned as producing a very nearly equal number of the two sorts of gametes, at one time fathered a litter consisting of 5 albinos by an albino female. Indeed, the record of this male shows that although in the long run the two sorts of gametes are produced in relatively equal numbers, yet, as pointed out in other cases, separate litters show that there is considerable variation in the proportions of the two sorts of gametes which are fertilized at each mating.

To sum up, it would seem to be conclusively proved that among house mice, fancy mice, and rats, complete albinism is inherited strictly in accordance with the Mendelian principles. Investigations have up to this time shown no clear cases of deviation from the common 3 : 1 and 1 : 1 ratios of pigmented to albino individuals when any considerable number of offspring is taken into consideration.

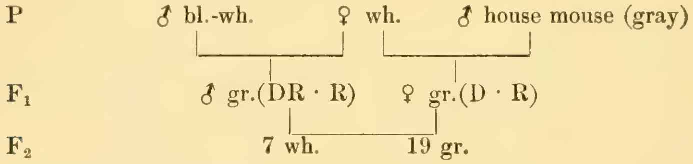
(3) *Complete albinism and two other coat characters.* In the cases

TABLE L.

(Based on observations of Parsons and Copeman.)

No. of Exp.	Parents.	Black	Black-white.	Brown.	Brown-white.	Albinos.	Expected No. of Albinos
1	♂ bl.-wh. × ♀ wh.	..	1	..	..	3	2
3	♂ br.-wh. × ♀ wh.	..	..	..	1	3	2
4	♂ bl.-wh. × ♀ wh	..	2	..	1	4	3.5
6	♂ br.-wh. × ♀ wh.	..	..	..	..	5	2.5
7	" "	..	..	..	4	1	2.5
10	♂ bl.-wh. × ♀ wh.	2	..	1	..	1	2
12	♂ br.-wh. × ♀ wh.	..	..	2	..	6	4
13	" "	2	..	2	..	4	4
14	♂ bl.-wh. × ♀ wh.	3	..	1	..	2	3
15	" "	..	2	..	2	4	4
16	" "	..	1	..	1	2	2
18	" "	..	2	..	..	3	2.5
19	" "	..	2	..	..	1	1.5
21	" "	..	4	..	..	3	3.5
23	" "	..	5	..	..	2	3.5
28	♂ wh. × ♀ br.-wh.	..	1	..	4	1	3
35	♂ wh. × ♀ bl.	1	3	1	..	2	3.5
39	♂ wh. × ♀ br.	1	1	..	1	2	2.5
40	♂ bl.-wh. × ♀ wh.	..	2	..	..	3	2.5
41	" "	..	..	..	..	5	2.5
47	♂ wh. × ♀ bl.	1	2	..	..	2	2.5
		10	28	7	14	59	59.0

thus far considered the heredity of complete albinism has been studied with respect to its behavior when brought into association with either total pigmentation or partial pigmentation. It has been shown that, when the wild house mouse or the wild brown rat is crossed with an albino, heterozygotes are produced in which the character, total pigmentation, is dominant, and complete albinism is recessive. In the case of the spotted varieties of mice and rats it is also found that the pigmented condition is dominant over complete albinism. In both cases the heterozygotes produce, on interbreeding, the Mendelian proportions of pigmented and of albino individuals. This is true, furthermore, where two heterozygotes are mated, of which the one had a wild totally pigmented mouse as the colored parent, and the other a spotted or partially pigmented one. To illustrate: the writer bred house mice to albinos and obtained gray heterozygotes. Black-white mice were also bred to the same stock of albinos, and gray heterozygotes were obtained similar in outward appearance to those from the other cross. The following diagram shows this parentage:



The heterozygotes of generation F_1 were then interbred, and as each contained complete albinism recessive, it is expected that 25 per cent of the young of generation F_2 would be albinos. The actual result was that, of 26 young produced, 19 were gray, and 7 were albinos. The albinos are thus but .5 of an individual in excess, so that, within the possibilities of experimentation, the expectation is realized. Some of the pigmented mice showed small flecks of white on tail and feet, but unfortunately no exact record of this was kept.

A different result is to be expected on Mendelian principles, if, instead of *both* animals of generation F_1 having complete albinism recessive, only *one* is of this nature. If we postulate that the other parent shall be a homozygote, two conditions are now possible which will allow the occurrence of three coat characters in generation F_2: either (1) the heterozygote must produce gametes representing respectively the *spotted* and the *albino* conditions, while the homozygote's germ cells contain only the character total pigmentation; or (2) the gametes of the heterozygote must represent in equal numbers the characters total pigmentation and albinism, while those of the homozygote represent only the spotted character. If matings such as these be made, the offspring will be of two sorts, *half* of them having complete albinism recessive, while the other half are heterozygous with respect to total and partial pigmentation. If these young be now intercrossed in all possible ways, the final result will be that only 1 in 16 individuals will be albino, instead of 1 in 4. To make this more clear, the two possible cases are worked out below, D representing the dominant condition of total pigmentation as seen in the wild house mouse or rat; DR, the mosaic or spotted condition in which albinism occurs in the soma between pigmented patches; and R representing the albino condition. A period between two of these signs used together shows that segregation takes place at that point and the resulting gametes are of the nature expressed by the symbols on the right and the left of the period respectively.

Case I.

Heterozygote = $DR \cdot R$; homozygote = $D \cdot D$. This mating gives the following result:

$$\begin{array}{ll} DR + R & = \text{gametes of heterozygote,} \\ D + D & = \text{`` \quad `` homozygote,} \\ \hline 2D \cdot DR + 2D \cdot R & = \text{offspring, or generation } F_1. \end{array}$$

If, now, these F_1 animals be allowed to breed *inter se*, three combinations (*a.*, *b.*, *c.*) are possible, and the smallest number of individuals representing all the possible unions in their probable proportions is 16, thus:

a.

Parents both $D \cdot DR$

$$\begin{array}{c} D + DR \\ D + DR \\ \hline D + 2D \cdot DR + DR \end{array}$$

b.

Parents both $D \cdot R$

$$\begin{array}{c} D + R \\ D + R \\ \hline D + 2D \cdot R + R \end{array}$$

c.

Parents one $D \cdot DR$, the other $D \cdot R$

$$\begin{array}{c} D + DR \\ D + R \\ \hline 2(D + D \cdot DR + D \cdot R + DR \cdot R) \end{array}$$

There is one chance each for mating *a* and *b*, where there are two chances for mating *c*. If all these unions take place in their chance frequencies, there will result six sorts of animals as follows:

$$4D + 1DR + 4D\cdot DR + 4D\cdot R + 2DR\cdot R + 1R.$$

Of the 16 individuals, 9 (or those of the first three classes just enumerated) do not have albinism recessive; 6 (or the next two classes) do have that character recessive, while the 16th animal is a pure recessive.

Case II.

Here the heterozygote is D·R, and the homozygote is DR. The result is the same as in Case I, save that there is a slightly different proportion of certain of the pigmented classes, as may be seen from the following:

$$\begin{array}{cl} D + R & = \text{gametes of heterozygote,} \\ DR + DR & = \text{`` \quad `` homozygote,} \\ \hline 2D\cdot DR + 2DR\cdot R & = \text{offspring, or generation } F_1. \end{array}$$

The possible unions among the animals of generation F_1 are:

$$\begin{array}{c} a. \\ D + DR \\ D + DR \\ \hline D + 2D\cdot DR + DR \end{array} \qquad \qquad \begin{array}{c} b. \\ DR + R \\ DR + R \\ \hline DR + 2DR\cdot R + R \end{array}$$

$$\begin{array}{c} c. \\ D + DR \\ DR + R \\ \hline 2(D\cdot DR + DR + D\cdot R + DR\cdot R) \end{array}$$

Total of F_2:

$$1D + 4DR + 4D\cdot DR + 2D\cdot R + 4DR\cdot R + 1R.$$

The writer has not been able fully to test experimentally the relations here worked out, to see if they actually occur under proper crossing. In one instance, however, a chocolate-white male, a pure DR, was bred to a gray heterozygote whose parents were a house mouse and an albino. This mating is therefore of the sort explained under Case II, above. Two litters were obtained from this pair, the one of 5, the other of 6 young. All these 11 young were gray. The individuals of the second litter were allowed to grow up and breed together, in order that they might show, if possible, what result would come of their random mating. When about five months old these six heterozygotes produced two lots of young. The first lot consisted of a chocolate and an albino mouse;

the second was of 6 entirely gray mice. Referring again to the example given under Case II, it is seen that the six gray mice of generation F_1 are theoretically of two sorts, D·DR and DR·R. The only union by which albinos could be obtained from these parents is that of two DR·R individuals. That it *was* this union which produced the litter of 1 chocolate and 1 albino, seems to be additionally proved by the occurrence of the chocolate individual. No case is as yet recorded in which the house mouse in the second generation from a cross with albinos has produced chocolate young, hence it is more than likely that the young one of this color inherited that character directly from the chocolate-white grandfather ; that is, either two DR gametes or a DR and an R gamete united to produce it, so that its parents were both of the DR·R class. The fact that there were no albinos among the 6 individuals of the second litter probably indicates that the parents were in this case either two D·DR individuals or a D·DR and a DR·R., neither of which combinations would result in the production of albino offspring.

As a means of testing the relations of the three characters involved in a case of this sort, some of the statistics given by Crampe ('85) appear to be applicable. This investigator details a number of litters obtained by inbreeding the members of a single family, wherein there is evidence that the three characters, which have just been considered, were present. Thus on page 604 he gives a table containing the results of inbreeding derivatives of "species" by albino (Compare Table M). Since there occur numerous spotted young in the progeny, it is hardly to be doubted that what Crampe thought was the "species" was really a heterozygous (D·DR) individual containing both the character total pigmentation and that of partial pigmentation. This interpretation is rendered still more probable by the fact that these animals were of "variety 6," *i.e.* black with traces of white on toes or tail, a condition which is often seen in mice from wholly pigmented and spotted parents. Another table of the same kind (see Table N) is given on p. 603 of the "Farben-Inzucht, Farben-Reinzucht und Farbenkreuzung" in Family II. This family also had the three characters D, DR, and R, represented among its members, as the table of ancestry shows, for albinos, "species," and spotted animals are hopelessly intermingled. A third table, on p. 612 (see Table O), furnishes additional data of the same sort, where gray-whites are inbred from a cross of "species" with a heterozygous black-white rat (DR·R). The exact nature of all the matings cannot be known, but it appears that "families" of rats in many cases, at least, were kept together, so that there was every chance for random mating, and the totals given may represent in some cases a number of litters from various parents. These three tables may now be examined with a view to observing whether or not complete albinism is present in one in sixteen of the offspring, as is expected.

By way of explanation it may be stated that Crampe used abbreviations for denoting the colors of his rats. Thus, in the tables which follow, "var. 5" denotes a black-white rat, "var. 6" is a black animal with small flecks of white, usually confined to the feet, forehead, middle of belly, and tip of tail, and "var. 7" is wholly black.

TABLE M.

(Based on observations of Crampe.)

	Male Parent.	Female Parent.	Black-white Young.	Black with White Flecks.	Black.	Albino.
1	var. 6	var. 6	..	8	1	..
2	"	"	5	4	1	..
3	"	"	5	5	2	..
4	"	"	1	4	1	..
5	"	var. 7	..	10	6	..
6	"	"	..	7	4	..
7	"	"	..	2	3	2
8	"	var. 6	3	9	2	1
9	"	"	2	5	2	2
10	var. 7	var. 7	..	6	26	..
11	"	var. 6	..	17	10	3
12	var. 6	"	5	6	..	2
13	"	"	6	13	..	6
14	var. 7	var. 7	..	7	8	..
15	"	"	..	1	10	2
16	var. 6	var. 6	15	15	5	..
17	var. 7	var. 7	..	..	13	..
			42	119	94	18

In a total of 273 rats, it is expected that 17$\frac{1}{16}$ will be albinos, and there are actually 18. It is evident that albinos can result from any one of three matings, DR·R × DR·R, D·R × D·R, or DR·R × D·R, and in each case the albinos are to the pigmented rats as 1 : 3. In cases 7, 9, 12, 13, and 15 this ratio obtained, but in cases 8 and 11 the albinos are more nearly as 1 : 16, and it is not improbable that the totals in the two latter cases represent the results obtained where entire families containing two sorts of heterozygotes were bred *inter se*.

Table N is reproduced from the data given by Crampe ('85) on page 604.

TABLE N.

(Based on observations of Crampe.)

	Male Parent.	Female Parent.	Black-white Young.	Black with White Flecks	Black.	Albino.
1	var. 6	var. 6	3	10	. .	
2	"	var. 7	. .	7	7	
3	"	"	. .	1	4	
4	"	var. 6	11	18	4	3
5	"species" ×	gr.-wh. 2d. gen.	6	28	12	4
6	"	"	14	32	. .	2
			34	96	27	9

This lot gives a total of 166 young, and 9 are albinos where 10$\frac{3}{8}$ are expected. If the matings represented by this table are those given under our "Case II," we expect that $\frac{3}{16}$ or 31 of the animals produced will be unspotted (i. e. of classes D and D·R), and there are actually 27 entirely black. It is impossible to say which of the remaining individuals belong to the DR, D·DR, or DR·R classes, since these combinations may give a varying amount of spottedness.

The last of Crampe's ('85) tables to be considered (Table O) is that given by him on page 612, which contains the totals obtained by inbreeding, for five generations, families of gray-white rats among which were the three characters D, DR, and R, as appears from Crampe's statement. The young of "family K" are also included, being of a similar nature.

TABLE O.

(Based on observations of Crampe.)

Generation.	"Family."	Gray-white Young.	Black-white Young.	Albinos	Expected No. of Albinos.
3	P 230	59	. .	5	4
4	P 330	59	. .	2	$3\frac{13}{16}$
5	P 430	33	. .	2	$2\frac{3}{16}$
6	P 530	20	. .	3	$1\frac{7}{16}$
7	P 630	41	. .	2	$2\frac{11}{16}$
. .	K 20	19	2	2	$1\frac{5}{16}$
		231	2	16	$15\frac{7}{16}$

This table is particularly instructive because it is certain that the total for each generation represents the sum total of all the unions that took place among the various heterozygotes of a single family. The closeness with which the numbers of albinos obtained approximate the theoretical $\frac{1}{16}$ is certainly remarkable.

It will be noticed also that this table contains no class showing total pigmentation, — a fact which may mean a partial change of the D character through long inbreeding and association with the characters DR and R. The degree of white marking among the gray-whites is of course very variable, and it is assumed that in the present instance gray rats showing only a few flecks of white were included with the rest, though probably resulting from a cross of wild gray and either partial or total albinism. For Crampe states that the latter cross did sometimes give gray individuals with whitish feet or white tips of tail. This fact is referred to in the discussion of partial albinism.

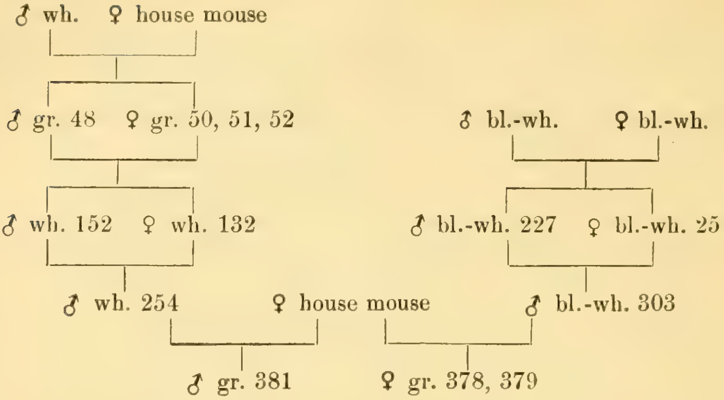
To sum up the results of the three tables, we have, by the interbreeding of heterozygotes, among which three allelomorphic characters are present, a total of 688 young. By the Mendelian principle of segregation, $\frac{1}{16}$ of these, or 43, are expected to be albinos, and there are actually 43 of this sort. That the observed and the theoretical results should closely approximate was to be expected, but their exact agreement to a unit is, of course, more or less fortuitous.

Still other combinations of three allelomorphic characters are possible. Bateson (: 03^a) has indicated one such with respect to comb character among fowls. The characters "rose comb" (R) and "pea comb" (P) are each dominant with respect to the character "smooth comb" (S). A "pea comb" is defined as "a comb consisting of three fairly regular longitudinal ridges, along each of which are several more or less lumpy tubercles." "A rose comb consists of a great number of papilliform elevations, standing in the same horizontal plane. It is wide in the middle, and contracts to a point or 'pike' behind." Two heterozygotes, the one by parents having respectively pea comb and smooth or single comb, the other by parents having respectively rose comb and smooth comb, were bred together. The two heterozygotes thus had each a different dominant comb character, though both had the same recessive character. The result of their union was the production of four sorts of young. The following scheme shows the nature of the unions:

$$\begin{array}{rl} R + S = & \text{gametes of rose (smooth) fowl,} \\ \underline{P + S = \quad ``} & \text{pea (smooth)} \quad `` \ , \\ F\ 2:\ RP + PS + RS + SS = & \text{possible unions.} \end{array}$$

These four classes were found to be actually present in F_2, each well characterized. The RP combination was peculiar, resembling the "walnut" comb of some Malayan fowls. In generation F_3, the results of which are not fully published as yet, Bateson found evidence of imperfect segregation in some of the RP class.

A series of somewhat similar experiments has been made by the writer, using mice. The character common to both heterozygotes was that of total pigmentation, derived from a cross with the wild gray house mouse. The other two characters were in the one case complete albinism, and in the other partial albinism, the latter from a black-white male known to be pure with respect to that character. The following scheme will serve to show the derivation of the two sorts of heterozygotes used:



From the union of these two sorts of heterozygotes (♂ gr. 381 and ♀ gr. 378, 379) there were obtained thirteen young, of which nine were entirely gray and four were gray with small white flecks on the forehead, tail, or belly, one having also white feet. According to the Mendelian principles of segregation, the unions taking place to produce these thirteen young are the following (using the letters D, DR, and R as before, to express respectively total, partial, and no pigmentation);

$$\begin{array}{rl} D + R & = \text{gametes of } ♂, \\ D + DR & = \quad \text{``} \qquad ♀, \\ \hline D + D{\cdot}R + D{\cdot}DR + DR{\cdot}R & = \text{offspring} \end{array}$$

No albinos should result in F_1, nor were there any produced. It now became necessary to test the individuals of this generation severally in order to determine their nature. As previously stated, four of the young had small flecks of white on their otherwise gray coats. One of these slightly spotted mice was bred to an albino of spotted ancestry (the importance of this latter fact is elsewhere pointed out), and six young were produced, three of which were gray with white spots, and three were albinos. Evidently, then, the pigmented animal being tested was either of the class DR·R, or D·R, these being the only two in which complete albinism was present as a recessive allelomorph. That it was actually of the class DR·R, however, is sufficiently shown from the fact that it not only showed partial albinism in the soma, but its three spotted young by the albino showed a still greater amount of white. It is interesting further to note the exact 1 : 1 ratio of the two classes, pigmented and completely albino young, in this litter of six. A second individual was tested in like manner. This time a gray mouse, showing no trace of white in the pelage, was chosen, and bred to an albino of spotted ancestry. Of the four young resulting, two were entirely gray and two were spotted, the one being gray-white, the other black-white. It is plain that in this case the gray parent was of the class D·DR. For the possible unions of such an animal with the albino would result in two animals of the character D·R (here presumably represented by the two all-gray young) and two of the character DR·R (the two spotted individuals). Moreover, these two sorts of young are actually present in the ratio 1 : 1 as expected.

It is interesting to observe that of the four possible classes of young by the heterozygous parents, viz.: D, D·DR, D·R, and DR·R, only one, the class DR·R, is expected to show a spotted coat. In the thirteen young obtained representing these four classes, we therefore look for three, or one-fourth the total number, to be spotted, and there are actually four. It is unsafe, however, to assume that all these spotted young are of the class DR·R, for a mouse of the class D·DR may sometimes show white flecks on the tail. The case appears to be an instructive one, however, and so far as worked out, conforms to the Mendelian laws. No attempt has been made to interbreed at random the four sorts of individuals of F_1 which have been considered, viz.: D, D·R, D·DR, and DR·R. It may readily be calculated, however, that if all the possible unions were to take place, the smallest total number of individuals representing all such combinations would be sixty-four. The same six classes as previously worked out would be present, and the total albinos would still be one in sixteen. The relations of these classes to one another would be: 16 D: 16 D·DR: 16 D·R: 4 DR: 8 DR·R: 4 R. It may be seen that an actual statistical test of a case of this sort is far from being a simple matter.

4. *Albinos bred inter se.* So far as certainly known, two albinos when interbred produce only albinos, or, as is commonly said, albinos breed true. It must not be assumed, however, as some recent writers have done, that any given albino is the exact equivalent of any other albino, in so far as concerns its influence in the heredity of characters other than albinism. The writer has obtained a large number of young both from interbreeding mice of albino parentage and from interbreeding those which Bateson has termed "extracted recessives," that is, the young of two pigmented heterozygotes which each contain the albino

character recessive. No union of two albino mice, regardless of their parentage, has ever given any but albino young in the present writer's experience. This result is corroborated by the observations of a number of other investigators. Thus, von Guaita ('98 ; : 00), Cuénot (: 02), and Bateson (: 03^{b}) record crosses of two albino mice in which only albino young were produced. Crampe ('85) obtained a similar result in his experiments with albino rats. In the case of other mammals, also, a like result has been obtained. Both Cumberland (: 01), and Castle and Allen (: 03) state that albino guinea pigs breed true to that character, and Woods (: 03) has made a similar record in the case of albino rabbits of colored parentage.

In man it has been shown that complete albinism is a recessive character (see Farabee, : 03, and Castle, : 03^{a} ; also Bateson, : 03^{b}), and it is to be expected that two human albinos would have albino children only. The two authors last mentioned, however, have shown that in the negro a marriage of pigmented and albino persons has resulted in the production of pigmented children only, and in the case of a marriage of two other normally colored persons (one being a known heterozygote) albino as well as pigmented children were born. Mr. A. H. Clark, in a letter to the writer, makes the interesting observation that in a case which came to his notice while in the West Indies, an albino negress gave birth to only normally pigmented children, her husband being a normal negro.

A further illustration of the recessive nature of the albino character is recorded by Mlle. Barthelet (: 00). This observer made three series of experiments with gray house mice and albinos, for the purpose of testing the supposed phenomenon of telegony. In the first series, four virgin albinos were covered by as many gray house mice. In each case only gray young resulted. Each female was then bred to an albino, and only albinos resulted. In the second series of experiments, a white female that had never borne young, was bred to a gray house mouse for three successive times, and produced only gray young. Being then covered by an albino male, she gave birth to albinos only. In the third series of experiments, a similar albino female was bred alternately to an albino and to a gray male, with the result that by the albino male only albino young were produced, and by the gray male only gray young. Some such result as this in other cases may have been the foundation for J. von Fischer's ('69) erroneous statement that in crosses between a normal and an albino form the young are like the male parent. Mlle. Barthelet's results might have been precisely predicted by one acquainted with the Mendelian principles of heredity. The work was apparently performed without a knowledge of these laws, which were then in process of rediscovery; but it shows clearly that a previous cross with a pigmented animal will not in any way interfere with the albino's power of transmitting its albino character when paired with a second albino.

Bateson (: 03^b) calls attention to the statement of Dr. C. Carter Blake, printed in the fourth edition of a booklet entitled "Fancy Mice" (Anonymous), to the effect that in two litters obtained from two pairs of albinos, a large percentage of the young was variously pigmented. This result, if true, is unique, and quite at variance with all other known cases. While under certain conditions a reversal of dominance may conceivably take place, it seems more than likely that either an error was made in regard to the true father of the litters in question, or the animals used were not true albinos. The period of gestation for house mice has been shown by Sobotta ('95) to be about 21 days, and the same is also true in the case of the common brown rat (Mailles, '87). If proper precautions were not taken, it might well have been that in the instances cited by Dr. Blake, the albino female had been covered by a pigmented male just before being put with the albino male. Hence a litter containing pigmented young might be born at any time within three weeks after the two had been placed together, and though really fathered by a colored animal, would have been attributed to the albino male. Certainly such a case as Dr. Blake describes should be most carefully established before receiving credence in the light of our present knowledge. Another possible explanation, however, may be suggested, viz., the albinos may not have been *complete* albinos. The occurrence of mice having pigment in the eyes only has been established, and these, though they might pass as albinos, are really of a very different nature. This is discussed further under the head of partial albinism.

Although pure albino mice "breed true" with respect to albinism, this does not imply that all albinos have the same influence in heredity when bred to pigmented animals. That this supposition is clearly erroneous seems to have been first pointed out by Darbishire (: 03), and Cuénot has made observations leading to the same conclusion. These results are more fully treated in the discussion of the behavior of colors (*vide infra*).

5. *Summary on heredity of complete albinism.* To sum up the results of experiment and observation on the manner of inheritance of complete albinism, it is clear that this character is recessive in the Mendelian sense to either total or partial pigmentation, so that a union of an albino and a pigmented parent (the latter being understood in this case to be

homozyogous) results in the production of individuals showing the pigment character only, whether that be total or partial. This is true not only for mice and rats but for other mammals also.

When two mice or rats, each having the character complete albinism recessive, are mated, there result 25 per cent of albino individuals. This relation sometimes occurs in single litters of young, but obviously can do so only in lots of 4 or of some multiple of 4. In the long run, however, the totals for a large number of such litters show a very close approximation to the Mendelian ratio of 3 pigmented mice to 1 albino.

That, further, Mendel's law of segregation holds in the inheritance of albinism, may be seen in cases where albinos are crossed with pigmented heterozygotes having albinism recessive. In such crosses the expectation is that pigmented and albino young will be produced in the ratio of 1 : 1. This is found true experimentally, as an average result. In the writer's experiments, it is found that of 13 litters of mice produced by heterozygotes mated *inter se*, in which the number of young was either 4 or 8, 7 litters, or practically one half, show the precise Mendelian ratio of 3 pigmented mice to 1 albino; and of 25 cases taken also at random, in which, from back-crossing of a heterozygote with an albino, an equality of these two classes is expected (the selected litters containing an even number of young), this ratio appears in 12 cases, or a trifle less than one-half.

This result is, on mathematical grounds, about what we should expect. For, in cases where equality of white and of pigmented offspring is expected in the long run, the chances for their distribution in individual litters of four young each are as follows:

All white:	1 chance in 70
3 white: 1 pigmented . . .	16 chances in 70
2 white: 2 pigmented . . .	36 chances in 70
1 white: 3 pigmented . . .	16 chances in 70
All pigmented	1 chance in 70

Consequently in litters of 4 we expect equality of albino and pigmented young in scarcely more than half the individual cases.

Similarly, when the litters consist of eight young each, the chances for the distribution of white and pigmented animals in individual litters are as follows:

All white :	1 chance in 12,870
7 white : 1 pigmented . .	64 chances in 12,870
6 white : 2 pigmented . .	784 chances in 12,870
5 white : 3 pigmented . .	3136 chances in 12,870
4 white : 4 pigmented . .	4900 chances in 12,870
3 white : 5 pigmented . .	3136 chances in 12,870
2 white : 6 pigmented . .	784 chances in 12,870
1 white : 7 pigmented . .	64 chances in 12,870
All pigmented	1 chance in 12,870

The experimental results obtained indicate, therefore, that the production *on the average* of an equal number of gametes of both sorts actually takes place, although separate litters are quite as apt to exhibit a discordant ratio as the theoretical one.

A ratio of 1 albino to 15 pigmented animals is to be expected in cases where the F_1 individuals are interbred at random, the one parent being heterozygous with respect to albinism and either total or partial pigmentation, the other parent having that one only of the last two coat-characters which is not possessed by the heterozygote; where, in other words, three different allelomorphic characters are brought to the union, and the F_1 generation consists of two classes of young, one of which still contains albinism as a recessive character, while the other contains only the two pigment allelomorphs. Generation F_2, obtained by interbreeding at random the young of F_1, should contain six classes of individuals. The actual occurrence of some of these classes has been experimentally demonstrated.

A third possible combination, in which one parent is heterozygous with respect to one pigment character and albinism, and the other is heterozygous with respect to two pigment characters, is theoretically capable of producing four sorts of individuals in F_1, none of which is unpigmented. The actual occurrence of such a result is indicated by the present series of experiments.

Complete albinism, so far as certainly known, is a recessive character in the Mendelian sense, when brought into competition with a pigment character. Two pure albinos when mated, produce albinos only, so far as at present tested, but it must not from that be assumed that any two albinos are physiologically equivalent as regards their influence on the heredity of pigment characters, in crosses with pigmented animals.

B. Partial Albinism.

1. *Manner of occurrence.* Partial albinism is a phenomenon of not infrequent occurrence among mammals and birds, and other organisms as well. In many species the unpigmented areas form part of a definite color pattern and may be in the form of stripes or spots variously disposed. Such markings, however, appear to come under a category different from that of the white areas occurring abnormally in animals that naturally are pigmented over all parts of the body. In the case of such animals, spots or unpigmented areas, when they occur, tend to appear at certain definite places, and the increase of the white areas takes place in such a way as to give the impression that the pigment is shrinking away toward certain centres more or less definitely situated. The writer's attention was first called to this matter by Professor Castle in the case of guinea pigs, where these centres are especially conspicuous. From a study of a number of spotted mammals the writer has made out in several of them the presence of ten such centres (see Figures 3–6). These are bilaterally arranged five on either side, as follows: (1) A genal centre for the pigment patch which embraces the eye, ear, and side of the face; (2) a nuchal centre for the neck patch; (3) a scapular centre for the patch which covers the shoulder and fore leg; (4) a pleural centre for the side patch; (5) a sacral centre for the large patch which includes the tail, hind limbs, and sacral region. Figure 3 shows in a diagrammatic way the location of these centres in the house mouse. On account of the shortness of the neck in the

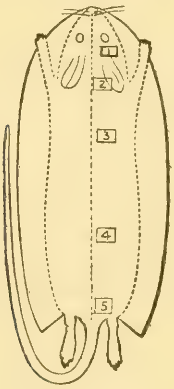


Figure 3.

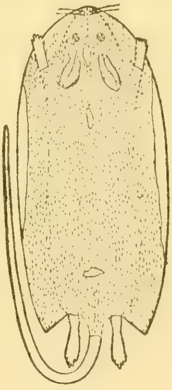


Figure 4.

mouse, the genal and nuchal centres are close together, and the two patches are generally not distinctly marked off from each other. From a comparative study of other mammals, however, it is found that the genal patch is rather definitely limited in its backward extension to the posterior part of the head proper, so that it is not difficult to assign it limits in cases where the two are fused. On the subject of the arrangement and variation of these patches in piebald mammals, Professor Castle and the writer have in preparation an extended paper based on the examination of a large number of animals.

In a mammal such as the mouse, which normally has total pigmentation, partial albinism seems to occur but seldom in the wild state, and when depigmentation does take place, it is most apt to occur in a small spot on the forehead, at the tip of the tail, or on the mid-ventral line. It is as if the pigment patches emanating from the two nuchal centres, or the two pleural centres, had not quite met, leaving a spot unpigmented on forehead or belly respectively, or as if the sacral patches had not spread to the end of the tail. There is great variation in the degree of isolation of the various pigment patches delimited by the white areas. Figure 4 shows the pattern of a mouse reared by the writer in which appear the beginnings of separation (1) of the genal patches (giving rise to the white streak in the forehead), and (2) of the nuchal and scapular patches (the small white spot in the median line at the nape indicating the boundary of the two). The fore feet are white, showing that the scapular patch has not reached its greatest possible extension.

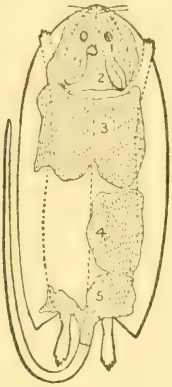


FIGURE 5.

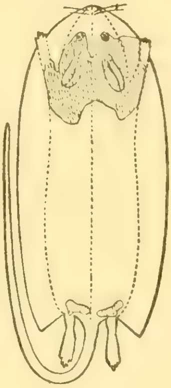


FIGURE 6.

(3) The pleural patches are separated ventrally, leaving a narrow white belly streak. (4) The pleural and the sacral patches have not fully coalesced on the dorsal part of the rump, leaving thus a median white spot, and the restriction of the sacral patches has given a white tail and white hind feet. Various degrees of increased restriction of pigment toward the different centres are possible, and the ultimate extreme condition is the complete absence of pigment about a centre. Thus, Figure 5 shows the condition obtaining in the writer's black-white male 303. The genal and nuchal pigment patches are practically intact, though a break in the middle of the forehead and a white streak along the throat show that the two former are suffering restriction. A white collar nearly separates the nuchal from the scapular patches, and the latter have shrunk away, as it were, from the belly and fore limbs, while posteriorly a notch in the median dorsal line indicates that a further restriction would divide the color area into right and left patches. One of the pleural patches, that on the left side, has disappeared altogether; the other is entirely free from the scapular patch, and nearly so with respect to the sacral patch of its own side. It will be observed that the single pleural patch tends to encroach very slightly on the area which its fellow normally would cover. The two sacral patches are fairly distinct, the one on the right being more extensive. The two fuse posteriorly and include the root only of the tail. Figure 6 represents a still more extreme case of restricted pigmentation; it is the color pattern of a black-white Japanese dancing mouse obtained from a dealer. The genal patches are reduced so as to leave a wide white blaze on the forehead, and the shoulder patches, too, are very much reduced, though they have not broken from the neck patches. The sacral patches are represented by two very small centres, one on either side of the base of the tail. The pleural patches have disappeared entirely. The writer has seen similar mice in which the shoulder patches also were gone. Hence it is not difficult mentally to picture a mouse in which all the patches have become eliminated, by selection or otherwise, giving a pure white individual with black eyes as the condition of partial albinism whose opposite extreme is an animal totally pigmented except at the tip of the tail. The actual occurrence of a white mouse with black eyes has been established by Bateson (:03^b, p. 74), who states that to his knowledge "strains of this type have been independently produced twice." Among other animals also individuals of this sort are known. Thus among horses, cattle, dogs, cats, guinea pigs, hens, and even doves, white individuals with pigmented eyes are not uncommon. It seems difficult to obtain such a race

in the case of mice, as the patches about the head are not easy of elimination.

To carry the reduction of pigmentation still farther would be to bring about a reduction of the pigment in the eyes of a black-eyed white animal. Bateson (:03[a]) mentions a strain of black-eyed white mice whose eyes were a "blackish red," so that apparently it is possible to reduce the eye pigment in a black-eyed white animal. Darbishire's pink-eyed, fawn-white mice show that depigmentation of the eye may take place independently of elimination of the coat patches. If the reduction of the patches by proper selection is possible, we may conceive of a total albino mouse derived from a pigmented stock, and so potentially a mosaic, which, nevertheless, if bred to an albino might give totally pigmented young. Evidence of this is found in Bateson's (:03[a]) statement that the black-eyed white mice do not breed true to that condition even *inter se*, and a cross with an albino of dissimilar ancestry would probably have the effect of upsetting the condition of restricted pigment still further. This indeed is the result which, as I am authorized by Professor Castle to state, has usually been obtained by him with guinea pigs. For on breeding together an albino and a black-eyed white animal, the young are more or less extensively pigmented, depending somewhat on the parentage of the albinos used. Other evidence showing the comparative instability of this condition of extreme pigment reduction is afforded by the white cattle of Scotland, the so-called "White Forest breed." According to E. L. Sturtevant ('74) this breed is characterized by the absence of pigment from the entire surface of the body, save the hoofs, horns, and nose-skin, which are black, and the ears, which are usually red. The eyes also are pigmented. The herds of this animal formerly kept in certain private parks of the British Isles show evidence of long inbreeding, and it is known that a severe process of selection has been practised to keep the breed up to the desired standard. Frequently pigment patches occur on the neck or sides of the offspring of two typically marked animals, and such blemishes in the herd are customarily removed at once.

Evidently a case of this sort among plants is that cited by Naudin ('65), in which two varieties of Datura were crossed. The one was an albino race with white flowers (*D. laevis*), while the other was also white flowered, but showed a slight pigmentation of the stem in the region of the nodes (*D. ferox*). This union always resulted in a blue flowered plant similar to the usual *D. tatula*. Evidently the form *ferox* was a partial albino in which the pigmentation was extremely reduced in the soma, though the cross with the albino served to call forth the production of a greater amount of coloring matter. The precise mechanism by which this result is obtained is not clear ; it is uncertain whether there is simply an increase of a pigment-producing ferment, as Cuénot's explanation would imply, whether the cross with an albino serves to upset an unstable condition, or whether some other unknown influence brings about the result. At all events, the facts thus far collected seem to indicate that the albinos used in these cases transmit a character which makes for total pigmentation, and this asserts itself in the first hybrid generation as a character dominant over partial pigmentation.

What is probably an analogous result is recorded by Tschermak (:02) in the case of peas (*Pisum*). The usual type of *P. arvense* is characterized by the presence of red pigment spots at the axils of the leaves. A Swedish variety was obtained which lacked this spot and bred true to the peculiarity. But hybrids obtained by crossing this pea with an albino stock were found to exhibit the red axillary spot. This phenomenon may also be looked upon as a case in which a character emerges from latency under the influence of cross breeding, the character missing in one parent having been transmitted through the albino parent to the offspring, in which it becomes active.

It is clear, then, that a white animal with black eyes is not an albino, but simply a *partial* albino, and care must be taken to recognize this fact in studying the heredity of albinism.

In practically all domesticated animals partial albinism occurs to a greater or less extent, and it is quite possible that the conditions incident to domestication favor its appearance, or at least its propagation. Among animals living under natural conditions also partial albinism is not infrequent, often appearing among small mammals in the form of a white tip to the tail. Thus Miller ('93) records having found white-tipped tails repeatedly in the case of the two eastern species of white-footed mouse (*Peromyscus*), and occasionally in *Mus musculus*, likewise in the vole (*Microtus pennsylvanicus*), and in the common Blarina (*Blarina brevicavda*). In the common jumping mouse (*Zapus hudsonius*) Miller has twice found specimens, otherwise perfectly typical, having a white tip to the tail, 8 mm. in extent. He points out that in the nearly related woodland jumping mouse (*Napaeozapus*) this character has become a distinctive mark. It would thus appear that the character is capable of inheritance and fixation. What causes bring about such a restriction of pigment seems as yet past finding out, but if the ideas already advanced are correct concerning the development of the pigment from certain centres on the body, the appearance of albinism in certain body regions would seem to be due to a weakening of certain of the centres for pigment development. The writer inbred house mice for four generations in the hope of bringing about by this means some trace of albinism, but none was produced.

2. *Heredity of partial albinism.* Among mice and rats, partial albinism tends to be inherited as a distinct character, so that when two purebred spotted animals are mated, all the offspring are spotted, though they may differ from the parents in respect to the precise degree of partial pigmentation. In a paper already mentioned, Castle and Allen (:03) have referred to these spotted mice as mosaics in which the characters, pigmentation and albinism, are *both* visible side by side in distinct areas of the animal's body, though when the two characters are brought together by the mating of a *totally pigmented* and an *albino* animal, albinism is recessive and does not appear in the soma. For this reason, such a spotted animal might be regarded as a mosaic in which the two characters are so united as not to be segregable in the formation of gametes. As Professor Castle has suggested to the writer, it is conceivable that *incomplete* segregation may take place in a heterozygote containing both the pigment and the albino characters. Such partial segregation would give rise to gametes partaking of both these characteristics, and the union of two such gametes would produce a partial albino. That spotted mice have really had such an origin, however, remains to be proved. It is clear from the experiments detailed in the preceding chapter that there is no evidence for the general occurrence of any but the normal Mendelian segregation of the two characters concerned. Moreover, the fact that two pure-bred spotted animals breed true and produce no albinos, shows that the albino character cannot become, as far as present experience goes, again segregated into a separate germ. If this view be tenable, the spotted or mosaic individual contains both the dominant and the recessive elements side by side in its mature germ cells as well as its soma, and may be symbolically expressed by the term DR. It may, however, with equal propriety be regarded as simply a "mutation" (de Vries) in which the production of pigment is for some reason more limited than it is in the normal individual. Among plants, that variety of Datura in which the fruits have prickles grouped in patches, instead of being uniformly disposed, may perhaps be similarly regarded as exhibiting a mosaic condition, since the thorny character is found to be dominant in heredity over the smooth condition of fruit (Bateson, :02, p. 23). The fact that in mice the spotted condition is one which cannot be pro-

duced experimentally by simply crossing pigmented with albino individuals is well known (Darwin, '76), and although Crampe appears to have been able to do so with his rats, nevertheless his results are capable of another interpretation, in some cases at least. Thus Crampe ('85) states that the wild gray rat or "species" bred to albinos gave animals entirely gray or else animals with white in the forehead, tail, or belly. Rats of the latter sort, if interbred, gave not only unicolored but spotted young as well, and also albinos. In this case, however, the possibility is not excluded that the so-called species was *not* an actual wild gray rat, but simply one which was gray through reversion, and had had one spotted parent.

These facts lead to a consideration of the behavior of the spotted or partial albino condition in crosses. As previously stated, when two pure-bred spotted individuals are mated, the young are all spotted. The spotted character is quite independent of color, however, so that it is possible to mate piebald mice, which differ in the color of their pigmented areas, and yet to obtain offspring all spotted. This shows that the mosaic, if such it be, retains its integrity strongly, or in other words *the localization of the pigment at certain points is a condition which is heritable.* Yet the degree of this localization varies, and in a single litter individuals may occur having pigment patches more or less extensive than those of their parents. The important point is that breaks between the pigmented areas do occur in the young of partially albino mice, be these breaks large or small.

A different result is obtained by breeding partial albinos to certain stocks of complete albinos. In the writer's early experiments a large number of young was obtained by breeding black-whites to albinos of a stock whose ancestry was unknown. In all cases the resulting young were practically totally pigmented. Some showed no trace of albinism in the pelage, though others had white toes, or a white tip to the tail, or even a few scattered white hairs. Only one had a white spot on the belly. This is much the same result as that recorded by Haacke ('95), who, in crosses of a Japanese spotted stock with albinos, obtained mice of a uniform color over the entire body, save that in occasional animals small flecks of white appeared in the usual places (i. e., on the forehead, tail, or belly). Von Guaita ('98) by a similar cross obtained a like result. The influence of the albinos in these cases seems to be to upset the condition of localization of the pigment, so that the pigment patches become more extensive, tending to cover the entire body surface as in totally pigmented animals. In explanation of this observation, it is suggested that the character total pigmentation may be transmitted by albinos, and when so transmitted dominates over the spotted condition. How this may possibly take place, an ingenious suggestion made by Cuénot (:03) indicates. This investigator believes that the formation of pigment is due to the action of some ferment upon a chromogenic substance, and that an albino may transmit the former apart from the latter. It might be that the gamete of such an albino brings in a quantity of the ferment sufficient to produce a greater amount of pigment, and so the resulting individual is much less marked with white than the spotted parent. Such considerations are, however, purely theoretical.

If the character total pigmentation be dominant, in the Mendelian sense, over partial pigmentation, it should be possible by interbreeding heterozygous individuals to obtain 25 per cent of offspring having the spotted character. Of the 75 per cent of self, or wholly pigmented animals, one-third are expected to be pure in respect to the character total pigmentation, and the remainder to be heterozygous.

From the writer's self-colored heterozygotes (F_1) obtained by crossing black-white mice with albinos which transmitted the character total pigmentation, a second generation was bred, consisting of 10 spotted, 27 nearly unicolor, and 15 albino young. As the albino young were not tested to discover which of the two pigment characters they transmitted, it is necessary to disregard them. Of the 37 pigmented animals, therefore, one-fourth, or 9, are expected to be spotted, and there were actually 10 of that nature. Von Guaita in his generation F_2, which was obtained in a similar manner, got 10 spotted and 20 self-colored individuals from the heterozygotes, an excess of 2.5 spotted young over the expected number. Darbishire (:03) records a similar case. He intercrossed spotted Japanese dancing mice with albinos of two stocks, the one pure bred, the other cross-bred. The latter sort had "appeared from time to time in the litters of piebald mice kept in the Oxford Laboratory for embryological purposes"; the former were purchased of a dealer. The cross-bred albinos, since they are presumably of spotted parents, will transmit the spotted character, so that Darbishire's result, 25 spotted and 4 unspotted young, is quite what might be expected. The four unspotted individuals probably came from an albino which was heterozygous with respect to the two characters, total and partial pigmentation. From his pure-bred albinos, Darbishire obtained 36 spotted and 61 unspotted mice, a fact indicating that at least some of these albinos transmitted the character making for total pigmentation. The result is due apparently to the transmission of pigmentation characters through the albinos, not to the relative degree of in-bredness or pure-bredness of the albino stocks. Moreover, the characters total and partial pigmentation are inherited quite apart from the character complete albinism, so that in the F_2 generation it is possible to obtain both spotted and self-colored animals having the character complete albinism recessive. An albino having the spotted character may be regarded as a spotted mouse in which the spots do not show, perhaps because of the absence of chromogenic substance, if we adopt the theory of Cuénot (:03).

Darbishire's mice offer another point of interest in this connection. The spotted Japanese individuals were of a peculiar strain, having *pink* eyes. All the F_1 heterozygotes had dark eyes, so that we may suppose the pigmented parents of the albinos to have had dark eyes. From his second generation, obtained by interbreeding the dark-eyed heterozygotes, Darbishire had 17 pink-eyed spotted mice, 36 dark-eyed individuals, and 13 albinos. These last doubtless represent both the pink- and the dark-eyed condition potentially, but until tested they may be disregarded. It is expected, therefore, that 25 per cent of the pigmented individuals will show also the pink-eyed character. This, in fact, is practically the outcome, for 17 young are pink-eyed, 13+ being expected. The character of the albinos could be tested by back-crossing them with the pink-eyed pigmented stock, and some would probably give pigmented mice, all of which would have pink eyes, while others would give young some with pink, some with dark eyes.

To test further the character partial pigmentation, the writer has back-crossed with spotted mice both albino and pigmented individuals which might be supposed to be heterozygous with respect to the two characters. The derivation of the animals used is,

spotted × non-spotted or self

self (spotted recessive),

and the expectation is that by the cross with a spotted mouse, self-colored and spotted offspring will result in the ratio, 1 : 1. This was practically the result obtained. Table P contains a list of crosses of this nature.

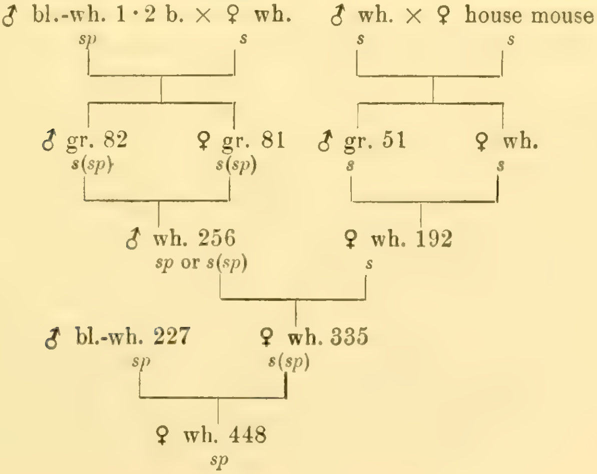
The expectation is that the two classes will each consist of 14.5 individuals, so that the deviation amounts to but 0.5 for each class, — as close an approximation as is possible in an odd number of offspring. The writer purposely selected as the heterozygotes for these tests *albinos* coming of a cross between spotted and self-colored mice, in order to throw light on the suggestion of Bateson (:03^b) that a segregation of pigment characters

TABLE P.

Parents.	Spotted Young.	Self-colored Young.
♂ gr.-wh. 352 × ♀ wh. 389 . .	2	2
♂ ch.-wh. 392 × ♀ wh. 447 . .	2	3
♂ ch.-wh. 423 × ♀ wh. 433 . .	4	2
♂ bl.-wh. 454 × ♀ wh. 439 . .	3	2
" "	3	1
♂ bl.-wh. 304 × ♀ gr. 422 . .	1	4
	15	14

occurs among the albino animals as well as among those which are pigmented. It is clear that a segregation did take place among the albinos with respect to the two pigmentation characters under discussion.

The writer also has an albino mouse, an extracted recessive, which appears to transmit the spotted character, but not the character giving total pigmentation. The following scheme shows the pedigree of this individual. The letters *s* and *sp* in connection with each animal indicates its character, whether *self* or *spotted*, the combination *s* (*sp*) indicating a self-colored heterozygote.



Two spotted mice, each having albinism recessive, must transmit to their *albino* young, as well as to their pigmented offspring, the character partial pigmentation; hence an albino of spotted parentage should give only spotted young on being bred to a spotted animal. This supposition the writer has tested and found to be correct.

It remains to discuss more in detail the relation of total to partial pigmentation. The experiments thus far performed do not show an absolute dominance of either condition over the other, though the tendency is in favor of the dominance of total pigmentation in the offspring of a spotted mouse and a mouse unmarked with white, as the following experiments indicate. The writer obtained three young by crossing a spotted black-white mouse (No. 303, Figure 5) with a gray, wholly pigmented house mouse. Two of the young were entirely gray, while the third was gray all over except for a white ring near the tip of the tail. In another case a black-white male was bred to a totally black mouse of house-mouse extraction, and of the three pigmented young obtained all were entirely black, save that the toes were rather lighter colored. They differed in no respect, however, from their mother. In another case a totally gray mouse resulted from the union of a D and a DR gamete, and when bred to an albino of spotted extraction, it gave two plainly spotted and two unicolor young. The result is perhaps similar to that obtained in the cross of the spotted with an albino stock not known to come of spotted parents, for the young tend to be wholly pigmented, though frequently they show a white tip or ring on the tail. Moreover, further intercrossing of these heterozygotes often gave "golden-agouti" and chocolate mice, which show no trace of white, but might have offspring with the white tail tip or forehead stripe. Both Haacke ('95) and von Guaita ('98) speak of obtaining only unicolor mice from their first cross of spotted with albino mice, but Haacke adds that these hybrid animals sometimes showed a fleck of white on forehead or belly.

Crampe ('85) states that by breeding the wild gray rat ("species") to the black-white variety, he obtained 4 totally gray animals and 36 which were gray with white "Abzeichen," in the shape of flecks on the tail, belly, or forehead. By interbreeding these latter, he obtained gray rats and black rats exhibiting all the six classes of coloration which he describes, viz.; gray, gray with white markings, gray-white, black-white, black with white flecks, and black with no markings whatever. The unions of gametes in the first described mating would be

$$\begin{array}{rcl} D + D & = & \text{gray "species,"} \\ \underline{DR + DR} & = & \text{black-white,} \\ 4\ DR\cdot D & = & F_1\text{, mostly with white flecks.} \end{array}$$

This generation, F_1, interbred, should give gametic unions as follows:

$$\begin{array}{l} \qquad DR + D \\ \underline{\qquad DR + D \qquad} \\ DR + 2DR\cdot D + D = \text{theoretical classes,} \\ 77 \quad : 133 \quad : \quad 25 = \text{observed numbers,} \\ 58.75 : 117.5 : 58.75 = \text{calculated numbers.} \end{array}$$

Crampe obtained the three theoretical classes: plainly spotted (DR), hardly spotted (DR·D), and unspotted (D). These classes occurred in something like the anticipated proportions, though it is clear that there were more spotted rats than expected. Moreover, as his first cross (F_1) showed, the union of DR and D does not always give spotted animals, and this fact might serve to reduce still further the class D, which is less than 50 per cent of what it was expected to be. These facts may indicate incomplete segregation of the two characters, which, indeed, differ only in degree.

In general, however, it is clear that in mice and rats the cross of a wholly pigmented variety with one having partial pigmentation serves to reduce the extent of the white areas in the offspring, so that the latter may either show very slight traces of albinism at the places where albinism is most apt to occur, or they may show no white at all.

3. *Summary on heredity of partial albinism.* To sum up, partial albinism among mammals which normally are wholly pigmented occurs in certain definite places in the shape of white patches or breaks between pigmented areas. These areas, in suffering reduction, become diminished toward certain centres, of which there are ten arranged symmetrically, five on either side of the longitudinal axis.

According as the pigment patches are little or more reduced, or disappear entirely, various degrees of partial albinism may be recognized. The extreme on the one hand is that condition in which albinism occurs at the extremities of the appendages, or as small flecks at the points where two adjacent pigment patches had just failed to meet. The opposite extreme is that condition wherein pigment production is found only in the eye, and even here a certain amount of reduction may take place in the coloring matter.

A partial albino may be regarded as a mosaic exhibiting, in its soma, two characters which, when segregated, are to each other as dominant (pigment) to recessive (albinism). Two partial albinos breed true to that (partial albino) condition, even though each contains an albino character *recessive* (compare formula $2a$, p. 73). Necessarily, therefore, two-thirds of the spotted offspring of such *heterozygous* animals (DR·R) come of the union of a gamete containing the character partial albinism (DR) with one bearing the completely albino character (R). Yet the union of a spotted animal with an albino *not of spotted parents* produces young which tend to be totally pigmented. An albino coming of *two spotted* parents, on the other hand, when bred to a *spotted* individual, probably produces only *spotted* offspring, for this union is manifestly similar to that producing spotted young, where both spotted parents have albinism recessive.

Complete pigmentation in mice tends to be dominant over partial pigmentation, although the dominance is not always perfect. In rats, Crampe seems to have found the tendency to be toward an intermediate condition. In general, the result of a union of a pure *spotted* and a pure *unspotted* (pigmented) individual is to greatly reduce the amount of albinism in the young of the F_1 generation or to eliminate albinism altogether, the outcome being, in one case, an intermediate condition, in the other, dominance of the character total pigmentation.

Partial albinism, then, tends to be inherited as a distinct character and in accordance with the Mendelian principles; in mice it behaves as a recessive in relation to total pigmentation.

VI. Heredity of Pigment Characters.

1. *The gray coat.* The "wild gray" or typical house-mouse coat has been shown by Miss F. M. Durham (Bateson, :03[b]) to owe its color to the presence in the hairs of three pigments, black, chocolate, and yellow; these may be present in slightly different proportions, thus producing a lighter or a darker gray, or one with more than the usual amount of yellow (sable). For the purposes of this paper, however, "gray" is understood to mean that sort of coat in which all three pigments are present, no particular regard being paid to the slight variations of lighter or darker tint. This gray coat under cross-breeding may undergo resolution, giving several distinct color types characterized by an absence of one or two of the three pigments. In the writer's experiments the types thus obtained were: (1) black mice, whose pigments were black and chocolate, (2) golden agoutis, containing only the chocolate and the yellow

pigments, and (3) chocolate mice, containing the chocolate alone. Yellow mice in whose pelage is only the yellow pigment are a well-known type, but the writer was unable to obtain any thorough segregation from his other stocks. To avoid confusion, these types are here referred to by the terms used in Bateson's paper (: 03^b) on the types of fancy mice and rats. The hairs of the mice bred by the present writer have been examined with the microscope and the pigment elements found to be as stated by Miss Durham.

Wild gray house mice breed true to that color, so far as observed. The writer bred a considerable number from a pair captured in Cambridge, and four generations of inbreeding failed to produce any but gray animals.

Four house mice were bred to albinos obtained of a Boston dealer, and by reciprocal crosses 67 young were produced, all of which were gray. Among these was a litter of three of a very light gray, but they unfortunately died while only a fortnight or so old. Subsequently the same parents produced normal gray young only.

The gray heterozygotes obtained from this cross were interbred, and produced, in addition to the expected albinos (extracted recessives), a number of pigmented animals, 40 in all, which with a single exception were gray like their parents. The exception was a mouse entirely black. Here evidently was a case of segregation, the yellow pigment being entirely lacking. Although this individual appeared coal black, the presence of the chocolate pigment was manifest at times when the pelage was moulted, the new hair appearing brownish. Bateson states that he has not seen any black mouse whose hair did not contain also the chocolate pigment. To account for the presence of this black mouse it may be assumed that among the gametes of both its hybrid gray parents there were present some which lacked the yellow pigment character. Two such gametes must have united to produce the black individual. The black character was probably transmitted through the *albinos* used, for, as will be shown farther on, this stock seemed regularly to produce about 25 per cent of gametes bearing the black pigment character, the remainder having the gray character. In the formation of gametes, therefore, the gray heterozygotes must have produced some sex cells which lacked the yellow pigment character. It is conceivable that this result may have been brought about by a resolution of the gray character in such a way that yellow and chocolate passed into one gamete, black and chocolate into another. A part of the gray heterozygotes of generation F_1 were back-crossed with the albino parent, and of the 31 pigmented animals so obtained, 3 were black. This fact points to a segregation of the color characters both in the gametes having the albino character and in those having the pigment character. In crosses with heterozygotes from this house-mouse-albino stock no colors other than gray and black were obtained.

Of the black mice obtained from this stock by the process of back-crossing already described, some proved to have albinism recessive, while others were pure in respect to albinism, showing that the black condition was not one depending for its production on the union of a recessive with a dominant gamete, though the former (recessive gamete) was probably effective in transmitting pigment. The black mice were found to produce only black pigmented mice and albinos; their case is more fully discussed under the next heading.

Gray heterozygotes obtained by crossing house mice with albinos were bred to animals similarly colored but descended from black-whites and albinos. All of their pigmented young, 19 in number, were likewise gray.

The reversionary gray coat of mice derived from a cross between black-white and albino mice is not always transmitted intact, but commonly undergoes more or less resolution of its pigment characters.

Thus, three pairs of gray-white mice, whose grandparents on the side of each parent were respectively a black-white and an albino, and whose parents were gray heterozygotes, produced gray young in large proportion, but also a few golden-agouti and a few black individuals. The total offspring from three such pairs was 14 gray-white, 1 black-white, 4 golden-agouti-white, and 1 albino. This seems plainly to be a case of resolution of the gray pigment character, as there was no golden-agouti ancestor, so far as known.

Gray house mice, when bred to fancy stocks of other colors, usually give gray young only. Thus, only gray mice were obtained by the writer from a cross of a female house mouse with a black-white male. These gray young were then bred to gray heterozygotes derived from a cross of a house mouse with an albino, and only gray offspring resulted. Dehne ('55) records crossing a yellow mouse with a house mouse, hoping thereby to propagate the yellow variety, but all the young were gray, and the experiment was given up. Cuénot (:03) finds that the cross of wild gray mice with black mice always results in gray offspring in the first generation, but in the second generation the black character reappears under close breeding, in the proportions demanded by the Mendelian laws for recessive characters, i. e., 3 gray to 1 black.

For rats, Crampe ('77) found that the wild gray type was dominant over both the albino and the black-white variety in the first generation, so that only gray rats were produced. A "Mestiz" (apparently a gray heterozygote), one of whose parents was a wild gray rat, when bred to a black-white rat, gave 4 gray and 5 black offspring, a result comparable to that of Cuénot, in which black acted as a recessive character.

It is therefore clear that the wild gray coat of the species type is strongly dominant over those types obtained from it through resolution of the three pigment characters. The wild gray coat also shows less tendency to break up into black, chocolate, and golden-agouti than does the reversionary gray coat seen in cross-breds from fancy black-white and albino stock. This will be shown in detail farther on.

2. *The black type.* According to the researches of Miss Durham (Bateson, :03^b), the black coat contains two sorts of pigments, – a black and a brown, or chocolate. No case was found in which black occurred alone. This result I am able to confirm. The brown pigment is visible in the new hairs when the black animals are moulting, but it is much less conspicuous in the fully grown hairs. Since only these two pigments are found in the black coat, it is natural to inquire what has become of the third or yellow pigment. Two explanations are possible: either yellow is absent because of its having become latent, or else through segregation it has been lost entirely. The latter alternative seems the more probable; the evidence for and against it will be discussed later.

In general, two black mice breed true to that color, though under some circumstances it is possible to obtain chocolate animals also. The writer has reared a considerable number of black-white mice and they have never produced any but black-white young (except, of course, where albinos appeared in consequence of the recessive occurrence of albinism in both parents). Mice wholly black, which came of a house-mouse stock, also behaved in a similar way, giving blacks only, among the pigmented individuals. A black-white mouse bred to a black mouse of house-mouse extraction produces only black young, some of which may at times show a trace of white at the tip of the tail or on the toes. Bateson (:03^b) notes a similar result in the case of self-colored blacks bred to black-whites, and asks therefore if black-white may not be recessive to black. As the writer has shown in the chapter on albinism, it is more properly a case of dominance of complete pigmentation over partial pigmentation.

In looking over the records given by von Guaita ('98, :00) it is found that in all cases where black mice were paired, the pigmented young were black (or black-white), and a similar result was recorded by Haacke

('95) for the black-white Japanese waltzing mice which he bred. In general, therefore, the black type seems to be a definite one which breeds true to that pigment character, and does not show the presence of the yellow pigment.

The cross of the black with the albino type is one of considerable interest. Haacke ('95) bred black-white mice of a stock imported from Japan to some albinos whose ancestry is unrecorded, and obtained usually gray mice indistinguishable from the wild gray ones. More rarely, a wholly black mouse came of this same union. Von Guaita ('98, :00) repeated the experiment, crossing black-white Japanese dancing mice with an inbred stock of albinos. He obtained twenty-eight young, all uniformly gray like the house mouse. A like result is recorded by Davenport (:04), only gray young having resulted in two litters from black mice bred to albinos.

In the writer's experiments, as previously recorded (Castle and Allen, :03), two black-white mice were crossed with albinos, the result being that some of the offspring were gray, others black. The spotted animals were both males and were known to give only black-white young when bred to mice of that color character. Their gametes, then, did not transmit the yellow pigment character, and the above result shows the same to have been true also of some at least of the gametes of the albinos, for there were in all 15 black young in addition to 28 gray ones. An explanation of this result is suggested as follows: Since the black mice produced gametes having only the black character (or rather, black-chocolate), it is clear, on the present hypothesis, that some of the albinos were producing gametes with only the black character, while a large number were producing gametes whose union with those of the black mice brought together all three pigment characters, resulting in the production of gray young. If we suppose that the albinos used came of a mixed ancestry, as seems probable, and that one-half the gametes formed by them contained latent the complex gray character, while in the other half the latent gray element underwent resolution, the following combinations of pigment characters would result from the mating described:

a.

$$\begin{array}{rl} \text{bl.-ch.} + \text{bl.-ch.} & = \text{gametes of ♂,} \\ \text{wh. (gr. latent)} + \text{wh. (gr. latent)} & = \text{gametes of ♀,} \\ \hline \text{4 gr.} \;=\; F_1. & \end{array}$$

b.

bl.-ch + bl.-ch. = gametes of ♂,
wh. (yell.-ch. latent) + wh. (bl.-ch. latent) = gametes of ♀,
2 gr. + 2 bl. = F_1.

In the one case (*a*), the gray character transmitted by the albino is sufficient to produce only gray offspring. In the other case (*b*), the gray character, latent in the albino, is supposed to undergo resolution so that gametes are formed one-half of which contain the yellow-chocolate character and one-half the black-chocolate character. In the two cases wherein the yellow character unites with the black of the black-white parent, gray offspring result, since all three of the pigment elements are thus brought together. In the other two cases, however, both unions are of gametes having only the black-chocolate elements, and so only black young result. If this hypothesis be correct, there should be on the average 3 gray to 1 black young produced by the cross described. There were actually obtained 43 young, so that the expected result is ($32\frac{1}{4}$ or) 32 gray to ($10\frac{3}{4}$ or) 11 black mice. The observed result was 28 gray to 15 black, a fairly close approximation.

In a second case a black-white male having albinism recessive was bred to these same albino females. Assuming, as before, that in one-half the cases the gametes of the albinos contained the gray character, and that in the remaining cases the gray had been resolved, the combinations of gametes to be expected are:

a.

bl.-ch + bl.-ch. = gametes of ♂,
wh. (gr. latent) + wh. (gr. latent) = gametes of ♀,
2 gr. + 2 wh. (gr. latent) = F_1.

b.

bl.-ch. + bl.-ch. = gametes of ♂,
wh. (yell.-ch. latent) + wh. (bl.-ch. latent) = gametes of ♀,
1 gr. + 1 wh. (gr.) + 1 bl. + 1 wh. (bl.) = F_1.

The expected result is therefore: $\left\{\begin{array}{l}\text{1 white (black latent)}\\ \text{3 white (gray latent)}\end{array}\right\}$ + 3 gray + 1 black. The actual result was 21 albinos + 10 gray + 2 black. The albinos are here in considerable excess, but if the 12 pigmented individuals are alone considered the expected result would be 9 gray to 3 black mice, so that the agreement between theoretical and observed is reasonably close. The albino offspring were not all tested, hence it is not possible to say whether there were any among them which contained the black character only, as this hypothesis would demand. On the whole, it seems reasonable to believe that segregation did take place among the albinos used, in the manner indicated.

That albinos differ in their ability to transmit pigment characters has been shown by Darbishire (:02) and Cuénot (:03). The latter had three sorts of albinos: (1) from gray ancestors, (2) from black parents, (3) from two "yellow" mice of a complex ancestry. The same black male bred to the first sort, produced always gray mice; bred to the second sort, always black mice; bred to the third sort, gray, black, or yellow mice. To explain this result Cuénot has formulated the ingenious and attractive theory that pigments are produced by the action of specific ferments on a chromogenic substance, and that an albino may transmit the former but not the latter. He supposes that the albino of gray parentage transmits the ferments corresponding to the different sorts of pigments which constitute the gray coat, and so the union of such an albino with a pigmented animal produces gray offspring. In like manner the albino of black parentage transmits only the ferments which make for black. Assuming that there is some such process as Cuénot has indicated, it is easy to conceive of segregration taking place in such a way that certain of the gametes of an albino provide one sort of ferment and other gametes another sort. Thus, if the albinos used by the writer should in half of the cases produce gametes with the ferments that give the gray coat, while the other half had the ferments either for black-and-chocolate or for yellow and chocolate, such a result as was obtained is readily explainable. At present, however, such an hypothesis does not admit of positive proof, although it seems clear that the albino of mixed ancestry does in some way contribute the third pigment element to produce gray offspring.

The gray heterozygotes resulting from this cross of black-whites and albinos of mixed ancestry, when interbred, show a further resolution of the pigment characters. Those used by the writer were practically of a uniform gray all over, with only an occasional trace of albinism, so that it is evident that the albino parents transmitted the character total pigmentation. Therefore, in generation F_2 about $\frac{1}{4}$ of the animals are spotted. The colors obtained from the gray heterozygotes are: gray (32), gray-white (11), black (8), black-white (1), chocolate (1), and golden-agouti (2), as well as albinos (21).

The *black* heterozygotes of this same (F_2) generation, when interbred, produced only black-whites (4), blacks (7), and albinos (6). Thus the black failed to undergo further resolution, and it contained no other pigment in a recessive condition. This result is like that recorded by von Guaita, whose black mice, when interbred, produced no other color type than black.

When, however, gray heterozygotes were bred to black heterozygotes, both of the F_1 generation, the result was similar to that given by the gray animals bred *inter se.* For from this mating there resulted: gray (14), gray-white (5), black (5), black-white (2), chocolate (4), golden-agouti (4), pied golden-agouti (1), albinos (5).

These results show that gray with its *three* pigment elements is more readily resolvable than is the black, which consists of but *two* pigments. As will be shown presently, it is sometimes possible to obtain chocolate mice from black parents, hence it might have been expected that individuals of this color should occur among the offspring of the gray and black heterozygotes. But the presence of golden-agouti among these offspring seems plainly due to the introduction of the third or yellow pigment character through the *gray* parent. Moreover, when resolution does take place, the yellow element is the first to disappear, so that black individuals are next in point of numbers to the gray ones, and are nearly twice as numerous as either the golden-agouti or the chocolate mice. Leaving the albino young out of consideration, it is seen that in the cross of gray heterozygotes, resolution of the pigments took place with such frequency as to produce 12 individuals lacking one or two of the three pigments, as against 43 which possessed all the pigments and so were gray. In the cross of gray with black mice of the same (F_1) generation, 15 showed resolution of color, as against 19 which were gray. It is difficult to draw any definite conclusions as to the proportions of germ cells in which the pigment characters undergo resolution. For it is hardly probable that all the gray individuals result from the union of gametes each of which has the gray character, since this color might be produced by the union of any two gametes which *together* contained the three elements; thus gray × black, gray × chocolate, gray × yellow, as well as black-chocolate × yellow, or yellow-chocolate × black might result in the production of gray individuals. The writer has been unable to obtain mice with the yellow pigment alone by this process of segregation, but this negative result is not surprising when we recall that in the experiments of von Guaita (:00) neither chocolate nor golden-agouti types were produced, though these have appeared repeatedly in the writer's experiments.

Possibly this difference in results is due to a difference in the nature of the albinos used, and in the readiness with which the pigment elements undergo resolution. If, as is apparently the case, all the black animals in the writer's experiments contained both black and chocolate pigments, it follows that when the black or the yellow pigment elements became segregated from the other two in gamete formation, they must regularly have been eliminated, so that the individual resulting from two gametes of a similar pigment nature was either golden-agouti or black (i. e., black-chocolate). Similarly if the black and the yellow elements remain coupled, they are regularly eliminated, leaving only the chocolate element in the gamete, for no black-yellow type of mouse is certainly known. The chocolate element on the other hand may occur either alone or in combination with black or with yellow, giving in the one case, on union with a similar gamete, a chocolate individual, and in the other, either a black (i. e., black-chocolate) or a golden-agouti.

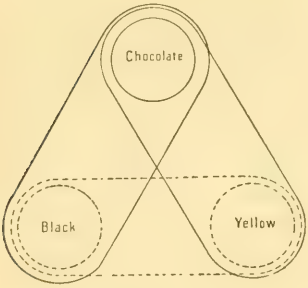


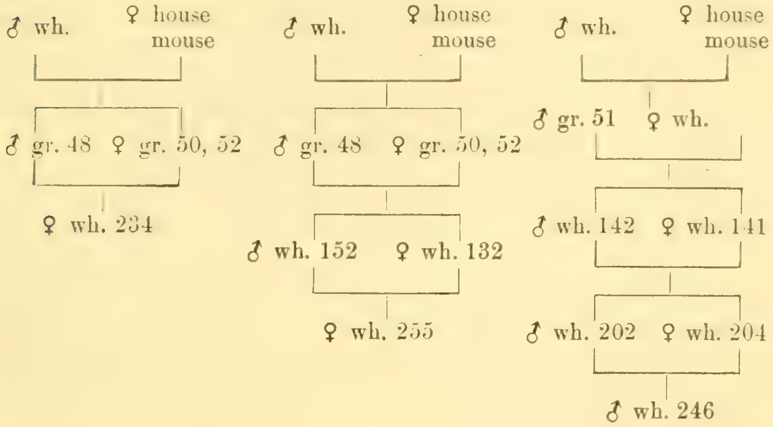
FIGURE 7.

Solid line indicates colors or combinations of colors which may result by segregation.

Dotted line indicates colors or combinations of colors which did not result by segregation in the writer's experiments, though theoretically possible.

The researches of Cuénot (: **03**), as well as those of other experimenters along similar lines, show clearly that in crosses between albino and pigmented individuals, the former, although showing no pigment colors, have nevertheless an equal share in the transmission of pigment characters. Whatever may be the nature of the pigment-producing elements, it is plain that the albino possesses these, and that they undergo segregation as in pigmented animals. The occurrence of the black heterozygotes, in the cross of black-white with albino mice, was certainly due to the union of gametes each containing the black-chocolate characters alone. For black mice regularly produce gametes with these pigment characters only, and it is therefore evident that the albino parent produced a certain number of gametes bearing the black-chocolate pigment character (or none). These uniting with those produced by the black-white mouse, resulted therefore in black animals having albinism recessive. Yet with the albino character went also the ability to transmit black-chocolate, *but no other colors.*

In a few cases albinos were crossed with black mice derived from the heterozygotes of house-mouse and albino parentage. Two of the black mice had albinism recessive, the third, though born in the same litter, produced no albinos. The ancestry of the albinos used in this cross is shown in the following diagram:



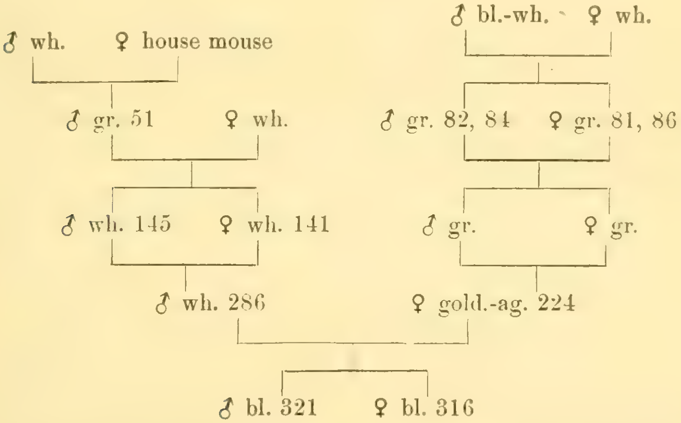
From each of these three pairs of black and albino mice, both gray and black young were obtained, the total being 10 gray, 4 black. There were also 16 albinos, as well as two which were dark-eyed but died before attaining the coat pigments. Here, again, if it be assumed that one-half the gametes of the albino contain the gray character and that in the other half segregation of this character takes place, so that at least one-quarter of all the gametes produced have the black character, then one in four of the pigmented offspring should be black, the others gray. In the experiment, 4 out of 14 mice *were* black, the number demanded by the hypothesis being 3.5.

In a similar case, where black animals from spotted stock were bred to the writer's original lot of albinos, there were produced 4 gray mice and 1 black one in addition to 3 albinos. The evidence therefore favors the idea that the albino may transmit the gray character entire in a certain proportion of its gametes, while the rest transmit regularly only certain components of that character.

In the course of these experiments, it was found that two black mice, if paired, produced only black young, even though one parent had the character albinism recessive. Since one-half the unions in this case take place between gametes containing respectively the albino and the black-pigment character, it follows that the albino gametes of a black animal contain only that pigment character which gives black. Consequently if two black animals, each having albinism recessive, are crossed, the albinos that result contain only that pigment character that makes for black. The truth of this hypothesis may be readily tested by breeding these albinos to pure black animals, the expected results being that only black offspring will be produced. This expectation has been fully realized in the experiments of the writer. Albino mice both of whose parents were black, have, in numerous crosses, produced no other color type than the black, when bred to black mice. This held true irrespective of the origin or degree of relationship of the black animals used. Whether black-white mice were used, or those wholly black derived from the house-mouse cross, or even a black stock purchased of a dealer, the result was the same, so far as the production of black pigment was concerned. This result is the same as that obtained by Cuénot (:03), who had albinos from a cross of two black mice, and found that they gave only blacks when bred to black animals. A similar case is recorded in "Taf. 2," of von Guaita's (:00) later contribution. His pair 3, consisting of an albino and a black animal, produced, in 11 litters, 16 black-white mice, 7 black, and 20 albinos. The parents of the albino used in this cross were a black and a black-white mouse, so that their albino young would possess the black pigment character only.

Additional evidence that the writer's black mice and the albinos obtained from them possessed only the two pigment characters that give black (i. e., black and chocolate) is afforded by the cross of black with chocolate animals. This cross, even supposing the pigment character of the black animal in some cases to become resolved into black and chocolate, would then afford but two combinations of pigments, namely, black with chocolate giving black, or chocolate with chocolate giving chocolate. If, however, the yellow character is present in a latent state in either parent, the cross should bring it into activity and a proportion of gray young should result. Thus far the writer has known of no case where gray young did occur in a mating of this sort. Two crosses were made in which a black-white female (403) was bred to two chocolate-white males, and each of the two litters thus obtained consisted of an albino and four black-white mice. That is, black was dominant over the chocolate. Again, an albino, both of whose parents were black, and which therefore should possess the black character, was bred to a chocolate female, and of the four young produced, two were albinos and two were black. Another albino mouse, also of black parentage, was bred to a chocolate male, and all the seven young were black. A like result was obtained in other crosses of black with chocolate mice. These experiments further indicate that the *albino actually transmits pigment characters* and is not, as might perhaps be thought, simply purified of pigment characters. For if the white parent transmitted no pigment, the pigmented young in this last case would have been chocolate.

If, then, black be dominant over chocolate, it should be possible to obtain both black and chocolate offspring by interbreeding the heterozygous black animals. This result was actually obtained by the writer in case of two black animals which came of a cross between an albino and a golden-agouti. As will be shown farther on, the black character doubtless came through the albino, which was of a stock known to transmit this character. The golden-agouti parent probably supplied the chocolate character. The following diagram shows the ancestry of the two black mice :



In three litters from this pair of black mice, there were obtained 6 black, 5 chocolate, and 1 white young. If each black parent produced, by segregation, gametes half of which were of the black, and half of the chocolate character, one-fourth of the young would be chocolate and the rest black as regards their pigment qualities. Of the 12 young actually obtained 5 were chocolate (3 being expected). The single albino which occurred was not tested to determine in which pigment category it should be placed. Additional evidence that chocolate is recessive toward black is afforded by the following cross. A black mouse, itself a cross between a black and a chocolate, and hence assumed to be heterozygous with respect to these two characters, was back-crossed with a chocolate individual. The expected result, on Mendelian principles, is an equality of black and of chocolate young. The single litter of three young obtained from this pair were two of them chocolate and one black, hence fulfilling the expectation as closely as is possible with an odd number of offspring. The following additional cases also point to the same relation of black to chocolate. Bateson (: 03^{b}. p. 83) includes, among a series of matings supplied by a Mr. F. G. Parsons, two cases in which black mice produce chocolate young. These additional cases are:

"(8) bl.-w. ♀ C (? DD) . . . × bl.-w. ♂ *a* (DR) = 3 bl.-w., 1 br.-w.
(37) bl. ♀ D^2(D × *a*) (DR) × bl.-w. ♂ *a* (DR) = 2 bl.-w., 2 bl., 1 br."

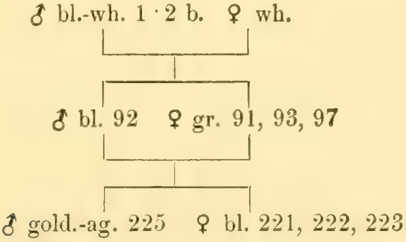
Thus the same male gave, by two black females, in the one case 3 black-whites to 1 chocolate, and in the other, 2 blacks and 2 black-whites to 1 chocolate, a total of 7 black to 2 chocolate individuals. If in each case both black parents had the chocolate character recessive, this result is approximately the 3 : 1 ratio expected on Mendelian principles. Moreover, the black-white male ♂ *a* was subsequently bred to two chocolate females, with the result that there were 4 black, 4 chocolate, and 6 albino young.

As regards the transmission of pigment characters, the gametic unions in this case were doubtless as follows:

bl.-ch. + ch. = gametes of ♂ bl.-wh. *a* [DR·R,]
ch. + ch. = " ♀ chocolate,
2 bl. + 2 ch. = F_1.

Both parents also had the character albinism recessive, but the albino young resulting from the cross would contain one or both of the two pigment characters latent, as could be shown by back-crossing them with the chocolate parent. If the chocolate character be recessive, it is expected that one-half the young will have the black, and one-half the chocolate character. If the albinos possess the same equal distribution of the pigment characters as do the pigmented animals, the Mendelian expectation is fully realized here. In the crosses given by Bateson, black and chocolate are the only pigments produced, although albinos, chocolates, and blacks are freely interbred. The albinos are doubtless from two chocolate or two black parents, or from a chocolate and a black, and thus do not contain the yellow element necessary to produce the gray coat. In the cases cited by Bateson, therefore, chocolate seems to be present as a character recessive toward black; for, except in the cases noted, black breeds true and is dominant over chocolate. Bateson's white females, A and B, were probably from chocolate parents, and so transmitted that pigment character only, for when bred to the heterozygous male, A, they both gave black as well as chocolate offspring, but when bred to chocolate males, chocolate pigment alone was produced. A third albino female, D, was probably heterozygous with respect to the black and the chocolate characters, for she gave young of both these colors by a chocolate male. In all cases where chocolate mice were interbred, chocolate was the only pigment obtained. The possibility of an alternative dominance of the one color over the other should not be lost sight of, but the writer's experiments do not clearly indicate such a relation.

The relation of the black to the yellow character was tested by breeding black mice to golden-agoutis. Each animal in this mating had the chocolate pigment present in the hairs, and the union of the two brought together all three of the pigment elements. Hence the cross might be expected to produce gray animals, and perhaps those of other colors as well, if the pigment characters undergo resolution. In the course of the writer's experiments three black mice, sisters, were bred to a golden-agouti male, their brother. The ancestry of these animals, so far as known, was as follows:



Three litters were obtained by this mating, giving a total of 7 gray and 2 black mice.

In a second case a golden-agouti male (485) was bred to an albino of black parentage, so that the union of the pigment characters was similar to that in the previous case. The three young obtained from this pair were all gray. Davenport (:**04**) records a somewhat similar case, in which a cross of a black with a yellow mouse resulted in a single gray young. Bateson (:**03**^{b}) states, further, that Miss Durham by crossing yellow with black mice obtained sables (i. e. gray with brighter flanks), dingy fawns (i. e. predominately yellow associated with dark pigment), and also some fully black individuals, but the yellow mice used by Miss Durham were presumably not golden-agoutis.

In the case of the gray and the black young obtained by the writer, it may have been that the former represented those animals in which a combination of all three pigment characters had taken place. The case of the few black animals, however, requires a different explanation. If they represent a combination of black-chocolate and of yellow-chocolate characters, then the yellow must have become latent. If, however, in the case of the yellow-chocolate character chocolate has been segregated in certain instances, then in those instances equal numbers of black and of gray mice should result. This is expressed by the following scheme:

bl.-ch. + bl.-ch. = gametes of black parent,
yell.-ch. + ch. = " " golden-agouti,
2 gr. + 2 bl. = F_1.

If this segregation takes place in half of the gametes produced by the golden-agouti, the total result is 3 gray to 1 black mouse, which is a relation not greatly different from that obtained, viz.: 7 gray, 2 black. But if we assume this segregation on the part of the golden-agouti, it seems reasonable that it should take place among the gametes of the black mice too, giving thus a certain number of chocolate individuals. Yet none occurred. Miss Durham's result also failed to show such a resolution on the part of the black character.

It is much more probable that the golden-agouti here employed may have contained the chocolate character recessive, or, at any rate, that the segregation resulted in gametes some of which had the yellow-chocolate, and others the chocolate character. The result would be the same in either case.

Unfortunately the writer was unable to obtain a second generation from the black offspring produced by the cross between a black and a golden-agouti. A few of the gray young were tested, however, with the following results. In one case, two of the gray young were bred together. If the gametes formed were of two sorts, corresponding to those produced by the black and the golden-agouti parents respectively, the following combinations would be possible:

bl. + gold.-ag. = gametes of gray ♂,
bl. + gold.-ag. = " ♀,
1 bl. + 2 gray + 1 gold.-ag. = offspring.

The expected result, provided that the gametes retain their purity with respect to pigment characters, is that one-fourth the offspring will be black, and one-fourth golden-agouti, while the remaining one-half will be gray, since they represent unions involving the three different sorts of pigment characters. The actual result was of this nature, for the three classes were present in the two litters obtained in the following numbers of individuals: 2 black, 2 gray, 8 golden-agouti. This, then, has the appearance of a simple case of Mendelian inheritance, wherein two pigment characters (black and golden-agouti) are produced in separate gametes equally numerous.

A further test of this hypothesis was made by back-crossing one of the gray (black × golden-agouti) mice with a golden-agouti female. If the gray mouse produces gametes whose pigment characters have kept their integrity, the two sorts of unions possible would give equal numbers of gray and of golden-agouti young, thus:

$$\begin{array}{ll} \text{bl.} + \text{gold-ag.} & = \text{gametes of gray male,} \\ \text{gold.-ag.} + \text{gold.-ag.} & = \quad \text{“} \quad \text{gold.-ag. female,} \\ \hline 2 \text{ gray} + 2 \text{ gold.-ag.} & = \text{offspring.} \end{array}$$

If, however, as in certain cases previously described, it be assumed that one-half the unions only are of this sort, while in the other half the gametes of the golden-agouti parent undergo resolution, the result in this latter case would be:

$$\begin{array}{ll} \text{bl.} + \text{yell.-ch.} & = \text{gametes of gray male,} \\ \text{ch.} + \text{yell.-ch.} & = \quad \text{“} \quad \text{gold.-ag. female,} \\ \hline 1 \text{ bl.-ch.} + 2 \text{ gold.-ag.} + 1 \text{ gr.} & = \text{offspring.} \end{array}$$

The total result would be four (50 per cent) of golden-agouti young, three (or $37\frac{1}{2}$ per cent) of gray young, and one (or $12\frac{1}{2}$ per cent) black, if the union of black with yellow results in black. These three classes of colors were all present in the single litter obtained, but the proportions were: one golden-agouti, three gray, one black. This experiment is not conclusive, therefore, but indicates that in some cases the gametes may retain their pigment characters intact, whereas in other cases these characters may undergo resolution. On the other hand, it is conceivable that the gray animal represents merely the coexistence of the three characters in the germ-cells as well as in the soma, and that it is a matter of chance whether at gamete formation the three pigment elements become variously segregated or remain together in one gamete. Such a state of affairs, however, seems improbable, since, if it existed, it would be possible to obtain from two pure gray animals offspring some of which would show merely the black, the yellow, or the chocolate pigment characters. No case of this sort is known to the writer. Further and more extensive tests will be necessary to a full understanding of the processes here taking place, but it seems for the present that pigment segregations occur chiefly, if not exclusively, under the influence of cross-breeding with other color varieties or with albinos.

Among rats, the black pigment character appears to behave much as in the house mouse. Black rats breed true, as observed by Crampe ('**85**) and others. Crampe also found that black or black-white rats, if they contained albinism recessive, produced only black pigmented young, in addition to the Mendelian proportion of albinos.

In the case of rabbits, however, black seems to be of a somewhat different nature. Woods (: **03**) finds that on interbreeding black rabbits a small proportion come gray or yellow. His figures are: 105 black, 9 gray, 8 yellow. This may indicate simply an impure black, or the yellow element may be latent or recessive in certain individuals. In the case of the albino 112, *both of whose parents were black*, it was found that, with three exceptions, only black and albino young were produced when it was mated with a black heterozygote. The three exceptions were all gray young, and came of a black animal whose parents and four grandparents were black. However, since black animals bred *inter se* may produce a small proportion of gray young, the result from breeding black to black is practically the same as that obtained by breeding black to albino of black parentage, as has been found true of mice.

3. *The chocolate type.* The chocolate mice used in these experiments were derived from the gray heterozygotes of the cross between a black-white and an albino stock. According to Bateson (: **03**^{b}), his pupil, Miss Durham, finds the hairs of chocolate mice to contain only the chocolate pigment, and this observation the writer has been able to confirm by a microscopical examination of hairs from his chocolate mice.

Of the mice of this type obtained from the gray heterozygotes above mentioned, some were found to have the albino character recessive, while others were pure with respect to albinism. Mice of the latter sort, when interbred, produced only chocolate offspring. A similar result was obtained by mating a pure chocolate animal to one having albinism recessive. In this case the gametic unions (formula 5, p. 72) are of two sorts:

$$\begin{array}{ll} \text{ch.} + \text{ch.} & = \text{gametes of pure chocolate,} \\ \text{ch.} + \text{wh.} & = \text{“ \quad heterozygous chocolate,} \\ \cline{1-1} 2 \text{ ch.} + 2 \text{ ch. (wh.)} & = \text{offspring.} \end{array}$$

Consequently, one-half the chocolate young from this sort of a cross result from the union of a gamete of the chocolate pigment character with one of the albino character. The latter, however, carries potentially the chocolate character, so that the result of the union is to produce a chocolate animal, since no other color element is introduced. It follows that if two chocolate mice of this sort are interbred, the 25 per cent of albinos resulting will in like manner possess the chocolate character.

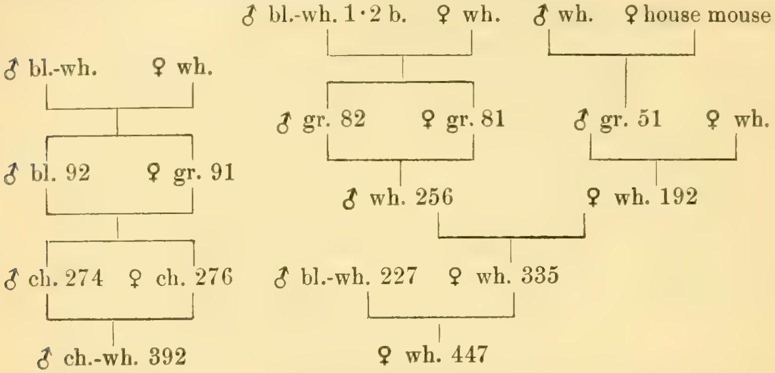
The unions of gametes are (formula 2):

$$\begin{array}{c} \text{ch.} + \text{wh.} \\ \text{ch.} + \text{wh.} \\ \hline \text{ch.} + 2 \text{ ch. (wh.)} + \text{wh.} \end{array}$$

This the writer has been able to demonstrate by actual experiment. Such albinos from two heterozygous chocolate parents, when bred to chocolate mice, gave chocolate young only. The obvious inference from results of this sort is that *the albino gametes formed by a pigmented animal which has the albino character recessive, possess the same pigment potentiality as the pigment bearing gametes formed by the same animal.* An albino, therefore, both of whose parents had the chocolate character only, will produce young of that character, when mated with a chocolate animal. This fact is doubtless the explanation of Bateson's (:**03**^{b}) statement that "Miss Durham has found chocolate a simple dominant over albino," for her albino was presumably of chocolate parentage.

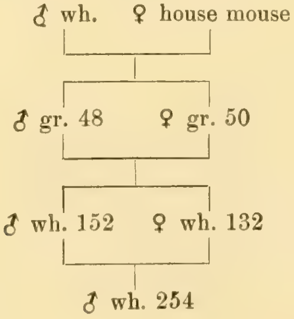
A different result is obtained by mating chocolate mice with albinos of mixed ancestry. The writer made several matings between chocolate mice and albinos, the latter being extracted recessives from crosses between gray house mice and the original stock of albinos previously mentioned. Four different albinos were paired with as many chocolate mice, and of the 22 resulting young, 20 were gray and 2 black. The black was presumably transmitted through the albino stock, which, as has been shown, seemed to produce a certain number of gametes having the black character. The predominance of the gray character is strong. Bateson (:**03**^{b}) records a cross of similar result, wherein a chocolate and an albino gave gray young.

An interesting and different result was had by breeding to a chocolate-white mouse an albino of a yet different extraction. The following scheme shows the ancestry of the pair so far as known:



The single litter from this pair consisted of 5 young, of which 1 was chocolate, 2 were gray, and 2 black. Assuming that the chocolate parent produced gametes having the chocolate character only, it is necessary to suppose that the albino transmitted through some of its gametes the black character only (producing the black offspring), and through others the gray character (from the gray grandparents three generations back). If some of the germ cells having the gray character should produce by resolution gametes having the chocolate character only, the chocolate class would be produced also. At all events, it seems clear that the third or yellow element was contributed by the albino parent, which in turn had inherited it from the gray mice on the maternal side.

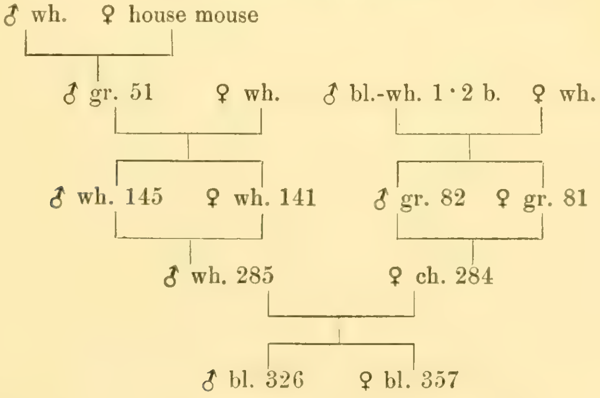
In order to test further the heredity of the chocolate character, the writer mated together two of the gray heterozygotes coming from a cross between chocolate mice and albinos, which in turn were the descendants of a house mouse and the writer's original albino stock. The ancestry of the albino parent of the two gray animals is as follows:



The albino male, 254, gave gray animals on being bred to the chocolate female, 264. These gray offspring, bred *inter se*, produced in three litters, 1 chocolate, 2 golden-agouti, 2 black, 12 gray, and 5 albino young.

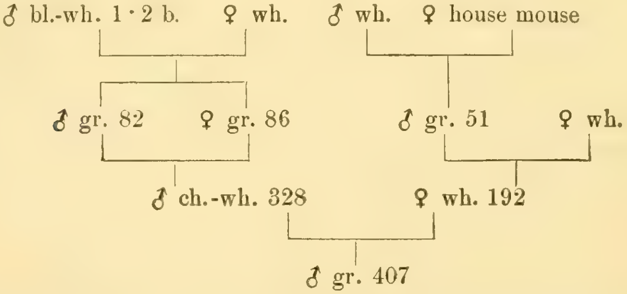
It is difficult, in view of the small number of young obtained, to formulate a wholly satisfactory hypothesis to account for the combinations of color characters which here took place. The chocolate offspring are to the other pigmented young as 1 : 16 instead of being as 1 : 3, the expected ratio. Apparently, a certain number of gametes bearing the black character were produced by the gray heterozygotes, and some of these gametes may have united with those bearing the chocolate character, thus producing black offspring, while at the same time reducing the number of pure chocolate individuals. The large proportion of gray young may indicate either the presence of unresolved gray in some of the gametes, or the recombination of the three elements of a gray which underwent resolution at gamete formation. The white male, 254, must have contributed the characters other than chocolate which appear in his grandchildren.

The occurrence of a few black mice in crosses between chocolates and certain albinos has been mentioned. The ancestry of two of these black individuals, which were saved for further testing, is as follows:



If the chocolate parent produced only gametes having the chocolate pigment character, these black individuals must have resulted from the union of such a chocolate gamete with one from the albino containing the black character, and hence must be heterozygous with respect to the two characters. The albino was descended from a stock of albinos known to transmit the black character in a certain proportion of cases, hence this supposition seems plausible. Moreover, from the writer's crosses of black and chocolate mice, as well as from the data given by Bateson for similar crosses, it seems fairly clear that the chocolate under these circumstances may behave as a simple recessive towards black. In this case, the black heterozygotes, on being back-crossed with chocolate mice, should give both black and chocolate individuals in the ratio, 1 : 1. This, in fact, was approximately the result obtained. In six litters a total of 10 black and 13 chocolate mice was obtained, each class being thus within two of the expected relation, 11.5 : 11.5.

A different result came from back-crossing with a chocolate mouse a *gray* heterozygote whose parents were respectively a chocolate and a white mouse. The ancestry of the gray animal follows :



This gray mouse gave, in two litters by a chocolate mouse, 3 black, 3 gray, and 3 golden-agouti young. If the chocolate mouse produced only gametes having the chocolate pigment-character, while those of the gray mouse in half the cases underwent resolution and in the other half retained the gray character unresolved, the gametic unions would be :

gr. + gr. = gametes of gray parent (unresolved),
ch. + ch. = " chocolate parent,
4 gr. = offspring.

bl.-ch. + yell.-ch. = gametes of gray parent (resolved),
ch. + ch. = " chocolate parent,
2 bl. + 2 golden-agouti = offspring.

The result is that three classes of young, gray, black, and golden-agouti, are produced, in the proportions 2 : 1 : 1. In the experiment these three classes were present, but in the proportions 1 : 1 : 1.

One of the three gray young from the cross of the gray heterozygote and the chocolate female was back-crossed with the same chocolate mouse, its mother. The single litter obtained consisted of 2 gray, 1 black, and 1 chocolate offspring, a fact which probably indicates that the gray animal produced a certain number of gametes with the black character, and others with the chocolate character. The number of young obtained is, however, too small to allow of generalization.

This same gray male was also mated with an albino female which transmitted the black pigment character. If chocolate always goes with black, the combinations of pigment characters represented by this mating are probably as follows:

$$\begin{array}{rl} \text{bl.-ch.} + \text{yell.-ch.} & = \text{gametes of gray parent,} \\ \text{bl.-ch.} + \text{bl.-ch.} & = \text{gametes of albino parent,} \\ \hline 2 \text{ black} + 2 \text{ gray} & = \text{offspring.} \end{array}$$

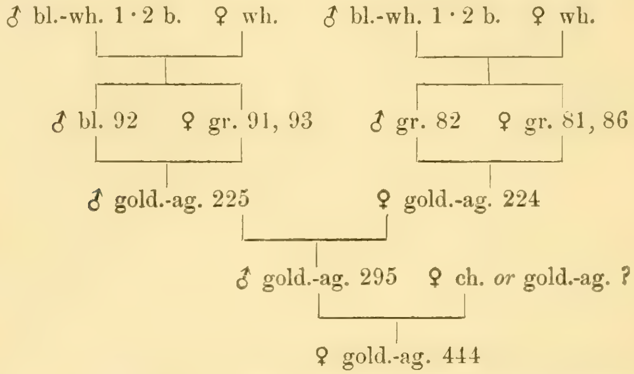
The observed result was 3 black, and 4 gray mice, so that, although the totals obtained are small, it seems probable that the gray character did undergo resolution in the manner suggested.

Evidence has been adduced from the experiments cited by Bateson and from those performed by the writer to show that the black coat character may be a simple dominant over chocolate. The relation of the chocolate character to golden-agouti may next be considered.

4. *The golden-agouti type.* This is the type of mouse which Bateson describes as possessing the chocolate and the yellow pigments, but not the black. It therefore has a dingy or tawny appearance. Microscopical examination of the hairs of certain mice bred by the writer confirms this description.

These mice occurred in small proportion among the young of gray heterozygotes, themselves the offspring of black-white and albino mice, and hence are evidence that a resolution of color characters took place in the gametes of their gray parents. Since the golden-agouti contains but the two pigments, yellow and chocolate, the utmost number of varieties obtainable by interbreeding two animals of this sort is manifestly three, assuming that both pigment characters undergo resolution in the same manner. These would be, a pure yellow, a pure chocolate, and the combination of the two, or golden-agouti. In the writer's experiments, only the last two types have appeared. No pure yellow mouse has been obtained among all the various crosses, which have produced hundreds of offspring. This fact may point to some physiological peculiarity in case of the writer's stock, by reason of which the yellow pigment character was unable to assert itself to the same degree as did the black and the chocolate, or perhaps the complete resolution never took place.*

A number of matings were made between golden-agouti mice, and the usual result was offspring all golden-agouti in color. In case of two pairs of golden-agouti mice, however, chocolate young resulted, as well as those of the parents' color. The ancestry of the first of these two pairs is as follows:



The golden-agouti female, 444, was bred back to her father of the same color, 295. The writer is not absolutely certain which of two animals was the mother of the female 444, a chocolate or a golden-agouti, both of which happened to be with the ♂ 295 at the time. More probably, however, a chocolate mouse was the mother. At all events, on back-crossing the golden-agouti daughter with her father, 3 golden-agouti and 2 chocolate young resulted. If both animals in this cross had the chocolate character recessive, this result is readily explicable. It is not entirely clear, however, that this was the case. If chocolate be recessive to pure golden-agouti, it is plain that ♀ 444 is heterozygous with respect to the two characters. It therefore remains to show that her father, 295, was of a similar nature. The parents of ♂ 295 were both golden-agoutis, the one from two gray parents, the other from a gray and a black parent. The black parent could produce, by resolution of its pigment characters, only the black or the chocolate. Assuming that the latter character only was present in some of the gametes of the black animal, and that one of these united with a gamete from the gray animal that contained both the chocolate and the yellow characters, but not the black, then the golden-agouti, 225, may well have retained the chocolate character recessive. Golden-agouti ♀ 224, from two *gray* parents, may be assumed to contain no chocolate *recessive*. The mating of 224 and 225 will then result in golden-agoutis of two sorts, the one pure, the other with chocolate recessive, and 295, of course, must be taken as of the latter class. This explanation is put forward tentatively, pending test by further experiment, but another possibility is also not to be overlooked, namely, that the color characters of a golden-agouti may undergo resolution. If the yellow and the chocolate pigment characters become segregated into separate gametes, then it is clear that golden-agouti as well as chocolate young would result, but in addition there should be some pure yellow offspring as well. The fact that none have yet occurred, speaks against a complete resolution of the yellow-chocolate combination. Possibly, however, the resolution of the yellow-chocolate takes place, only in such a way that gametes are formed which are respectively yellow-chocolate and chocolate, as regards their pigment characters.

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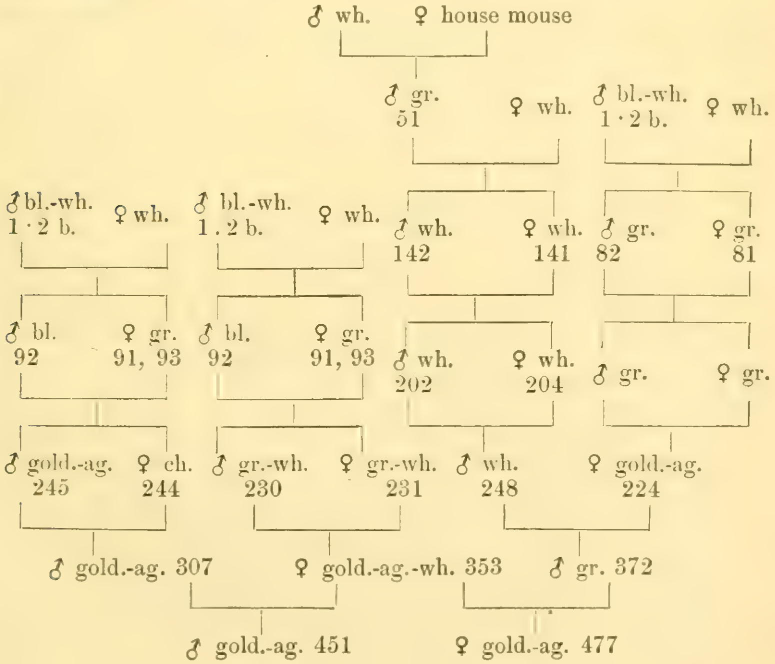
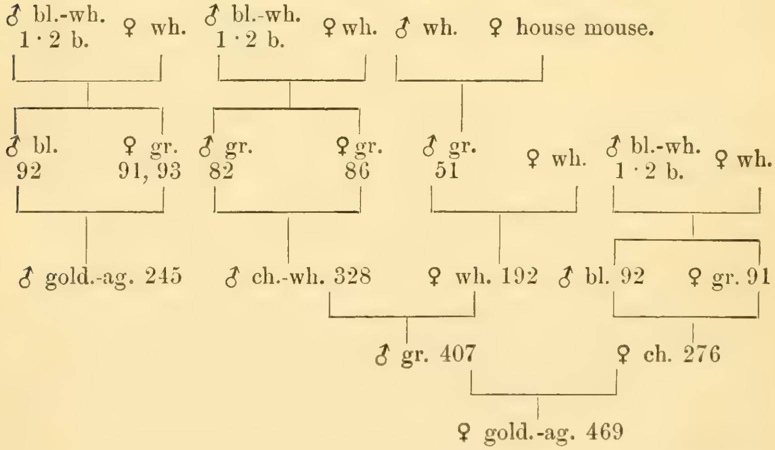
* Bateson, in a letter to Professor Castle, received some time since, suggests that the yellow color of certain fancy mice may be due, not to resolution of the wild gray coat, but to crossing with *Mus sylvaticus*, "which has almost certainly taken place."

The second pair of golden-agoutis that produced chocolate offspring as well as those of their own color, were of the following rather complicated descent:



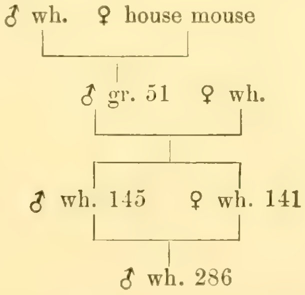
Both animals of this pair, numbers 451 and 477, had albinism recessive, that character having been handed on in this condition through four generations in case of the ♂ 451. For in the first litter obtained from them there were 3 albino, 2 chocolate, and 1 golden-agouti young. It is difficult to see how the female of this pair could have inherited a recessive chocolate character, though the male might conceivably have done so, through the chocolate grandparent.

If chocolate be recessive toward golden-agouti, a cross between "pure" animals of these two colors should result in offspring of the latter pigment type only. Only two crosses have thus far been made by the writer between these two varieties. In the one, a golden-agouti male, 245, was bred to a chocolate female, 244, and in three litters there resulted 10 chocolate to 7 golden-agouti offspring. In the second case, a golden-agouti female, 409, was paired with an albino of chocolate parentage, known by tests to have the chocolate pigment-character only. Two young, a chocolate and a golden-agouti, resulted, so that these two pairs of mice are comparable with respect to the pigment characters they transmit. If, then, the golden-agouti parent of each pair can have possessed chocolate in a recessive condition, the approximate equality of the two sorts of young among the offspring is readily explicable on Mendelian principles. The ancestry of the two golden-agoutis is as follows:



If the golden-agouti ♂ 245 contains the chocolate character recessive, it must have come from the black parent through resolution. The gray parent probably supplied a gamete with the chocolate-yellow character, so that a golden-agouti offspring with chocolate recessive was produced. A somewhat similar process may have taken place in the case of golden-agouti ♀ 469. One parent was chocolate, the other gray. A resolution of the gray character into black and yellow-chocolate may have taken place, and a gamete of the latter pigment-character, uniting with one from the chocolate female 276, would produce a golden-agouti with the chocolate character in double strength, perhaps segregable. In any event, it seems clear either that the two golden-agoutis here considered produced gametes, some with the pigment character which they themselves showed and others with the chocolate character, or else that there is no necessary dominance of either pigment character over the other, but that sometimes one and sometimes the other dominates.

A number of crosses were made between golden-agoutis and albinos, the results of which varied according to the nature of the albinos used. Four golden-agouti mice were bred to albinos descended from gray house mice and the writer's original stock of albinos. In a total of thirty-five pigmented offspring, two were black and the others gray. The albino parent of the two black young produced also four gray mice in the same litter. Her ancestry is as follows:



The black character probably came through the albino stock, which, as was pointed out in discussing the cross of black mice with that stock, seemed to produce about 25 per cent of gametes with the black character. Moreover, a segregation of pigment characters must also have taken place in case of the golden-agouti female, probably into chocolate and either yellow or yellow-chocolate, else the black young could hardly have resulted. For any other combination of the pigments indicated would have given gray, not black. The latter color, then, probably resulted from the combination of a gamete having the black character with one having the chocolate character, the former produced by the albino, the latter by the golden-agouti. The black offspring, on this hypothesis, may contain the chocolate in a recessive, or at least in a segregable condition. Fortunately the two black young were a male and a female, and they were therefore mated. In three litters, they produced 6 black, 5 chocolate, and 1 albino offspring. On Mendelian principles, 25 per cent or 3 of these young are expected to be chocolate, as regards pigment characters. There are actually 2 more chocolates than expected, disregarding the albino, whose character was not determined. It therefore seems possible that these two black mice did arise in the manner suggested, and produced gametes, some of which were black and some chocolate, as regards pigment characters. This same pair has been previously mentioned in discussing the relation of chocolate to black.

The gray heterozygotes resulting from the cross of golden-agoutis with albinos may have originated either (1) by the dominance of the gray character which came from the house mouse through the albinos, or (2) by the union of the yellow-chocolate character furnished by the golden-agouti parent with the black character, which, as well as gray, the albinos transmitted, though to a lesser degree. If, now, at the formation of gametes the color characters undergo segregation to the extent only of undoing the original (heterozygous) combination of gray and chocolate-yellow, or black and chocolate-yellow, it is plain that generation F_2 will consist of offspring exhibiting three types of color, as seen in the following diagram of the possible unions:

gr. + yell.-ch. = gametes of gray hybrid of first sort,
gr. + yell.-ch. = gametes of gray hybrid of first sort,
3 gr. + 1 gold-ag. = offspring.

bl.-ch. + yell.-ch. = gametes of gray hybrid of second sort,
bl.-ch. + yell.-ch. = gametes of gray hybrid of second sort,
1 bl. + 2 gr. + 1 gold.-ag. = offspring.

gr. + yell.-ch. = gametes of gray hybrid of first sort,
bl.-ch + yell.-ch. = gametes of gray hybrid of second sort,
3 gr. + 1 gold.-ag. = offspring.

The chances in favor of the third combination are twice as many as for either of the other two, hence generation F_2 should consist of young of three color types in the proportions: 1 black, 11 gray, 4 golden-agouti. The writer actually obtained, in five litters, 1 black, 14 gray, 4 golden-agouti young, as well as 5 albinos, which were not tested. The close agreement of the observed with the calculated total seems to corroborate the conclusion previously stated, that the writer's original albino stock produced gametes some with the gray, some with the black pigment character. It also is good evidence of the segregation of the several pigment characters of the heterozygote in such a manner as simply to unloose them from their combination practically unchanged. Five males and four females contributed to the result just given, but only one of the females was proven to have the black character among her gametes, while the male by which she had the black mouse subsequently died before further tests could be made. The same female by three other of her brothers gave only albino, gray, and golden-agouti young.

To test the matter still further, however, three of the same lot of gray males were back-crossed with golden-agouti females. Assuming as before that some of the gray animals produce gametes with the *black* and the golden-agouti characters respectively, and others those with the *gray* and the golden-agouti characters respectively, the possible combinations are as follows:

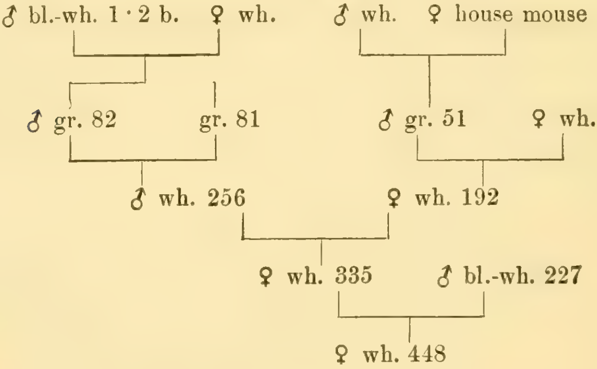
$$\begin{array}{l}\text{gr.} + \text{yell.-ch.} \\ \underline{\text{yell.-ch.} + \text{yell.-ch.}} \\ 2\ \text{gr.} + 2\ \text{gold.-ag.}\end{array}\begin{array}{l} = \text{gametes of gray parent,} \\ = \text{gametes of golden-agouti parent,} \\ = \text{offspring.}\end{array}$$

$$\begin{array}{l}\text{bl.-ch.} + \text{yell.-ch.} \\ \underline{\text{yell.-ch.} + \text{yell.-ch.}} \\ 2\ \text{gr.} + 2\ \text{gold.-ag.}\end{array}\begin{array}{l} = \text{gametes of gray parent,} \\ = \text{gametes of golden-agouti parent,} \\ = \text{offspring.}\end{array}$$

The expected result is that only gray and golden-agouti young will be produced, and that these two classes will be equally numerous. This result was realized qualitatively, but not quantitatively, for there were actually produced 19 gray and 7 golden-agouti young. In other words, there are nearly one-quarter fewer golden-agoutis than expected. If this discrepancy be simply the result of chance, the explanation suggested may still be considered valid, yet the excess of gray animals is so great that it seems hardly proper to ignore it entirely.

Having thus followed through the results of crossing golden-agoutis with albinos of mixed extraction, it remains to mention two other crosses in which the albinos used were of a yet different nature as regards pigment characters. A cross was made between a golden-agouti male and an albino female of black parentage, which therefore had only the black pigment character. This union brings together in the heterozygote all three pigments, and consequently only gray offspring are expected. All the eight young from this pair were, in fact, gray. The result is, therefore, quite similar to that obtained by mating black with golden-agouti mice, for, as previously stated, gray young usually are produced. Black individuals may result, however, from such a cross as this. Their occurrence is due probably to the union of a gamete having the black character (from the black parent) with one having the chocolate character (from the golden-agouti parent).

Finally, a golden-agouti was mated with an albino female of the extraction indicated by the following diagram:



This albino, 448, when mated with a golden-agouti male, 484, produced 2 gray and 2 golden-agouti young. The ancestry of the black-white grandparent, 227, is unknown, beyond the fact that it was of a stock which had been crossed with chocolate as well as with black individuals. In this mating, it is necessary to assume a segregation of characters in such a way that one-half the gametes of the albino female, 448, contain no black element. Union of such gametes with the chocolate-yellow gametes of the golden-agouti male would produce golden-agouti young. The gray character may have come wholly through the white female, 335, certainly its black constituent did, so that it seems necessary to assume also that the black-white ♂, 227, had chocolate recessive.

After many crosses, it was at last possible to obtain albinos both of whose parents were golden-agoutis. The same mating produced also golden-agouti and chocolate young. It is therefore expected that these albinos, on being back-crossed with mice of their parents' type, will give either chocolate or golden-agouti young, but not gray or black offspring. This experiment is still in progress.

5. *Comparison with Mirabilis.* The behavior of pigments in the case of mice seems to be somewhat similar to that recorded by Correns (:02) in certain varieties of Mirabilis. Several types of blossoms are recognized which, as regards pigment characters, may probably be reduced to three, viz.: (1) red, with which is to be included the rose-type, a "dilute" form of red; (2) yellow, which occurs also in a dilute or *yellowish* form; and (3) the albino. These several types breed true as a rule. Red corresponds to gray in mice, and is probably a combination of red with yellow chloroplasts. Pied varieties occur, as with mice. Correns found that red × albino gave all red blossoms, which shows either that red is a simple dominant over whatever pigment-character the albino may have contributed, or that the latter also contained the red character. The ancestry of the albinos is not given. A cross of albino with rose resulted, however, in rose flowers, so that the former may have transmitted red in the dilute form. At all events, no yellow flowers were produced. A third cross was made between the yellowish stock and the albino. The latter doubtless had the red pigment-character present in dilute form, and was dominant over yellow, for rose flowers were produced. This result is like that obtained by pairing albino mice of gray ancestry with other types lacking one or two of the three color elements, so that the gray results either through dominance or through combination of all the color characters. When the albino type of Mirabilis was crossed with the bright (i. e., undiluted) yellow type, *full red* flowers resulted. The albino used in each of these crosses with the yellow sort must have transmitted the red character in dilute form, for with the *dilute* yellow type dilute red or rose flowers resulted, while with the *full* red type full red flowers were produced. The former cross is therefore quite the same as that in which dilute red and dilute yellow are crossed, giving blossoms of the former or rose type. It is therefore fairly clear that the albino forms of these plants may transmit pigment-characters just as do the albino mice. The data given by Correns as to the behavior of the spotted character do not show very conclusively whether or not it is recessive toward the character total pigmentation. One cross of the albino with mottled white-and-red gave some of the spotted type and a less number that were all red.

VII. Conclusions.

From the experiments here recounted, the following conclusions as to the heredity of color characters in mice are drawn :

(1) *Albinism.* Complete albinism is a character recessive toward either partial or complete pigmentation. Heterozygotes, from a cross of pigmented with albino individuals, produce gametes having the albino character and those having the pigment-forming character in approximately equal numbers. The chance combinations of gametes from two such heterozygotes give rise, in the long run, to pure dominants, heterozygotes, and recessives, in the proportions, 1 : 2 : 1. The pure dominants and the pure recessives breed true *inter se* to the pigment-forming and the albino characters respectively. A heterozygote bred to an individual of either pure class produces, as an average result, equal numbers of heterozygotes and pure animals.

Partial albinism is a condition in which the pigment is reduced around definite body centres, so that unpigmented areas occur between the pigment patches or at their borders. Ten such centres may be distinguished in the mouse, arranged symmetrically five on either side of the median plane. These centres represent a cheek patch, a neck patch, a shoulder patch, a side, and a rump patch. Various degrees in the reduction of the pigment patches occur, or the patch may be eliminated altogether. Partial albino, or pied individuals, breed true to that condition, but partial albinism behaves toward complete albinism as a recessive character, although the heterozygote may sometimes show an intermediate condition. Moreover it is possible for albino mice to transmit either the character total pigmentation or partial pigmentation, or an albino may be heterozygous with respect to these two characters so that, when bred to a pied mouse, equal numbers of pied and of self-colored offspring result.

When pure self-colored mice or rats are bred to pied animals having albinism recessive, generation F_1 consists of two sorts of pigmented individuals both of which are heterozygous, one with respect to the characters total and partial pigmentation, the other with respect to total pigmentation and albinism. A random mating of the F_1 individuals results in the production of pigmented and albino offspring in the ratio, 15 : 1. A like result is obtained if the parental generation consists of a pure pied animal and a totally pigmented one having albinism recessive. Generation F_2, obtained by interbreeding the offspring of such parents, consists theoretically of six classes of individuals, viz. : D, DR, D·DR, D·R, DR·R, and R. Most of these classes have been demonstrated to be actually present.

If the hypothesis advocated by Cuénot be correct, that pigment is produced by the action of specific ferments on a chromogenic substance, and that albinos carry the former but not the latter, then it must be assumed that the two characters opposed in a cross of a pigmented with an albino form are: (1) presence of chromogenic substance, and (2) absence of chromogenic substance. At all events albinos may transmit pigment-characters, even though they themselves show no trace of pigment.

An albino that transmits the character partial pigmentation may be looked upon as potentially a spotted mouse in whose soma the pigmented areas do not show, perhaps because the necessary chromogenic substance is lacking. The spotted condition may be due to a decrease in the amount of both the chromogenic substance and the specific ferments as compared with the amounts of these substances in a totally pigmented animal.

2. *Pigments.* (1) The gray coat of the house mouse contains the three pigments, black, chocolate, and yellow, as has been stated by Bateson (:03^b), and this combination as found in the wild animal is strongly dominant over any of the colors shown by fancy varieties (with the possible exception of yellow, which the writer has as yet had no opportunity to test). Therefore when wild gray animals are bred to varieties of other colors, gray offspring result.

By interbreeding gray heterozygotes, themselves from a cross between gray house mice and albinos, a few black offspring were obtained, indicating that a resolution of the gray color may have taken place in a few cases. No further resolution of the gray from house mouse crosses was obtained.

In the case of gray heterozygotes resulting from a cross of fancy black-white with albino mice, resolution took place to the extent of producing the golden-agouti and the chocolate, as well as the black type.

(2) Animals of the black type breed true to that color. This statement holds good even if one or both of the parents contain albinism recessive. For in this case the gametes having the albino character carry also the specific "ferment" or whatever it may be, that stands for black.

It follows that an albino, both of whose parents are black, transmits the black character. Such an albino, bred to black animals, is found to give young having the black-chocolate pigment-characters only.

Black mice bred to albinos of gray or of mixed ancestry produce gray and sometimes also black offspring, according as the gray character present in the albinos remains unresolved or undergoes resolution.

The black-pigment character appears to be dominant over the chocolate character, so that a black mouse, or an albino bearing the black character latent, gives only black young when bred to a chocolate individual. These black heterozygotes bred *inter se* may produce both black and chocolate offspring, in approximately the ratio, 3 : 1, or if back-crossed with chocolate mice, the two classes may appear in nearly equal proportions among the young.

When black mice, or albinos bearing the black character, are bred to golden-agoutis, the three pigments constituting the gray coat are brought together by the union, and gray offspring result. Black young may also occur, in small numbers, indicating either a *resolution* of the yellow-chocolate character of the golden-agouti, or the *segregation* of the chocolate character present in either a recessive or a latent condition in the golden-agouti parent. It should, therefore, be possible to obtain chocolate young also from the cross of such a golden-agouti with a black animal having the chocolate character recessive.

No evidence has been obtained to show that black may be a simple dominant over golden-agouti, for gray offspring regularly result from mating pure black with pure golden-agouti mice. From such gray heterozygotes, a second generation was obtained, consisting of black, gray, and golden-agouti young, and indicating, therefore, that the black and the golden-agouti characters kept intact. One of the heterozygotes was back-crossed with the golden-agouti parent and produced the same three coat characters, black, gray, and golden-agouti. The golden-agouti parent, in this case, must have produced some gametes with the chocolate character only, so that black young resulted. (3) The chocolate type, represented by mice having pigment of that color only, breeds true, even if one of the parents contains albinism recessive. Chocolate mice, whether having albinism recessive or not, therefore, contain no other pigment-character. Albinos of two chocolate parents will, on being bred to chocolate mice, produce offspring having only the chocolate-pigment character.

Chocolate mice, bred to albinos of gray parentage, produce gray young. Black young have resulted, in a few cases, from breeding chocolate mice to albinos of mixed ancestry. This fact probably indicates a segregation of the color characters borne by the albino, in such a manner that the black character is present with chocolate, but without yellow, in some of the gametes, and is dominant over the chocolate character of the gamete with which it unites. In support of this hypothesis, it has been found that these black animals, on being back-crossed with chocolate mice, produce nearly equal numbers of black and of chocolate young.

The gray heterozygotes resulting from the cross of chocolate with albino mice, when interbred, did not produce the expected one-in-four of chocolate mice, but instead golden-agouti and black young were more numerous than those of the chocolate color, and these three classes were together about one-fourth as numerous as the gray. This result is probably due to a resolution of a portion of the gametes of the albinos having the gray character, so that black-chocolate and golden-agouti are present in different gametes. A similar resolution is also indicated by the back-cross of one of the gray heterozygotes with the chocolate parent, for the resulting offspring represented the gray, the black, and the golden-agouti types, but not the chocolate.

(3) Golden-agouti mice, characterized by having the yellow and the chocolate pigments, breed true to their type as a rule, but may give the chocolate type also. Chocolate as well as golden-agouti mice may be obtained by crossing the two types, but it is not yet certain whether this result is due to a resolution of the color of the golden-agouti or whether the latter contains chocolate in a recessive condition.

A cross of golden-agoutis with albino mice coming of mixed ancestry (including gray) resulted for the most part in gray young, though one golden-agouti female produced two black young, when bred to such an albino. These latter probably indicate that the female was producing some gametes with the chocolate character, which, uniting with the gametes from the albino having the black character, gave black young. This hypothesis is supported by the fact that the two black young bred *inter se* gave both black and chocolate offspring, and so must have been heterozygous with respect to the two pigment-characters. The albinos used were of a stock known to produce a proportion of gametes having the black character.

The gray young, resulting from this cross of golden-agouti with albino mice, gave, when interbred, about one-fourth of golden-agouti offspring, indicating that that character became segregated at gamete formation. Except for a single black young, the other progeny were gray.

Back-crossing the gray heterozygotes with golden-agouti animals reproduced the two types only, but the grays were present in slightly more than their expected proportion, although the total number of young was not large.

(4) In general, therefore, unresolved gray is dominant over all of the

other color types employed. Gray may also be produced synthetically by breeding together a black and a golden-agouti animal, for all three of the pigments composing the gray coat are thereby brought together in the young. So far as concerns the pigment-character of the young produced, either parent in this cross may equally well be replaced with an albino derived from parents of the pigment type indicated. Gray should also result from crossing the black type with the pure yellow, or with an albino of pure yellow ancestry, or from a cross between a pure yellow and an albino of black ancestry.

Black, chocolate, and golden-agouti mice as a rule breed true; but black may be a simple dominant over chocolate, and there is some evidence that a similar relation may exist between golden-agouti and chocolate. Both black and golden-agouti agree in consisting partly of chocolate pigment, so that the added recessive chocolate may have no appreciable effect on the appearance of the heterozygous individual.

All three of these color types may arise through a resolution of the compound character, gray. A similar resolution of compound pigment-characters occurs in the case of some plants, as in Mirabilis.

POSTSCRIPT.

Since the foregoing account was written, the author has seen Cuénot's third report on his experiments in breeding mice. It is gratifying to note that this investigator also finds the character partial albinism in mice to be recessive with respect to total pigmentation. The spotted or partial albino character, he finds, is separately heritable, and may be possessed potentially by an albino. Other albinos may possess the character producing total pigmentation, and both pigmented and albino individuals may be heterozygous with respect to the two coat characters. The results detailed in the present paper, therefore, completely confirm those obtained by Cuénot in regard to the heredity of the spotted character.

Cuénot also describes some experiments in crossing yellow mice with other color varieties, and finds, rather curiously, that the yellow character is dominant over both gray and black. This result indicates a strong individuality for the yellow character and strengthens the belief that the yellow type of mouse is in origin different from the black, chocolate, and golden-agouti types, and may, as Bateson suggests, be derived from another species, perhaps *Mus sylvaticus.*

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A NEW GENERAL THEORY OF ERRORS.

By William Edward Story.

A NEW GENERAL THEORY OF ERRORS.

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I. INTRODUCTION.

THE ordinary theory of errors — the method of least squares — has been developed in three ways (methods of Gauss, Hagen, and Crofton); not only is each of these developments based upon assumptions to which not every one will assent, but at certain stages in each of the two later developments exactness is replaced by approximation through the neglect of certain quantities (or terms) regarded as very small (or productive of very little effect) in comparison with others retained, whereas this neglect is not fully justified, as it seems to us.

The present paper is the outcome of an attempt to construct a theory of errors upon such simple principles as will be generally admitted to be necessary for the mathematical treatment of any such theory. The immediate object of the theory of errors is to determine the probability that the error made in the direct measurement of a physical magnitude shall lie between given limits. More complex applications of the theory (e. g. to functions of quantities directly observed and to quantities determined by calculations based upon direct observations) can be connected with the solution of the fundamental problem just stated by means familiar to all who have used the ordinary theory. Of course we consider only accidental errors, assuming the observations to have been corrected for all systematic errors before being subjected to our investigation; we suppose also that the observations were all made under the same conditions, so that they are equally reliable (their relative probabilities must be determined by the theory).

II. CASE OF A FINITE NUMBER OF OBSERVATIONS.

Let $z_1, z_2, z_3, \ldots, z_n$ be n observed values of a magnitude whose true value is Z, and let z denote an observed value in general. If A is any function of z and $A_1, A_2, A_3, \ldots, A_n$ are the values of this function for $z_1, z_2, z_3, \ldots, z_n$, respectively, we write

$$A_1 + A_2 + A_3 + \ldots + A_n = [A],$$

as is customary in the theory of errors. Let z_0 be the arithmetical mean of the observed values, so that

$$(1) \qquad z_0 = \frac{1}{n}[z].$$

Let x be the error of an observation in general, $x_1, x_2, x_3, \ldots, x_n$ the errors of the n given observations, and x_0 the arithmetical mean of these errors, so that

$$(2) \qquad \begin{cases} x = z - Z, x_1 = z_1 - Z, x_2 = z_2 - Z, x_3 = z_3 - Z, \ldots, x_n = z_n - Z, \\ x_0 = \frac{1}{n}[x] = \frac{1}{n}[z] - Z = z_0 - Z; \end{cases}$$

therefore, the arithmetical mean of the errors is the error of the arithmetical mean of the observations. Let ξ be the *residual* of an observation in general, — that is, the excess of the observed value over the arithmetical mean of the given observed values, — and let $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$ be the residuals of the several observations, so that

$$(3) \qquad \xi = z - z_0 = (x + Z) - (x_0 + Z) = x - x_0;$$

therefore, the residual of an observed value is the residual of its error; we may call it simply the *residual* of the observation. Let p_k be the mean k-th power of the residual, so that

$$(4) \qquad p_k = \frac{1}{n}[\xi^k];$$

in particular,

$$(5) \qquad \begin{cases} p_0 = \frac{1}{n}[\xi^0] = \frac{1}{n}[1] = 1, \\ p_1 = \frac{1}{n}[\xi] = \frac{1}{n}[z] - z_0 = 0. \end{cases}$$

The positive square root of the mean square of the residual shall be called the *mean residual* (it is called the *mean error* in the ordinary theory) and shall be represented by μ, so that

$$(6) \qquad \mu = \sqrt{p_2} \quad \text{or} \quad p_2 = \mu^2.$$

The ratio of the residual to the mean residual shall be called the *relative residual* and be represented by s, while the ratio of the mean k-th power of the residual to the k-th power of the mean residual shall be called the *relative mean* k-*th power* of the residual and be represented by ρ_k, so that

$$(7)\qquad \begin{cases} s = \dfrac{\xi}{\mu},\ \rho_k = \dfrac{p_k}{\mu^k}, \\[2ex] \rho_0 = \dfrac{p_0}{\mu^0} = 1,\ \rho_1 = \dfrac{p_1}{\mu} = 0,\ \rho_2 = \dfrac{p_2}{\mu^2} = 1. \end{cases}$$

A little consideration will make it clear that no observations, however many, can determine the *true* value of the observed magnitude, nor, therefore, the *errors* of the several observations, — they can determine only what is equivalent to their arithmetical mean and the residuals of the observations. The fundamental problem of the theory of errors must, therefore, be modified, and may now be stated to be the determination of the probability that the residual of an observation shall lie between certain limits according to the *law of the long run*, that is, in general, on the assumption that the distribution of errors over the whole range of possible error tends to follow a certain law, as the number of observations is increased without limit, and that the *proportion* of all the errors of *any* (finite) number of observations that fall between given limits is the same as if the number of observations was infinite. It is evident that the distribution of errors among different intervals in definite proportions implies an increase in the breadth of the intervals as the number of observations is diminished. The analytical theory of errors is based upon the supposition that the number of observations is infinite and that the interval for which the proportion of errors is to be determined may be taken to be as narrow as we please.

In connection with this statement of the fundamental problem it may be said that the determination of the value that may *best* be assigned to the observed magnitude is not explicitly a part of the fundamental problem, although immediately connected with it and perhaps the most desirable result of it from the practical point of view, but the criterion for the best value is by no means evident. Perhaps it will be generally considered that the best value is the *probable value*, that is, the value that is just as likely to be exceeded as not in the long run, so that the probability that the observed value shall fall short of the probable value is $\frac{1}{2}$ and the probability that the observed value shall exceed the probable value is $\frac{1}{2}$. Certainly the best value cannot be defined as the *most* probable value, that is, the value whose probability is a maximum, because it turns out that the probability is sometimes a maximum for more than one value; the probability may even have equal maxima for two or more values.

III. Case of an Infinite Number of Observations — Assumptions.

The assumptions upon which we shall base the general theory of errors (upon which the ordinary theory has always been based in part, explicitly or implicitly) are these:

I. *Possible errors form a practically continuous sequence from a certain lower limit to a certain upper limit;*

II. *The probability that the error of an observation lies between* x *and* x + dx, *where* dx *is infinitesimal, is* $\phi(\mathrm{x}) \cdot \mathrm{dx}$, *where* $\phi(\mathrm{x})$ *is an analytical function of* x *developable by Taylor's theorem throughout the whole range of possible error;*

III. *The probability that the error lies between given limits is independent of the unit of measurement.*

Assumption I forms the foundation for II, and II implies that $\phi(x + h)$ is developable according to powers of h by Taylor's theorem for all values of x and $x + h$ that lie within the range of possible error. The value of $\phi(x)$ for any given value of x is called the *relative frequency* of the error x.

The probability that the error of an observation lies between two given limits a (lower) and b (upper) is, evidently,

$$\int_a^b \phi(x) \cdot dx. \tag{8}$$

If $\underline{x}$ is the (algebraically) least possible error and $\bar{x}$ the greatest possible error, we have

$$\int_{\underline{x}}^{\bar{x}} \phi(x) \cdot dx = 1. \tag{9}$$

If we express x in terms of the residual ξ by (3), representing the lower and upper limits of possible residuals by $\underline{\xi}$ and $\bar{\xi}$, respectively (which correspond to the extreme errors $\underline{x}$ and $\bar{x}$), and writing

$$\phi(x) = \phi(\xi + x_0) = \psi(\xi), \tag{10}$$

we have from (9), because x_0 is constant,

$$\int_{\underline{\xi}}^{\bar{\xi}} \psi(\xi) \cdot d\xi = 1, \tag{11}$$

and the probability that the residual of an observation lies between a and b is

$$(12) \qquad \int_a^b \psi(\xi) \cdot d\xi.$$

The value of $\psi(\xi)$ for any given value ξ may be called the *relative frequency* of the residual ξ, and $\psi(\xi) \cdot d\xi$ is evidently the probability that the residual lies between ξ and $\xi + d\xi$; that is, $\psi(\xi) \cdot d\xi$ is the *proportion* of all the residuals that lie between ξ and $\xi + d\xi$. In calculating the mean powers of the residual by (4), the power of each residual is to be multiplied by the number of times it occurs, and the sum of the products thus formed for all the residuals is to be divided by the number of observations; if we assign all the residuals between ξ and $\xi + d\xi$ to the value ξ itself (or replace them all by their mean), the multiplier corresponding to this value is proportional to $\psi(\xi) \cdot d\xi$, while the whole number of observations is proportional to (11). We have then

$$(13) \qquad p_k = \int_{\underline{\xi}}^{\bar{\xi}} \xi^k \psi(\xi) \cdot d\xi,$$

and, in particular, by (5) and (6),

$$(14) \qquad \int_{\underline{\xi}}^{\bar{\xi}} \psi(\xi) \cdot d\xi = p_0 = 1,$$

$$(15) \qquad \int_{\underline{\xi}}^{\bar{\xi}} \xi \psi(\xi) \cdot d\xi = p_1 = 0,$$

$$(16) \qquad \int_{\underline{\xi}}^{\bar{\xi}} \xi^2 \psi(\xi) \cdot d\xi = p_2 = \mu^2.$$

Therefore, by (3), (14), and (15),

$$(17) \qquad \int_{\underline{x}}^{\bar{x}} x \cdot \phi(x) \cdot dx = \int_{\underline{\xi}}^{\bar{\xi}} (\xi + x_0) \cdot \psi(\xi) \cdot d\xi = x_0$$

Because $\psi(\xi)$ is essentially positive for any possible residual ξ, formula (15) shows that the possible values of ξ must include both positive and negative values, so that $\underline{\xi} < 0$, $0 < \bar{\xi}$, and 0 is a possible residual in all cases.

IV. EFFECT OF AN INFINITESIMAL SOURCE OF ERROR.

To determine the form of the function $\psi(\xi)$ we consider the effect of the addition of an *infinitesimal* source of error to the original source (however complex that may be), that is, of a source of error that produces infinitesimal errors with great frequency and finite errors of even moderate magnitude with infinitesimal frequency. We employ the same notation for all functions and quantities pertaining to the action of the infinitesimal source alone as for those pertaining to the original source alone, distinguishing them by an accent, and denote changes due to the addition of the infinitesimal source by the symbols for the functions and quantities changed with a δ prefixed.

A given error x produced by the simultaneous action of both sources is the result of a certain error x' produced by the infinitesimal source and an error $x - x'$ produced by the original source. The probability that the error produced by the combination shall lie between x and $x + dx$ is, then,

$$[\phi(x) + \delta\phi(x)]\, dx = \int_{x'}^{x'} \phi(x - x') \cdot \phi'(x') \cdot dx \cdot dx',$$

from which follows

$$\phi(x) + \delta\phi(x) = \int_{\underline{x}'}^{\overline{x}'} \phi(x - x') \cdot \phi'(x') \cdot dx'. \tag{18}$$

For the arithmetical mean of the error produced by the combination we have

$$x_0 + \delta x_0 = \int_{\underline{x}+\delta\underline{x}}^{\overline{x}+\delta\overline{x}} x \cdot dx \cdot \int_{\underline{x}'}^{\overline{x}'} \phi(x - x') \cdot \phi'(x') \cdot dx'$$

$$= \int_{\underline{x}}^{\overline{x}} x \cdot dx \cdot \int_{x'}^{\overline{x}'} \phi(x - x') \cdot \phi'(x') \cdot dx' + \overline{x} \cdot \phi(\overline{x}) \cdot \delta\overline{x} - \underline{x} \cdot \phi(x) \cdot \delta\underline{x}.$$

It is to be observed that, in whatever manner these integrals are evaluated, $x - x'$ must be taken over the whole range of values from $\underline{x}$ to $\overline{x}$. The limits of possible error ($\underline{x}$ and x) cannot be fixed *a priori*; some may think that large errors or residuals are impossible by the nature of accurate observation, but we prefer to regard the limits as determined by the frequency function $\phi(x)$, namely, we assume that the value of

$\phi(x)$ (which is positive for any possible error x) is 0 at each limit;* if $\phi(x)$ is not 0 for any *finite* value of x, all values of the error are to be regarded as possible. The results will show that $\phi(x) = 0$ for $x = \pm \infty$, and, in general, $\phi(x)$ has a very small value for any large value of x. We have, then,

$$(19) \qquad \phi(\overline{x}) = 0 \quad \text{and} \quad \phi(\underline{x}) = 0,$$

so that the last two terms of $x_0 + \delta x_0$ vanish, and, on reversing the order of integration, by (17) and (9),

$$\begin{aligned} x_0 + \delta x_0 &= \int_{\underline{x}'}^{\overline{x}'} \phi'(x') \cdot dx' \cdot \int_{\underline{x}+x'}^{\overline{x}+x'} x \cdot \phi(x - x') \cdot dx \\ &= \int_{\underline{x}'}^{\overline{x}'} \phi'(x') \cdot dx' \cdot \int_{\underline{x}}^{\overline{x}} (x + x') \cdot \phi(x) \cdot dx \\ &= \int_{\underline{x}'}^{\overline{x}'} (x_0 + x') \, \phi'(x') \, dx' = x_0 + x_0'; \end{aligned}$$

so that

$$(20) \qquad \delta x_0 = x_0'.$$

By (10), as applied to both component sources and their combination, we have

$$\begin{aligned} &\phi'(x') = \phi'(\xi' + x_0') = \psi'(\xi'), \\ &\phi(x - x') = \phi(\xi - \xi' + x_0 - x_0') = \psi(\xi - \xi' - x_0'), \\ &\phi(x) + \delta\phi(x) = (\phi + \delta\phi)(\xi + x_0) = (\psi + \delta\psi)(\xi - x_0') = \\ &\qquad\qquad \psi(\xi - x_0') + \delta\psi(\xi - x_0'). \end{aligned}$$

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* The relative frequency $\psi(\xi)$ of a *possible* residual ξ must be positive; if ξ varies continuously from zero, either through positive or through negative values, the limit $\overline{\xi}$ or $\underline{\xi}$ must be reached before $\psi(\xi)$ becomes negative; but if $\psi(\xi)$ passes through zero without changing sign as ξ passes through a certain value, — a value that corresponds to a point of contact of the curve represented by $y = \psi(\xi)$ with the axis of abscissae, — that value of ξ is not *necessarily* a limit to the possible values; such a value of ξ could never occur as a residual, although values on both sides of it might occur. However, if the conditions under which the observations are made change continuously in such a manner that the curve $y = \psi(\xi)$ assumes a form in which it touches the axis of abscissae, this must be the limiting (extreme) form, provided a continuation of the change would produce a curve having negative ordinates in the neighborhood of the point of contact; namely, on account of the permanence of the relation (14), the limits $\underline{\xi}$ and ξ cannot change discontinuously.

for any *given* value of ξ, because the arithmetical mean of the error corresponding to the function $\phi + \delta\phi$ is $x_0 + \delta x_0 = x_0 + x_0'$. Therefore, (18) becomes

$$\psi(\xi - x_0') + \delta\psi(\xi - x_0') = \int_{\underline{\xi}}^{\bar{\xi}} \psi(\xi - \xi' - x_0') \cdot \psi'(\xi') \cdot d\xi',$$

or, when ξ is replaced by $\xi + x_0'$,

$$\psi(\xi) + \delta\psi(\xi) = \int_{\underline{\xi}'}^{\bar{\xi}'} \psi(\xi - \xi') \cdot \psi'(\xi') \cdot d\xi' \tag{21}$$

for any possible value of ξ. Now, by assumption II, $\phi(x)$ is developable by Taylor's theorem and, therefore, so is $\psi(\xi)$, that is

$$\psi(\xi - \xi') = \sum_0^\infty{}_k (-1)^k \frac{\xi'^k}{k!} \frac{\partial^k \psi(\xi)}{\partial \xi^k};$$

therefore, by (13), (14), and (15),

$$\begin{aligned}
\psi(\xi) + \delta\psi(\xi) &= \sum_0^\infty{}_k (-1)^k \frac{1}{k!} \frac{\partial^k \psi(\xi)}{\partial \xi^k} \int_{\underline{\xi}'}^{\bar{\xi}'} \xi'^k \psi'(\xi') \cdot d\xi' \\
&= \sum_0^\infty{}_k (-1)^k \frac{p_k'}{k!} \frac{\partial^k \psi(\xi)}{\partial \xi^k} \\
&= \psi(\xi) + \sum_2^\infty{}_k (-1)^k \frac{p_k'}{k!} \frac{\partial^k \psi(\xi)}{\partial \xi^k};
\end{aligned}$$

so that

$$\delta\psi(\xi) = \sum_2^\infty{}_k (-1)^k \frac{p_k'}{k!} \frac{\partial^k \psi(\xi)}{\partial \xi^k}. \tag{22}$$

Also, by (13) and (21),

$$p_m + \delta p_m = \int_{\underline{\xi}+\delta\underline{\xi}}^{\bar{\xi}+\delta\bar{\xi}} \xi^m \cdot d\xi \cdot \int_{\underline{\xi}'}^{\bar{\xi}'} \psi(\xi - \xi') \cdot \psi'(\xi') \cdot d\xi'$$

$$= \int_{\underline{\xi}}^{\bar{\xi}} \xi^m \cdot d\xi \cdot \int_{\underline{\xi}'}^{\bar{\xi}'} \psi(\xi - \xi') \cdot \psi'(\xi') \cdot d\xi' + \bar{\xi}^m \cdot \psi(\bar{\xi}) \cdot \delta\bar{\xi} - \underline{\xi}^m \cdot \psi(\underline{\xi}) \cdot \delta\underline{\xi}.$$

Reversing the order of integration, observing that $\xi - \xi'$ must take all values from $\underline{\xi}$ to $\bar{\xi}$ and that, by (19), $\psi(\bar{\xi}) = 0$ and $\psi(\underline{\xi}) = 0$, we have

$$p_m + \delta p_m = \int_{\underline{\xi}'}^{\bar{\xi}'} \psi'(\xi') \cdot d\xi' \cdot \int_{\underline{\xi}+\xi'}^{\bar{\xi}+\xi'} \xi^m \cdot \psi(\xi - \xi') \cdot d\xi$$

$$= \int_{\underline{\xi}'}^{\bar{\xi}'} \psi'(\xi') \cdot d\xi' \cdot \int_{\underline{\xi}}^{\bar{\xi}} (\xi + \xi')^m \cdot \psi(\xi) \cdot d\xi$$

$$= \sum_0^m {}_k \binom{m}{k} \int_{\underline{\xi}'}^{\bar{\xi}'} \xi'^k \cdot \psi'(\xi') \cdot d\xi' \cdot \int_{\underline{\xi}}^{\bar{\xi}} \xi^{m-k} \cdot \psi(\xi) \cdot d\xi$$

$$= \sum_0^m {}_k \binom{m}{k} p_k' \, p_{m-k} = p_m + \sum_2^m {}_k \binom{m}{k} p_k' \, p_{m-k},$$

where $\binom{m}{k}$ denotes the coefficient of x^k in the development of $(1 + x)^m$ by the binomial theorem, that is

$$\binom{m}{k} = \frac{m\,!}{(m-k)\,!\,k\,!}. \tag{23}$$

Therefore,

$$\delta p_m = \sum_2^m {}_k \binom{m}{k} p_k' \, p_{m-k} \quad \text{for } 2 \leqq m. \tag{24}$$

In particular, for the smaller values of m,

$$\begin{aligned}
&\delta p_0 = 0, \quad \delta p_1 = 0 \quad (\text{because } p_0 = 1 \quad \text{and} \quad p_1 = 0),\\
&\delta p_2 = p_2', \quad \delta p_3 = p_3', \quad \delta p_4 = 6\,p_2 p_2' + p_4',\\
&\delta p_5 = 10\,p_3 p_2' + 10\,p_2 p_3' + p_5',\\
&\delta p_6 = 15\,p_4 p_2' + 20\,p_3 p_3' + 15\,p_2 p_4' + p_6', \quad \text{etc.},
\end{aligned}$$

and, therefore,

$$\begin{aligned}
&p_2' = \delta p_2, \quad p_8' = \delta p_3, \quad p_4' = \delta p_4 - 6\,p_2 \cdot \delta p_2,\\
&p_5' = \delta p_5 - 10\,p_3 \cdot \delta p_2 - 10\,p_2 \cdot \delta p_3,\\
&p_6' = \delta p_6 - 15\,p_4 \cdot \delta p_2 - 20\,p_3 \cdot \delta p_3 - 15\,p_2\,(\delta p_4 - 6\,p_2 \cdot \delta p_2)\\
&\quad = \delta p_6 - 15\,p_2 \cdot \delta p_4 - 20\,p_3 \cdot \delta p_8 - 15\,(p_4 - 6\,p_2^2)\,\delta p_2, \ \text{etc.}
\end{aligned}$$

Because the right member of (24) is linear in the p''s and p_m' has the coefficient 1 in the expression for δp_m but does not occur in the expression of any δp with suffix less than m, p_m' can be expressed as a linear function of the δp's with suffixes not greater than m whose coefficients are polynomials in the p's. The expression for p_6' in terms of δp_2, δp_3, δp_4, and δp_6 shows that the coefficients of the δp's in the expressions of the p''s are not generally linear. If we substitute the expressions of the p''s as linear functions of the δp's in (22) we obtain $\delta\psi(\xi)$ as a linear function of the δp's whose coefficients are polynomials in the p's multiplied by derivatives of $\psi(\xi)$ with respect to ξ; therefore, *$\psi(\xi)$ is a function of ξ and the* p's *with suffixes as great as 2 alone*, say *

$$(25) \qquad \psi(\xi) = F(\xi, p_2, p_3, p_4, p_5, \ldots).$$

This result confirms the statement made above that all theory can determine is the probability that the *residual* shall lie between given limits.

V. Change of the Unit of Measurement.

If we reduce the unit of measurement in the ratio 1 : λ, the numerical values of all measurements are multiplied by λ, so that, for given observations, x, x_0, ξ, $d\xi$, p_k (for $2 \overline{\overline{<}} k$) have to be replaced by λx, λx_0, $\lambda\xi$, $\lambda \cdot d\xi$, $\lambda^k p_k$, and, by assumption III, the probability that the residual lies between $\lambda\xi$ and $\lambda\xi + \lambda \cdot d\xi$ with the new unit is the same as that it lies between ξ and $\xi + d\xi$ with the old unit; therefore,

$$F(\lambda\xi, \lambda^2 p_2, \lambda^3 p_3, \lambda^4 p_4, \ldots) \cdot \lambda\, d\xi = F(\xi, p_2, p_3, p_4, \ldots) \cdot d\xi,$$

that is

$$(26) \qquad F(\lambda\xi, \lambda^2 p_2, \lambda^3 p_3, \lambda^4 p_4, \ldots) = \frac{1}{\lambda} F(\xi, p_2, p_3, p_4, \ldots)$$

for all values of ξ, p_2, p_3, p_4, . . . The relations (25) and (26) express that $\psi(\xi)$ is a homogeneous function of ξ, $\sqrt{p_2}$, $\sqrt[3]{p_3}$, $\sqrt[4]{p_4}$, etc. of degree −1, that is, with the notation of (6) and (7),

$$(27) \qquad \psi(\xi) = \frac{1}{\mu} f(s, \rho_3, \rho_4, \rho_5, \ldots);$$

for convenience, we shall sometimes abbreviate $\psi(\xi)$ to ψ and

$f(s, \rho_3, \rho_4, \rho_5, \ldots)$ to $f(s)$ or f.

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* This conclusion implies that every infinitesimal change in $\psi(\xi)$, for a given value of ξ, can be effected by the addition of an infinitesimal source of error, — which ought, perhaps, to be stated as a fourth assumption lying at the base of the general theory.

VI. DIFFERENTIAL EQUATION OF $f(s)$.

Formulae (22) and (24) give the variations of $\psi(\xi)$ and p_m due to the addition of *any* infinitesimal source of error; the mean square p_2' of the residual due to such a source is an essentially positive infinitesimal; in determining the mean powers of the residual for such a source by (13) we may confine our attention to values of the residual that lie within very narrow limits which, in accordance with the remark made in connection with (15), include 0, so that we need consider only very small values of ξ; the terms of the integral (13) for greater values of k are infinitesimals of a higher order than the corresponding terms of the integral for smaller values of k, so that the integral p_k' for any value of k greater than 2 is an infinitesimal of a higher order than p_2'. We specialize the infinitesimal source of error by assuming $p_k' = 0$ for $3 \overline{\gtrless} k$.* We have then, by (24),

$$\delta p_m = \binom{m}{2} p_{m-2} \cdot p_2' \quad \text{for } 2 \overline{\gtrless} m,$$

from which, together with (6), follows

$$p_2' = \delta p_2 = 2\,\mu \cdot \delta\mu,$$

so that
$$\delta p_m = m\,(m-1) \cdot p_{m-2}\,\mu \cdot \delta\mu,$$

while (22) gives

$$\delta\psi(\xi) = \tfrac{1}{2} p_2' \cdot \frac{\partial^2\psi(\xi)}{\partial\xi^2} = \frac{\partial^2\psi(\xi)}{\partial\xi^2} \cdot \mu \cdot \delta\mu.$$

We denote partial differentiation by the round ∂, regarding ψ as a function of $\xi, p_2, p_3, p_4, \ldots$ and f as a function of $s, \rho_3, \rho_4, \rho_5, \ldots$, so that, by (7) and (27),

$$\frac{\partial\psi}{\partial\xi} = \frac{1}{\mu^2}\frac{\partial f}{\partial s} \quad \text{and} \quad \frac{\partial^2\psi}{\partial\xi^2} = \frac{1}{\mu^3}\frac{\partial^2 f}{\partial s^2}.$$

The variations denoted by δ were all taken for a *definite* value of ξ: therefore, by (7), (27), and the formulae just written,

$$\delta s = \delta\left(\frac{\xi}{\mu}\right) = -\frac{\xi}{\mu^2} \cdot \delta\mu = -s \cdot \frac{\delta\mu}{\mu},$$

$$\delta\psi = \frac{\partial^2\psi}{\partial\xi^2} \cdot \mu \cdot \delta\mu = \frac{\partial^2 f}{\partial s^2} \cdot \frac{\delta\mu}{\mu^2},$$

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* This assumption does not in any way affect the generality of the results obtained, because p_k' enters into $\psi(\xi)$ only through the variations of the unaccented p's.

$$\delta f = \delta(\mu\,\psi) = \mu \cdot \delta\psi + \psi \cdot \delta\mu = \left[\frac{\partial^2 f}{\partial s^2} + f\right]\frac{\delta\mu}{\mu},$$

$$\delta p_k = k\,(k-1)\cdot \rho_{k-2}\,\mu^{k-1}\cdot \delta\mu,$$

$$\delta\rho_k = \delta\left(\frac{p_k}{\mu^k}\right) = \frac{\delta p_k}{\mu^k} - k\,\frac{p_k}{\mu^{k+1}}\cdot\delta\mu = [k\,(k-1)\cdot\rho_{k-2} - k\cdot\rho_k]\,\frac{\delta\mu}{\mu};$$

therefore,

$$\begin{aligned}\delta f &= \frac{\partial f}{\partial s}\,\delta s + \frac{\partial f}{\partial \rho_3}\,\delta\rho_3 + \frac{\partial f}{\partial \rho_4}\,\delta\rho_4 + \frac{\partial f}{\partial \rho_5}\,\delta\rho_5 + \ldots \\ &= \frac{\partial f}{\partial s}\,\delta s + \underset{3}{\sum}{}_k \frac{\partial f}{\partial \rho_k}\cdot\delta\rho_k \\ &= \left\{-\,s\,\frac{\partial f}{\partial s} + \underset{3}{\sum}{}_k\,[k\,(k-1)\,\rho_{k-2} - k\rho_k]\,\frac{\partial f}{\partial \rho_k}\right\}\frac{\delta\mu}{\mu}.\end{aligned}$$

Equating the two expressions for δf thus found, omitting the common factor $\frac{\delta\mu}{\mu}$, we obtain

$$\frac{\partial^2 f}{\partial s^2} + s\,\frac{\partial f}{\partial s} + f = \underset{3}{\sum}{}_k\,[k\,(k-1)\,\rho_{k-2} - k\,\rho_k]\,\frac{\partial f}{\partial \rho_k}, \tag{28}$$

a differential equation for f which may be written

$$\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial s} + s f\right) = 2\cdot Tf, \tag{29}$$

where

$$T = \tfrac{1}{2}\underset{3}{\sum}{}_k\,[k\,(k-1)\,\rho_{k-2} - k\rho_k]\,\frac{\partial}{\partial \rho_k} \tag{30}$$

is a differential operator that *acts upon* f *only through the* ρ*'s with suffixes as great as* 3; the first part of that term of T for which $k = 3$ vanishes because $\rho_1 = 0$. We have omitted the upper limit to the sum in T because that depends upon the number of ρ's involved in f, a number as yet unknown.

VII. THE OPERATOR T. THE P'S AND S'S.

The operator T is so important for the theory that it deserves particular study. By a *constant* we shall henceforth mean a quantity that is independent of the ρ's with suffixes as great as 3 as well as of s. If u and v are any two operands and C is any constant, we have

$$(31)\qquad \begin{cases} T\,(u+v) = T\,u + T\,v, \\ T\cdot C\,u = C\cdot T\,u, \\ T\cdot u\,v = u\cdot T\,v + v\cdot T\,u. \end{cases}$$

If u is a function of the ρ's with suffixes not greater than m, $T\,u$ is also a function of the ρ's with suffixes not greater than m; if u is a polynomial in the ρ's, $T\,u$ is a polynomial in the ρ's of the same degree as u; in particular, if u is a linear function of the ρ's with suffixes not greater than m, $T\,u$ is a linear function of the ρ's with suffixes not greater than m. If $3 \overset{=}{<} m$,

$$(32)\qquad T\rho_m = \tfrac{1}{2}\,m\,(m-1)\cdot\rho_{m-2} - \tfrac{1}{2}\,m\cdot\rho_m.$$

Therefore, if u is a linear function of ρ_m and the ρ's with suffixes less than m, where $3 \overset{=}{<} m$, $T^n\,u$ is a linear function of ρ_m and the ρ's with suffixes less than m in which the coefficient of ρ_m is the coefficient of ρ_m in u multiplied by $\left(-\frac{m}{2}\right)^n$, where n is any positive integer or 0; more generally, if $G(T)$ denotes any polynomial in T (linear function of powers of T with constant coefficients), $G(T)\cdot u$ is a linear function of the ρ's with suffixes not greater than m in which the coefficient of ρ_m is the coefficient of ρ_m in u multiplied by $G\left(-\frac{m}{2}\right)$; in particular, $G(T)\cdot u$ will not involve ρ_m if $G(T)$ contains $T+\frac{m}{2}$ as a factor, and only then. Formula (32) holds also for $m = 0$, 1, and 2, giving

$$T\rho_0 = 0\cdot\rho_0 = 0,\ T\,\rho_1 = -\tfrac{1}{2}\,\rho_1 = 0,\ T\,\rho_2 = \rho_0 - \rho_2 = 0,$$

because $\rho_0 = \rho_2 = 1$ and $\rho_1 = 0$, by (7); therefore,

$$(T+\tfrac{0}{2})\,\rho_0 = 0,\ (T+\tfrac{1}{2})\,\rho_1 = 0,\ (T+\tfrac{2}{2})\,\rho_2 = \rho_2 = 1;$$

so that $G(T)\cdot\rho_m$ does not involve ρ_m if $G(T)$ contains the factor $T+\frac{m}{2}$, where m is 0 or any positive integer excepting 2, and $G(T)\cdot\rho_2$ does not involve ρ_2 if $G(T)$ contains T as a factor. Of course, polynomials in T are commutative with each other, so that it is of no consequence what position in a product $G(T)$ the annihilating factor occupies.

It is convenient to employ the usual symbol $y^{(n)}$ to denote the product of n factors diminishing by successive units from y, whatever meaning y may have (it may be a symbol of quantity or of operation), so that

$$(33)\qquad y^{(n)} = y\,(y-1)\,(y-2)\,(y-3)\,.\,.\,.\,(y-n+1)$$

if n is any positive integer; in consequence of this definition,

$$(34) \qquad \begin{cases} y^{(1)} = y, \\ y^{(n+1)} = y^{(n)} \cdot (y - n), \\ y^{(n)} = \dfrac{y^{(n+1)}}{y - n}, \end{cases}$$

for $1 \leqq n$; we take the last formula as the basis of the definition of $y^{(n)}$ for $n = 0, -1, -2, -3, \ldots$, successively, and thus obtain

$$(35) \qquad y^{(0)} = 1, \quad y^{(-n)} = \frac{1}{(y+1)(y+2)(y+3)\ldots(y+n)}$$

for any positive integer n (that is, for any negative integer $-n$). With this notation we have

$$(36) \qquad \begin{cases} y^{(n)} = y^{(i)} \cdot (y - i)^{(n-i)} = (-1)^n \cdot (-y + n - 1)^{(n)}, \\ (y + n - 1)^{(n)} = (-1)^n \cdot (-y)^{(n)}, \\ n^{(n)} = n\,!, \quad n^{(i)} = \dfrac{n\,!}{(n-i)\,!}, \\ \dbinom{n}{i} = \dfrac{n^{(i)}}{i\,!} = \dbinom{n-1}{i} + \dbinom{n-1}{i-1}, \end{cases}$$

where each of the letters n and i represents any positive integer or 0; also,

$$(37) \qquad \begin{cases} (2\,i)\,! = 2^{2i} \cdot i\,!\,(i - \tfrac{1}{2})^{(i)} \\ (2\,i + 1)\,! = 2^{2i+1} \cdot i\,!\,(i + \tfrac{1}{2})^{(i+1)}, \end{cases}$$

because, evidently, $2^i\,i\,!$ is the product of the even numbers from 2 to $2\,i$, $2^i\,(i - \frac{1}{2})^{(i)}$ is the product of the odd numbers from 1 to $2\,i - 1$, and $2^{i+1}\,(i + \frac{1}{2})^{(i+1)}$ is the product of the odd numbers from 1 to $2\,i + 1$. Furthermore, we have

$$(38) \qquad \begin{aligned} (y-1)^{(n)} &= (y-1)^{(n-1)} \cdot (y - n) = y^{(n)} - n \cdot (y-1)^{(n-1)} \\ &= y^{(n)} - n \cdot y^{(n-1)} + n^{(2)} \cdot y^{(n-2)} - n^{(3)} \cdot y^{(n-3)} + \ldots \\ &= \sum_{0}^{n}{}_{k} (-1)^k \cdot n^{(k)} \cdot y^{(n-k)}, \end{aligned}$$

where n is any positive integer or 0.

From (32) it is evident that any polynomial in T changes ρ with any even suffix into a linear function of the ρ's with even suffixes and ρ with any odd suffix into a linear function of the ρ's with odd suffixes. Therefore, if u is a linear function of ρ_{2m} and the ρ's with even suffixes less than $2m$, where $2 \leqq m$, $(T + m)\,u$ does not involve ρ_{2m}, $(T + m)^{2}\,u$ involves neither ρ_{2m} nor ρ_{2m-2}, $(T + m)^{(3)}\,u$ involves neither ρ_{2m}, ρ_{2m-2}, nor ρ_{2m-4}, and so on, and $(T + m)^{(m-1)}\,u$ is a constant (or linear function of ρ_0 and ρ_2); therefore, by (36)

$$(39)\qquad \begin{cases} (T + m)^{(m+1)}\,u = (T + m)^{(m-1)}\,(T + 1)^{(2)}\,u \\ \qquad\qquad\qquad = (T + 1)\,T \cdot (T + m)^{(m-1)}\,u = 0\,; \end{cases}$$

that is, $(T + m)^{(m+1)}$ annihilates every linear function of the ρ's with even suffixes not greater than $2m$, where m is any positive integer or 0 (this is true for $m = 0$ or 1 because the operator contains T as a factor). But $(T + m)^{(m+1)}$ annihilates no linear function that involves a ρ with an even suffix greater than $2m$ or an odd suffix as great as 3; namely, if ρ_k is the ρ with greatest even suffix or greatest odd suffix (other than 1) that is involved in the linear function u (with a coefficient different from 0), the coefficient of ρ_k in $(T + m)^{(m+1)}\,u$ is the coefficient of ρ_k in u multiplied by $\left(m - \frac{k}{2}\right)^{(m+1)}$, whose factors are all negative if k is an even number greater than $2m$ and all odd multiples of $\frac{1}{2}$ if k is odd.

Similarly, if u is a linear function of ρ_{2m+1} and the ρ's with odd suffixes less than $2m + 1$, where $0 \leqq m$,

$$(40)\qquad (T + m + \tfrac{1}{2})^{(m+1)}\,u = 0,$$

so that $(T + m + \frac{1}{2})^{(m+1)}$ annihilates any linear function of the ρ's with odd suffixes not greater than $2m + 1$; but $(T + m + \frac{1}{2})^{(m+1)}$ annihilates no linear function that involves a ρ with an odd suffix greater than $2m + 1$ or an even suffix.

Applying (39) and (40) to ρ_{2m} and ρ_{2m+1}, respectively, we have

$$(T + m)^{(m+1)}\,\rho_{2m} = 0, \quad (T + m + \tfrac{1}{2})^{(m+1)}\,\rho_{2m+1} = 0,$$

so that, by (34),

$$T\,(T + m - 1)^{(m)}\,\rho_{2m} = -\,m \cdot (T + m - 1)^{(m)}\,\rho_{2m},$$

$$T\,(T + m - \tfrac{1}{2})^{(m)}\,\rho_{2m+1} = -\,(m + \tfrac{1}{2}) \cdot (T + m - \tfrac{1}{2})^{(m)}\,\rho_{2m+1},$$

or

$$(41)\qquad T\,Q_{2m} = -\,m \cdot Q_{2m}, \quad T\,Q_{2m+1} = -\,(m + \tfrac{1}{2}) \cdot Q_{2m+1}.$$

if we write

$$(42) \qquad (T + m - 1)^{(m)}\,\rho_{2m} = Q_{2m}, \quad (T + m - \tfrac{1}{2})^{(m)}\,\rho_{2m+1} = Q_{2m+1}\,;$$

and, evidently, formulae (41) hold for any constant multiples of Q_{2m} and Q_{2m+1}, respectively.

Now (32) shows that

$$(T + m) \cdot \rho_{2m} = 2\,m\,(m - \tfrac{1}{2}) \cdot \rho_{2m-2}$$

and, by continued application of the same formula,

$$(43) \qquad (T + m)^{(k)}\,\rho_{2m} = 2^k\, m^{(k)}\,(m - \tfrac{1}{2})^{(k)} \cdot \rho_{2m-2k}$$

for $0 \leqq k \leqq m - 1$; in particular,

$$(T + m)^{(m-1)}\,\rho_{2m} = 2^{m-1}\, m^{(m-1)}\,(m - \tfrac{1}{2})^{(m-1)}\,\rho_2$$

and

$$(44) \qquad (T + m)^{(m)}\,\rho_{2m} = 2^m \cdot m\,!\,(m - \tfrac{1}{2})^{(m)}\,\rho_0,$$

because, by (34) and (36),

$$\begin{aligned} m\,! = m^{(m)} &= m^{(m-1)} \cdot 1, \\ (m - \tfrac{1}{2})^{(m)} &= (m - \tfrac{1}{2})^{(m-1)} \cdot \tfrac{1}{2}, \\ (T + 1)\,\rho_2 &= \rho_2 = \rho_0\,; \end{aligned}$$

so that (43) holds also for $k = m$. Then, by (42), (38), (43), and (36),

$$(45) \qquad \begin{aligned} Q_{2m} &= \sum_{0}^{m}{}_{i}\,(-1)^i\, m^{(i)}\,(T + m)^{(m-i)}\,\rho_{2m} \\ &= 2^m\,(m - \tfrac{1}{2})^{(m)} \cdot m\,! \sum_{0}^{m}{}_{i}\,(-1)^i \binom{m}{i} \frac{\rho_{2i}}{2^i\,(i - \frac{1}{2})^{(i)}} \end{aligned}$$

for $0 \leqq m$.

Similarly, we have, from (32),

$$(T + m + \tfrac{1}{2}) \cdot \rho_{2m+1} = 2\,m\,(m + \tfrac{1}{2}) \cdot \rho_{2m-1}$$

and, by continued application,

$$(46) \qquad (T + m + \tfrac{1}{2})^{(k)} \cdot \rho_{2m+1} = 2^k\, m^{(k)}\,(m + \tfrac{1}{2})^{(k)} \cdot \rho_{2m-2k+1}$$

for $0 \leqq k \leqq m$. Therefore, by (42), (38), (46), and (36),

$$(47) \qquad \begin{aligned} Q_{2m+1} &= \sum_{0}^{m}{}_{i}\,(-1)^i\, m^{(i)}\,(T + m + \tfrac{1}{2})^{(m-i)}\,\rho_{2m+1} \\ &= 2^{m+1}\,(m + \tfrac{1}{2})^{(m+1)} \cdot m\,! \sum_{0}^{m}{}_{i}\,(-1)^i \binom{m}{i} \frac{\rho_{2i+1}}{2^{i+1}\,(i + \frac{1}{2})^{(i+1)}} \end{aligned}$$

for $0 \leqq m$.

If we write

$$(48)\qquad \begin{cases} P_{2m} = \sum_{0}^{m}{}_i (-1)^i \binom{m}{i} \dfrac{\rho_{2i}}{2^i\,(i-\frac{1}{2})^{(i)}} \quad \text{and} \\[2ex] P_{2m+1} = \sum_{0}^{m}{}_i (-i)^i \binom{m}{i} \dfrac{\rho_{2i+1}}{2^{i+1}\,(i+\frac{1}{2})^{(i+1)}}, \end{cases}$$

we have, by (45) and (47),

$$(49)\qquad \begin{cases} Q_{2m} = 2^m\,(m-\frac{1}{2})^{(m)}\,m\,!\,P_{2m} \quad \text{and} \\ Q_{2m+1} = 2^{m+1}\,(m+\frac{1}{2})^{(m+1)}\,m\,!\,P_{2m+1}, \end{cases}$$

and, because P_{2m} and P_{2m+1} are constant multiples of Q_{2m} and Q_{2m+1}, by (41),

$$(50)\qquad TP_{2m} = -\,m \cdot P_{2m}\,, \qquad TP_{2m+1} = -\,(m+\tfrac{1}{2}) \cdot P_{2m+1}\,,$$

for all values of m. For the P's with the smaller suffixes we have, from (48) and (7),

$$(51)\qquad \begin{cases} P_0 = 1, \quad P_1 = P_2 = 0, \quad P_3 = -\frac{1}{3}\,\rho_3\,, \quad P_4 = -\,1 + \frac{1}{3}\,\rho_4\,, \\ P_5 = -\,\frac{2}{3}\,\rho_3 + \frac{1}{15}\,\rho_5\,, \quad P_6 = -\,2 + \rho_4 - \frac{1}{15}\,\rho_6 \quad \text{etc.}\,; \end{cases}$$

so that equations (50) give

$$(52)\qquad TP_0 = 0 \cdot P_0 = 0, \quad TP_1 = -\tfrac{1}{2}\,P_1 = 0, \quad TP_2 = -\,P_2 = 0.$$

Because P_{2m} is a linear function of the ρ's with even suffixes up to $2\,m$ in which the coefficient of ρ_{2m} is different from 0, and P_{2m+1} is a linear function of the ρ's with odd suffixes up to $2\,m+1$ in which the coefficient of ρ_{2m+1} is different from 0, ρ_{2m} can be expressed as a linear function of the P's with even suffixes up to $2\,m$ and ρ_{2m+1} as a linear function of the P's with odd suffixes up to $2\,m+1$. In fact, if we multiply P_{2m} or P_{2m+1} by $(-1)^m \binom{k}{m}$ and sum for all values of m from 0 to k, we obtain

$$\sum_{0}^{k}{}_m (-1)^m \binom{k}{m} P_{2m} = \sum_{0}^{k}{}_i (-1)^i \frac{\rho_{2i}}{2^i\,(i-\frac{1}{2})^{(i)}} \sum_{0}^{k}{}_m (-1)^m \binom{k}{m}\binom{m}{i}$$

$$= \frac{\rho_{2k}}{2^k\,(k-\frac{1}{2})^{(k)}},$$

$$\sum_{0}^{k}{}_m (-1)^m \binom{k}{m} P_{2m+1} = \sum_{0}^{k}{}_i (-1)^i \frac{\rho_{2i+1}}{2^{i+1}\,(i+\frac{1}{2})^{(i+1)}} \sum_{i}^{k}{}_m (-1)^m \binom{k}{m}\binom{m}{i}$$

$$= \frac{\rho_{2k+1}}{2^{k+1}\,(k+\frac{1}{2})^{(k+1)}},$$

because

$$\sum_{i}^{k}{}_m(-1)^m\binom{k}{m}\binom{m}{i}=\binom{k}{i}\sum_{i}^{k}{}_m(-1)^m\binom{k-i}{m-i}$$

$$=(-1)^i\binom{k}{i}\sum_{0}^{k-i}{}_m(-1)^m\binom{k-i}{m}$$

$$=0 \text{ if } i<k \quad\text{and}$$

$$=(-1)^k \text{ if } i=k;$$

therefore

$$(53)\quad \begin{cases} \rho_{2k}=2^k\,(k-\tfrac{1}{2})^{(k)}\sum_{0}^{k}{}_m(-1)^m\binom{k}{m}P_{2m} \text{ for } 0 \mathrel{\overline{\gtrless}} k, \\ \rho_{2k+1}=2^{k+1}\,(k+\tfrac{1}{2})^{(k+1)}\sum_{0}^{k}{}_m(-1)^m\binom{k}{m}P_{2m+1} \text{ for } 0 \mathrel{\overline{\gtrless}} k. \end{cases}$$

As a consequence of (50) we have

$$(54)\qquad G\,(T)\cdot P_m=G\left(-\frac{m}{2}\right)\cdot P_m,$$

if $G(T)$ is any polynomial in T. The ease with which the result of applying any polynomial in T to the P's can be obtained by means of this formula makes the introduction of the P's in place of the ρ's very advantageous. We shall henceforth assume that all functions of the ρ's have been expressed as functions of the P's by (53), so that the degree of any polynomial in the P's with suffixes as great as 3 is the same as that of the corresponding polynomial in the ρ's. Equations (50) show also that the operator T, defined by (30) in terms of the ρ's, can be expressed in terms of the P's thus:

$$(55)\qquad T=-\tfrac{1}{2}\sum_{3}{}_k\,k\,P_k\frac{\partial}{\partial P_k}.$$

We assign to each P a *weight* equal to its suffix (leaving out of account P_1 and P_2, which are 0 and, therefore, nowhere occur) and define the weight of any product of P's to be the sum of the weights of its factors; any constant may be regarded as a multiple of $P_0=1$, and, therefore, as of the weight 0. The weight of any such product as $P_3{}^{n_3}\,P_4{}^{n_4}\,P_5{}^{n_5}\ldots$ is

$$(56)\qquad 3\,n_3+4\,n_4+5\,n_5+\ldots=w,$$

say, so that, by (55) and the last formula of (31),

$$(57)\qquad T\cdot P_3^{\,n_3}\,P_4^{\,n_4}\,P_5^{\,n_5}\ldots=-\frac{w}{2}\cdot P_3^{\,n_3}\,P_4^{\,n_4}\,P_5^{\,n_5}\ldots$$

and, evidently

$$(58) \qquad G(T) \cdot P_3^{n_3} P_4^{n_4} P_5^{n_5} \ldots = G\left(-\frac{w}{2}\right) \cdot P_3^{n_3} P_4^{n_4} P_5^{n_5} \ldots$$

where $G(T)$ is any polynomial in T. More generally, if Π is any *isobaric* polynomial of weight w in the P's, that is, any polynomial whose terms are *all* of one weight w,

$$(59) \qquad G(T) \cdot \Pi = G\left(-\frac{w}{2}\right) \cdot \Pi.$$

Therefore, if the coefficients of Π are not all 0, $G(T) \cdot \Pi$ is or is not 0 according as $G(T)$ does or does not contain $T + \frac{w}{2}$ as a factor. In particular, $(T+m)^{(m+1)}$ annihilates every polynomial in the P's whose terms are all of even weights not greater than $2m$ and annihilates no polynomial of other description, while $(T + m + \frac{1}{2})^{(m+1)}$ annihilates every polynomial whose terms are all of odd weights not greater than $2m+1$ and no others. By means of formula (59) any polynomial in T can be applied to any polynomial in the P's; namely, any polynomial in the P's can be expressed as the sum of isobaric polynomials of different weights and the polynomial in T can be applied to each isobaric part by (59).

The P's can be determined from the function $f(s)$; namely, from (7), (13), and (27) we have

$$(60) \qquad \rho_k = \frac{p_k}{\mu^k} = \int_{\underline{\xi}}^{\bar{\xi}} \frac{\xi^k}{\mu^k} \psi(\xi) \cdot d\xi = \int_{\underline{s}}^{\bar{s}} s^k \cdot f(s) \cdot ds \qquad \text{for } 0 \leqq k,$$

and obtain, by the substitution of these values of the ρ's in (48),

$$(61) \qquad P_{2m} = \int_{\underline{s}}^{\bar{s}} S_{2m} \cdot f(s) \cdot ds, \qquad P_{2m+1} = \int_{\underline{s}}^{\bar{s}} S_{2m+1} \cdot f(s) \,.\, ds,$$

where

$$(62) \qquad \begin{cases} S_{2m} = \displaystyle\sum_{0}^{m}{}_{i} (-1)^i \binom{m}{i} \frac{s^{2i}}{2^i \, (i - \frac{1}{2})^{(i)}}, \\[2ex] S_{2m+1} = \displaystyle\sum_{0}^{m}{}_{i} (-i)^i \binom{m}{i} \frac{s^{2i+1}}{2^{i+1} \, (i + \frac{1}{2})^{(i+1)}}, \end{cases}$$

for $0 \leqq m$. For the S's with the smaller suffixes we have

$$(63)\qquad \begin{cases} S_0 = 1, \quad S_1 = s, \quad S_2 = 1 - s^2, \quad S_3 = s - \frac{1}{3}\,s^3, \\ S_4 = 1 - 2\,s^2 + \frac{1}{3}\,s^4. \quad S_5 = s - \frac{2}{3}\,s^3 + \frac{1}{15}\,s^5, \\ S_6 = 1 - 3\,s^2 + s^4 - \frac{1}{15}\,s^6, \text{ etc.} \end{cases}$$

VIII. The Form of $f(s)$. The R's.

To simplify the integration of equation (29), we put

$$f(s) = \theta\,(s) \cdot u,$$

where u is an integral of the equation

$$\frac{\partial u}{\partial s} + s \cdot u = 0,$$

say

$$u = e^{-\frac{s^2}{2}},$$

where e represents the base of natural logarithms, so that

$$(64)\qquad f(s) = \theta\,(s) \cdot e^{-\frac{s^2}{2}}.$$

Writing θ as an abbreviation for $\theta\,(s)$, we have

$$\frac{\partial f}{\partial s} = \left[\frac{\partial \theta}{\partial s} - s \cdot \theta\right] e^{-\frac{s^2}{2}},$$

$$\frac{\partial^2 f}{\partial s^2} = \left[\frac{\partial^2 \theta}{\partial s^2} - 2\,s \cdot \frac{\partial \theta}{\partial s} - (1 - s^2)\,\theta\right] e^{-\frac{s^2}{2}},$$

so that (29) becomes

$$\left[\frac{\partial^2 \theta}{\partial s^2} - s \cdot \frac{\partial \theta}{\partial s}\right] e^{-\frac{s^2}{2}} = 2 \cdot T\theta \cdot e^{-\frac{s^2}{2}},$$

because $e^{-\frac{s^2}{2}}$ is independent of the ρ's (and P's); therefore,

$$(65)\qquad \frac{\partial^2 \theta}{\partial s^2} - s \cdot \frac{\partial \theta}{\partial s} = 2 \cdot T\theta,$$

which is a differential equation for $\theta\,(s)$.

The assumption II that $\phi\,(x)$ is developable by Taylor's theorem throughout the range of possible values of x implies that $\psi\,(\xi)$, $f(s)$, and $\theta\,(s)$, are similarly developable; we may therefore put

$$(66)\qquad \theta\,(s) = \sum_0^{\infty}{}_i\, \frac{A_i}{i\,!}\, s^i,$$

where A_i (for each value of i) is a function of the P's alone. This development implies that we shall approximate more and more nearly to the true form of $\theta(s)$, even if its complete expression is an infinite series, the more terms of (66) we take into account. Taking any finite number of terms, say for values of i up to κ, substituting in (64) and the resulting expression for $f(s)$ in (61), we obtain

$$(67) \qquad P_m = \sum_{0}^{\kappa}{}_i \frac{A_i}{i\,!} \int_{\underline{s}}^{\bar{s}} s^i R_m \cdot ds,$$

where

$$(68) \qquad R_m = S_m \cdot e^{-\frac{s^2}{2}} \text{ for } 0 \overline{\gtrless} m.$$

We see from (67) that, to the degree of approximation that corresponds to $\kappa + 1$ terms of (66), the P's are linear functions of the $\kappa + 1$ A's, whose coefficients are definite functions of the limits $\underline{s}$ and $\bar{s}$ alone, because the S's and, therefore, the R's are independent of the P's. Therefore, not more than $\kappa + 1$ of the P's (including P_0, P_1, and P_2) are *linearly* independent; the solution of such equations (67) as correspond to linearly independent P's gives the $\kappa + 1$ A's as linear functions of these P's, whose coefficients are definite functions of $\underline{s}$ and $\bar{s}$ derived from the integrals involved in (67); these expressions of the A's will involve a certain number of arbitrary constants if the number of linearly independent P's is less than $\kappa + 1$, but that is of no consequence. If the P's are subjected to infinitesimal variations, — say δP_m is the variation of P_m, — the A's, $f(s)$, and $\theta(s)$ will undergo variations, which may be represented by δA_i (for A_i), $\delta f(s)$, and $\delta\theta(s)$, respectively. Then, by (66) and (64),

$$\delta\theta(s) = \sum{}_i \frac{\delta A_i}{i\,!} s^i, \qquad \delta f(s) = \delta\theta(s) \cdot e^{-\frac{s^2}{2}},$$

and, by (61) and (68), because $f(s) = 0$ for each of the limits $\underline{s}$ and $\bar{s}$,

$$\delta P_m = \int_{\underline{s}}^{\bar{s}} S_m \cdot \delta f(s) \cdot ds = \int_{\underline{s}}^{\bar{s}} R_m \cdot \delta\theta(s) \cdot ds = \sum_{0}{}_i \frac{\delta A_i}{i\,!} \int_{\underline{s}}^{\bar{s}} s^i R_m \cdot ds\,;$$

that is, the relations between the δP's and the δA's are the same as if the coefficients of the A's in (67) were constants (whether $\underline{s}$ and $\bar{s}$ are functions of the P's or not); therefore, in determining the A's as functions of the P's that satisfy the differential equation (65) we may regard them as linear functions of the P's with *constant* coefficients. The necessary and sufficient conditions for these linear functions are those implied in (65) together with

(69) $$A_i = 0 \text{ for } \kappa < i$$

(to the degree of approximation to which we restrict our calculations).

Substituting the expression (66) for $\theta(s)$ in (65) we obtain

$$\sum_{2}^{\kappa}{}_{i} \frac{A_i}{(i-2)\,!}\, s^{i-2} - \sum_{1}^{\kappa}{}_{i} \frac{A_i}{(i-1)\,!}\, s^{i} = 2\sum_{0}^{\kappa}{}_{i} \frac{T A_i}{i\,!}\, s^i,$$

that is

$$\sum_{0}^{\kappa}{}_{i} \Big[A_{i+2} - i\, A_i - 2 \cdot T A_i \Big] \frac{s^i}{i\,!} = 0$$

for every possible value of s, where the first term of the multiplier of $\frac{s^i}{i\,!}$, is to be omitted for $i = \kappa$ and $i = \kappa - 1$ and the second term goes out of itself for $i = 0$. Therefore, the necessary and sufficient conditions for the satisfaction of (65) by (66) are

$$A_{i+2} = 2\left(T + \frac{i}{2}\right) \cdot A_i \text{ for } 0 \leqq i \leqq \kappa - 2$$

and

$$\left(T + \frac{i}{2}\right) \cdot A_i = 0 \text{ for } i = \kappa \text{ or } \kappa - 1\,;$$

that is

(70) $$A_i = 2\left(T + \frac{i-2}{2}\right) \cdot A_{i-2} \text{ for } 2 \leqq i \leqq \kappa$$

and

(71) $$\left(T + \frac{i}{2}\right) \cdot A_i = 0$$

for A_i with the greatest even suffix and A_i with the greatest odd suffix that occur in (66). On replacing i in (70) by $2\,i$ and $2\,i + 1$, respectively, we have, for the A's with even and odd suffixes separately,

$$A_{2i} = 2\,(T + i - 1) \cdot A_{2i-2} \quad \text{for} \quad 1 \leqq i \leqq \frac{\kappa}{2},$$

$$A_{2i+1} = 2\,(T + i - \tfrac{1}{2}) \cdot A_{2i-1} \quad \text{for} \quad 1 \leqq i \leqq \frac{\kappa - 1}{2},$$

and, by repeated application of these formulae,

(72) $$A_{2i} = 2^i\,(T + i - 1)^{(i)} \cdot A_0 \quad \text{for} \quad 1 \leqq i \leqq \frac{\kappa}{2}$$

and

(73) $$A_{2i+1} = 2^i\,(T + i - \tfrac{1}{2})^{(i)} \cdot A_1 \quad \text{for} \quad 1 \leqq i \leqq \frac{\kappa - 1}{2}\,;$$

that is, the A's with even suffixes are all derived from A_0 and all the A's with odd suffixes from A_1 by the application of definite polynomials in T. We may, then, determine the forms of the A's with even suffixes and of those with odd suffixes separately; that is, we may suppose our calculations restricted to a degree of approximation that corresponds to a development of $\theta(s)$ in which occur even powers of s up to $s^{2\alpha}$ and odd powers up to $s^{2\beta+1}$, without assuming any particular relation between α and β. Then (72) holds for $1 < i \leqq \alpha$, (73) holds for $1 \leqq i \leqq \beta$, while (71) is expressed by

$$(T+\alpha)\cdot A_{2\alpha}=0,\ (T+\beta+\tfrac{1}{2})\cdot A_{2\beta+1}=0,$$

that is, when $A_{2\alpha}$ and $A_{2\beta+1}$ have been determined by (72) and (73), respectively,

$$(T+\alpha)^{(\alpha+1)}\cdot A_0=0,\ (T+\beta+\tfrac{1}{2})^{(\beta+1)}\cdot A_1=0. \tag{74}$$

The only conditions imposed upon A_0 and A_1 so far are, then, that they shall be linear functions of the P's with constant coefficients that satisfy the equations (74). The only *linear* functions of the P's that are of a given weight w are constant multiples of P_w, while, by (54),

$$(T+\alpha)^{(\alpha+1)}\cdot P_w=\left(\alpha-\frac{w}{2}\right)^{(\alpha+1)}\cdot P_w,$$

$$(T+\beta+\tfrac{1}{2})^{(\beta+1)}\cdot P_w=\left(\beta-\frac{w-1}{2}\right)^{(\beta+1)}\cdot P_w;$$

therefore, in order that A_0 (as a linear function of the P's) shall satisfy the first equation (74) it is necessary and sufficient that the suffix w of any P involved in it shall satisfy the equation

$$\left(\alpha-\frac{w}{2}\right)^{(\alpha+1)}=0,$$

that is that w shall be an *even* integer not greater than 2α; and, in order that A_1 (as a linear function of the P's) shall satisfy the second equation (74) it is necessary and sufficient that the suffix w of any P involved in it shall satisfy the equation

$$\left(\beta-\frac{w-1}{2}\right)^{(\beta+1)}=0,$$

that is that w shall be an *odd* integer not greater than $2\beta+1$. We may, then, write

$$A_0=\sum_{0}^{\alpha}{}_k C_{2k}P_{2k},\ A_1=\sum_{0}^{\beta}{}_k C_{2k+1}P_{2k+1}, \tag{75}$$

where the C's are constants.

On applying (72) and (73) to the first and second of equations (75), respectively, we obtain, by (54) and (36),

$$(76)\begin{cases} A_{2i}=\sum_{0}^{\alpha}{}_{k} C_{2k}\,.\,P_{2k}\cdot 2^{i}\,(i-k-1)^{(i)}=\sum_{0}^{\alpha}{}_{k} C_{2k}\,P_{2k}\cdot(-1)^{i}\,2^{i}k^{(i)}, \\ A_{2i+1}=\sum_{0}^{\beta}{}_{k} C_{2k+1}\,P_{2k+1}\cdot 2^{i}\,(i-k-1)^{(i)}=\sum_{0}^{\beta}{}_{k} C_{2k+1}\,P_{2k+1}\cdot(-1)^{i}\,2^{i}k^{(i)}. \end{cases}$$

Substituting these values of A_{2i} and A_{2i+1} in (66) we have

$$\theta\,(s)=\sum_{0}^{\alpha}{}_{k} C_{2k}\,P_{2k}\cdot\sum_{0}^{k}{}_{i}(-1)^{i}\,\frac{2^{i}\,k^{(i)}}{(2\,i)\,!}\,s^{2i}$$
$$+\sum_{0}^{\beta}{}_{k} C_{2k+1}\,P_{2k+1}\cdot\sum_{0}^{k}{}_{i}(-1)^{i}\,\frac{2^{i}\,k^{(i)}}{(2\,i+1)\,!}\,s^{2i+1}$$

or, replacing $(2\,i)\,!$ and $(2\,i+1)\,!$ by their values in (37), observing the last line of (36), and employing the notation of (62),

$$(77)\qquad \theta\,(s)=\sum_{0}^{\alpha}{}_{k} C_{2k}\,P_{2k}\,S_{2k}+\sum_{0}^{\beta}{}_{k} C_{2k+1}\,P_{2k+1}\,S_{2k+1}\,;$$

again, by (64) and (68),

$$(78)\qquad f\,(s)=\sum_{0}^{\alpha}{}_{k} C_{2k}\,P_{2k}\,R_{2k}+\sum_{0}^{\beta}{}_{k} C_{2k+1}\,P_{2k+1}\,R_{2k+1}\,.$$

Substituting this expression of $f\,(s)$ in (61) we obtain

$$(79)\begin{cases} P_{2m}=\sum_{0}^{\alpha}{}_{k} C_{2k}P_{2k}\cdot\int_{\underline{s}}^{\bar{s}} S_{2m}R_{2k}\cdot ds+\sum_{0}^{\beta}{}_{k} C_{2k+1}P_{2k+1}\cdot\int_{\underline{s}}^{\bar{s}} S_{2m}R_{2k+1}\cdot ds, \\ P_{2m+1}=\sum_{0}^{\alpha}{}_{k} C_{2k}P_{2k}\cdot\int_{\underline{s}}^{\bar{s}} S_{2m+1}R_{2k}\cdot ds+\sum_{0}^{\beta}{}_{k} C_{2k+1}P_{2k+1}\cdot\int_{\underline{s}}^{\bar{s}} S_{2m+1}R_{2k+1}\cdot ds. \end{cases}$$

IX. THE COEFFICIENTS OF $f\,(s)$. CONDITIONS FOR THE LIMITS.

In all that contributed to the determination of the form of $f\,(s)$ as given by (78) no assumption has been made with regard to the *values* of the P's with even suffixes not greater than $2\,\alpha$ and the P's with odd suffixes not greater than $2\,\beta+1$ [excepting that, by (51), $P_0=1$,

$P_1 = P_2 = 0$], — in fact, they have been assumed to be entirely arbitrary; therefore, the first of equations (79) for $0 \leqq m \leqq \alpha$ and the second of these equations for $0 \leqq m \leqq \beta$ must be satisfied independently of the values of the P's involved; that is, for any one of the values of m in question, the coefficient of any P in the right member of the corresponding equation must be 0 excepting the coefficient of the P that occurs in the left member, which must be 1. Therefore,

$$(80)\quad \begin{cases} C_{2m} \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m}\, R_{2m} \cdot ds = 1 \quad \text{for} \quad 0 \leqq m \leqq \alpha, \ \text{excepting} \ \ m = 1\,; \\ C_{2m+1} \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m+1}\, R_{2m+1} \cdot ds = 1 \quad \text{for} \quad 1 \leqq m \leqq \beta\,; \end{cases}$$

$$(81)\quad \begin{cases} \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m}\, R_{2k} \cdot ds = 0 \quad \text{for} \quad 0 \leqq m \leqq \alpha,\ 0 \leqq k \leqq \alpha,\ k \lessgtr m\,; \\ \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m} R_{2k+1} \cdot ds = 0 \quad \text{for} \quad 0 \leqq m \leqq \alpha,\ 0 \leqq k \leqq \beta\,; \\ \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m+1}\, R_{2k} \cdot ds = 0 \quad \text{for} \quad 0 \leqq m \leqq \beta,\ 0 \leqq k \leqq \alpha\,; \\ \displaystyle\int_{\underline{s}}^{\bar{s}} S_{2m+1}\, R_{2k+1} \cdot ds = 0 \quad \text{for} \quad 0 \leqq m \leqq \beta,\ 0 \leqq k \leqq \beta,\ k \lessgtr m. \end{cases}$$

From the first, second, third, and fourth sets of equations (81) we ought to exclude cases for which $k = 1$, 0, 1, and 0, respectively, because $P_1 = P_2 = 0$ and the terms corresponding to these cases do not occur in (79); but, by (68),

$$(82)\qquad \int_{\underline{s}}^{\bar{s}} S_m\, R_k \cdot ds = \int_{\underline{s}}^{\bar{s}} S_m\, S \cdot e^{-\frac{s^2}{2}} \cdot ds = \int_{\underline{s}}^{\bar{s}} S_k\, R_m \cdot ds,$$

so that the cases that are to be excluded from any one set of equations (81) will occur (with an interchange of m and k) in other sets, excepting only the case of the integral

$$\int_{\underline{s}}^{\bar{s}} S_2\, R_1 \cdot ds = \int_{\underline{s}}^{\bar{s}} S_1\, R_2 \cdot ds,$$

which does not occur as the left member of any equation. The four sets of equations (81) can be represented, by virtue of (82), as the one set

$$(81') \qquad \int_{\underline{s}}^{\bar{s}} S_m\, R_k \cdot ds = 0$$

for any two *different* numbers m and k from among the even numbers not greater than $2\,\alpha$ (including 0) and the odd numbers not greater than $2\,\beta + 1$, excepting $m = 1$, $k = 2$, and $m = 2$, $k = 1$. The equations (80) determine all the constant coefficients C that actually occur in (78) (C_1 and C_2 do not occur), while equations (81) express relations that must exist between the limits $\underline{s}$ and $\bar{s}$ for any observations that conform to the theory.

To determine the values of the integrals that occur in (80) and (81), we consider the functions S and R defined by (62) and (68). We have

$$\frac{\partial S_{2m}}{\partial s} = \sum_{1}^{m}{}_i (-1)^i \binom{m}{i} \frac{i\, s^{2i-1}}{2^{i-1}\,(i-\frac{1}{2})^{(i)}}$$

$$= 2\,m \sum_{1}^{m}{}_i (-1)^i \binom{m-1}{i-1} \frac{s^{2i-1}}{2^i\,(i-\frac{1}{2})^{(i)}}$$

$$= -\,2\,m \sum_{0}^{m-1}{}_i (-1)^i \binom{m-1}{i} \frac{s^{2i+1}}{2^{i+1}\,(i-\frac{1}{2})^{(i+1)}},$$

$$\frac{\partial S_{2m+1}}{\partial s} = \sum_{0}^{m}{}_i (-1)^i \binom{m}{i} \frac{s^{2i}}{2^i\,(i-\frac{1}{2})^{(i)}}\,;$$

that is,

$$(83) \qquad \frac{\partial S_{2m}}{\partial s} = -\,2\,m \cdot S_{2m-1}\,, \qquad \frac{\partial S_{2m+1}}{\partial s} = S_{2m}\,,$$

for $0 \leqq m$. We have also, by (36),

$$S_{2m} = \sum_{0}^{m-1}{}_i (-1)^i \binom{m-1}{i} \frac{s^{2i}}{2^i\,(i-\frac{1}{2})^{(i)}} + \sum_{1}^{m}{}_i (-1)^i \binom{m-1}{i-1} \frac{s^{2i}}{2^i\,(i-\frac{1}{2})^{(i)}}$$

$$= S_{2m-2} - \sum_{0}^{m-1}{}_i (-1)^i \binom{m-1}{i} \frac{s^{2i+2}}{2^{i+1}\,(i+\frac{1}{2})^{(i+1)}}$$

$$(84) \qquad = S_{2m-2} - s \cdot S_{2m-1} \qquad \text{for } 0 \leqq m,$$

$$s \cdot S_{2m} = \sum_{0}^{m}{}_{i} (-1)^i \binom{m}{i} [2\,m + 1 - 2\,(m - i)] \frac{s^{2i+1}}{2^{i+1}\,(i + \frac{1}{2})^{(i+1)}}$$

$$= (2\,m + 1) \cdot S_{2m+1} - 2\,m \sum_{0}^{m-1}{}_{i} (-1)^i \binom{m-1}{i} \cdot \frac{s^{2i+1}}{2^{i+1}\,(i + \frac{1}{2})^{(i+1)}}$$

$$= (2\,m + 1) \cdot S_{2m+1} - 2\,m \cdot S_{2m-1}\,,$$

that is,

$$(85) \qquad S_{2m+1} = \frac{2\,m}{2\,m + 1} \cdot S_{2m-1} + \frac{1}{2\,m + 1} \cdot s \cdot S_{2m} \quad \text{for } 0 \leqq m.$$

From (68), (84), and (85) we obtain

$$(86) \qquad \begin{cases} R_{2m} = R_{2m-2} - s \cdot R_{2m-1} & \text{for } 0 \leqq m, \\ R_{2m+1} = \dfrac{2\,m}{2\,m + 1} \cdot R_{2m-1} + \dfrac{1}{2\,m + 1} \cdot s \cdot R_{2m} & \text{for } 0 \leqq m, \end{cases}$$

while from (68), (83), (84), and (85) follow

$$\frac{\partial R_{2m}}{\partial s} = -\,(2\,m \cdot S_{2m-1} + s \cdot S_{2m})\, e^{-\frac{s^2}{2}} = -\,(2\,m + 1)\, S_{2m+1} \cdot e^{-\frac{s^2}{2}}$$

$$(87) \qquad = -\,(2\,m + 1) \cdot R_{2m+1}\,,$$

$$\frac{\partial R_{2m+1}}{\partial s} = (S_{2m} - s \cdot S_{2m+1})\, e^{-\frac{s^2}{2}} = S_{2m+2} \cdot e^{-\frac{s^2}{2}}$$

$$(88) \qquad = R_{2m+2}\,.$$

From (83), (87), and (88) follow, after simple reductions by (84)–(86),

$$\frac{\partial}{\partial s} \{m \cdot S_{2m-1}\, R_{2k} - k \cdot S_{2m}\, R_{2k-1}\} = (m - k) \cdot S_{2m}\, R_{2k}\,,$$

$$\frac{\partial}{\partial s} \{S_{2m+1}\, R_{2k} - S_{2m}\, R_{2k+1}\} = 2\,(m - k) \cdot S_{2m+1}\, R_{2k+1}\,,$$

$$\frac{\partial}{\partial s} \{2\,m \cdot S_{2m-1}\, R_{2k+1} + S_{2m}\, R_{2k}\} = (2\,m - 2\,k - 1) \cdot S_{2m}\, R_{2k+1}\,.$$

Therefore, letting $[\ ]_{\underline{s}}^{\bar{s}}$ denote the excess of the value of the enclosed function of s for $s = \bar{s}$ over its value for $s = \underline{s}$, we have

$$(89) \qquad \int_{\underline{s}}^{\bar{s}} S_{2m}\, R_{2k} \cdot ds = \frac{m}{m - k} \left[S_{2m-1}\, R_{2k}\right]_{\underline{s}}^{\bar{s}} - \frac{k}{m - k} \left[S_{2m}\, R_{2k-1}\right]_{\underline{s}}^{\bar{s}}$$

for $k \lessgtr m$,

(90)

$$\int_{\underline{s}}^{\overline{s}} S_{2m+1}\, R_{2k+1} \cdot ds = \frac{1}{2\,(m-k)}\left[S_{2m+1}\, R_{2k}\right]_{\underline{s}}^{\overline{s}} - \frac{1}{2\,(m-k)}\left[S_{2m}\, R_{2k+1}\right]_{\underline{s}}^{\overline{s}}$$

for $k \lessgtr m$, and

(91)

$$\int_{\underline{s}}^{\overline{s}} S_{2m} R_{2k+1} \cdot ds = \frac{2\,m}{2\,m-2\,k-1}\left[S_{2m-1} R_{2k+1}\right]_{\underline{s}}^{\overline{s}} + \frac{1}{2\,m-2\,k-1}\left[S_{2m} R_{2k}\right]_{\underline{s}}^{\overline{s}}.$$

Also, from (83), (87), and (88) follow

$$\frac{\partial}{\partial s}\left(S_{2m}\, R_{2m-1}\right) = S_{2m}\, R_{2m} - 2\,m \cdot S_{2m-1}\, R_{2m-1}\,,$$

$$\frac{\partial}{\partial s}\left(S_{2m+1}\, R_{2m}\right) = -\,(2\,m+1) \cdot S_{2m+1}\, R_{2m+1} + S_{2m}\, R_{2m}\,,$$

so that

$$(92) \qquad \int_{\underline{s}}^{\overline{s}} S_{2m}\, R_{2m} \cdot ds = \left[S_{2m}\, R_{2m-1}\right)_{\underline{s}}^{\overline{s}} + 2\,m \cdot \int_{\underline{s}}^{\overline{s}} S_{2m-1}\, R_{2m-1} \cdot ds,$$

$$(93) \qquad \int_{\underline{s}}^{\overline{s}} S_{2m+1} R_{2m+1} \cdot ds = -\frac{1}{2m+1}\left[S_{2m+1}\, R_{2m}\right]_{\underline{s}}^{\overline{s}} + \frac{1}{2m+1}\int_{\underline{s}}^{\overline{s}} S_{2m}\, R_{2m} \cdot ds.$$

By alternate applications of (92) and (93) any integral of the type $\int_{\underline{s}}^{\overline{s}} S_m\, R_m \cdot ds$ can be reduced to a linear function of expressions of the type $\left[S_k\, R_{k-1}\right]_{\underline{s}}^{\overline{s}}$ for values of k descending from m to 1 and of the integral

$$(94) \qquad \int_{\underline{s}}^{\overline{s}} S_0 R_0 \cdot ds = \int_{\underline{s}}^{\overline{s}} e^{-\frac{s^2}{2}} \cdot ds.$$

X. Case in which all Values of s are possible.

For $\underline{s} = -\infty$ and $\overline{s} = +\infty$ (that is, if *all* values of the residual are possible), the integrals in question have simple numerical expressions. Namely, whatever integer k may be,

$$s^k\, e^{-\frac{s^2}{2}} = 0 \qquad \text{for } s = \pm\,\infty\,;$$

therefore,

$$R_k = 0 \qquad \text{for } s = \pm \infty\,; \tag{95}$$

therefore, equations (81) are all satisfied, by (89)–(91). Also, (92) and (93) become, for this case,

$$\int_{-\infty}^{+\infty} S_{2m}\, R_{2m} \cdot ds = 2\, m \int_{-\infty}^{+\infty} S_{2m-1}\, R_{2m-1} \cdot ds,$$

$$\int_{-\infty}^{+\infty} S_{2m-1}\, R_{2m-1} \cdot ds = \frac{1}{2\, m + 1} \int_{-\infty}^{+\infty} S_{2m}\, R_{2m} \cdot ds\,;$$

by alternate applications of these two formulae, we obtain

$$\int_{-\infty}^{+\infty} S_{2m}\, R_{2m} \cdot ds = \frac{2\, m}{2\, m - 1} \int_{-\infty}^{+\infty} S_{2m-2}\, R_{2m-2} \cdot ds,$$

$$\int_{-\infty}^{+\infty} S_{2m+1}\, R_{2m+1} \cdot ds = \frac{2\, m}{2\, m + 1} \int_{-\infty}^{+\infty} S_{2m-1}\, R_{2m-1} \cdot ds\,;$$

and again, by repetitions of these,

$$\int_{-\infty}^{+\infty} S_{2m}\, R_{2m} \cdot ds = \frac{m\,!}{(m - \frac{1}{2})^{(m)}} \int_{-\infty}^{+\infty} S_0\, R_0 \cdot ds,$$

$$\int_{-\infty}^{+\infty} S_{2m+1}\, R_{2m+1} \cdot ds = \frac{m\,!}{(m + \frac{1}{2})^{(m)}} \int_{-\infty}^{+\infty} S_1\, R_1 \cdot ds.$$

Now, by (93), (63), and (68),

$$\int_{-\infty}^{+\infty} S_1\, R_1 \cdot ds = \int_{-\infty}^{+\infty} S_0\, R_0 \cdot ds = \int_{-\infty}^{+\infty} e^{-\frac{s^2}{2}} \cdot ds = \sqrt{2\,\pi},$$

where π represents the ratio of the circumference of a circle to its diameter; therefore,

$$\int_{-\infty}^{+\infty} S_{2m}\, R_{2m} \cdot ds = \frac{m\,!}{(m - \frac{1}{2})^{(m)}} \sqrt{2\,\pi}, \tag{96}$$

$$\int_{-\infty}^{+\infty} S_{2m+1}\, R_{2m+1} \cdot ds = \frac{m\,!}{(m + \frac{1}{2})^{(m)}} \sqrt{2\,\pi}, \tag{97}$$

so that, by (80),

$$(98)\qquad C_{2m} = \frac{1}{\sqrt{2\pi}}\,\frac{(m-\frac{1}{2})^{(m)}}{m\,!}\ \text{for}\ 0 \leqq m \leqq a\ (\text{excepting}\ m=1)$$

and

$$(99)\qquad C_{2m+1} = \frac{1}{\sqrt{2\pi}}\,\frac{(m+\frac{1}{2})^{(m)}}{m\,!}\ \text{for}\ 1 \leqq m \leqq \beta.$$

It will be observed that C_{2m} is less and C_{2m+1} is greater than $\frac{1}{\sqrt{2\pi}}$; for very large values of m we have, by Stirling's formula,

$$(100)\qquad C_{2m} = \frac{1}{\pi\sqrt{m}}, \qquad C_{2m+1} = \frac{\sqrt{2m}}{\pi},$$

so that C_{2m} diminishes and C_{2m+1} increases without limit as m increases without limit; this fact will, perhaps, to a certain extent offset the conclusion that might be drawn from (61) that the values of the P's with odd suffixes are likely to be less than those of the P's with nearly the same even suffixes (on account of the presence of the odd functions S under the integral sign in the former and of the even functions S in the latter) in estimating the relative importance of the terms of $f(s)$ as given by (78) that contain R's with odd suffixes and R's with even suffixes.

For the C's with the smaller suffixes we have

$$(101)\qquad \begin{cases} C_0 = \dfrac{1}{\sqrt{2\pi}}, & C_4 = \dfrac{3}{8\sqrt{2\pi}}, \quad C_6 = \dfrac{5}{16\sqrt{2\pi}}, \ \text{etc.,} \\[2ex] C_3 = \dfrac{3}{2\sqrt{2\pi}}, & C_5 = \dfrac{15}{8\sqrt{2\pi}}, \quad C_7 = \dfrac{35}{16\sqrt{2\pi}}, \ \text{etc.,} \end{cases}$$

so that, by (27), (78), (68), (51), (63), and (101),

$$(102)\qquad \begin{cases} \psi(\xi) = \dfrac{1}{\mu}f(s) = \dfrac{1}{\mu\sqrt{2\pi}}\,[1 - \frac{3}{8}(1 - \frac{1}{3}\rho_4)(1 - 2s^2 + \frac{1}{3}s^4) \\ - \frac{5}{8}(1 - \frac{1}{2}\rho_4 + \frac{1}{30}\rho_6)(1 - 3s^2 + s^4 - \frac{1}{15}s^6) + \text{etc.} \\ - \frac{1}{2}\rho_3(s - \frac{1}{3}s^3) - \frac{5}{4}(\rho_3 - \frac{1}{10}\rho_5)(s - \frac{2}{3}s^3 + \frac{1}{15}s^5) + \text{etc.}]\,e^{-\frac{s^2}{2}}. \end{cases}$$

XI. Application to any Particular Case.

It seems reasonable to suppose that, in any case (whatever the values of $\underline{s}$ and $\overline{s}$) $f(s)$ will have small values for values of s beyond the limits of possible values, so that the extension of the limits to $-\infty$ and $+\infty$ will not make much difference in the values of the integrals by which the C's are determined; we may, then, take the values given by (98) and (99) as *approximate* in all cases.

The restriction of the development (78) of $f(s)$ to a finite number of terms is justified only by the fact that the P's that would occur in later terms are all negligible, that is that

$$(103) \qquad \begin{cases} P_{2m} = 0 & \text{for} \quad \alpha < m, \\ P_{2m+1} = 0 & \text{for} \quad \beta < m, \end{cases}$$

to the degree of approximation to which the calculations are restricted. In the special case for which $\underline{s} = -\infty$ and $\bar{s} = +\infty$ these equations (103) are *exactly* satisfied by (89)–(91), because the suffixes of the P's in (103) are different from those of any of the P's that occur in the right members of (79).

In applying the general method to any particular set of observations we have, then, to calculate, by (1)–(7) and (48) (from the observations), the values of z_0, the residuals ($\xi = z - z_0$), the p's, the ρ's, and the P's with so many successive suffixes as to satisfy ourselves that the P's with suffixes greater than a certain number are all negligible; the greatest even suffix and the greatest odd suffix of the P's that are not negligible are the values of $2\,\alpha$ and $2\,\beta + 1$, respectively; the values of the P's that are not negligible and the approximate values of the corresponding C's given by (98) and (99) being substituted in (77) determine an approximate form of $\theta(s)$; the positive and negative roots of $\theta(s) = 0$ that have the smallest absolute values are approximate values of $\underline{s}$ and $\bar{s}$, with which more accurate values of the C's can be calculated by (80); substituting these values of the C's in (77) we obtain a more exact form of $\theta(s)$, from $\theta(s) = 0$ more exact values of $\underline{s}$ and $\bar{s}$, and from (80) more exact values of the C's; and so we continue until the variations of the values of the C's are reduced to negligible quantities; the final values of the C's being substituted in (78) give the most exact form of $f(s)$ and, by (27), the most exact form of $\psi(\xi)$ that can be derived from the observations. The closeness with which the conditions (81) are satisfied by the final values of $\underline{s}$ and $\bar{s}$ may be regarded as a test of the *goodness of the observations*, that is of the closeness with which the residuals follow the law that they follow most closely.

It may be observed that there is no case for which $\alpha = 1$; more properly, the cases for which $\alpha = 1$ coincide with the cases for which $\alpha = 0$.

If $P_{2m+1} = 0$ for $1 \overset{=}{<} m$, $f(s)$ as given by (78) is an even function of s and the relative frequencies of positive and negative residuals that have the same absolute magnitudes are equal, that is, *positive and negative residuals are equally probable.*

The calculation of the true values of the C's by (80), (92), and (93) will be facilitated by tables of the values of $S_m R_{m-1}$ and of the integral (94). The values of $S_m R_{m-1}$ will be required for such values of m as are suffixes of the P's that are not negligible, which will be small, undoubtedly, in all actual cases; tables of $S_m R_{m-1}$ for $m = 3, 4, 5, 6$ will probably be sufficient. For the values of the integral (94) the ordinary tables of

$$(104) \qquad \Phi(\gamma) = \frac{2}{\sqrt{\pi}} \int_0^{\gamma} e^{-t^2} \cdot dt$$

can be used; namely,

$$\int_0^{\gamma} e^{-\frac{s^2}{2}} \cdot ds = \sqrt{2} \int_0^{\frac{\gamma}{\sqrt{2}}} e^{-t^2} \cdot dt = \sqrt{\frac{\pi}{2}} \cdot \Phi\left(\frac{\gamma}{\sqrt{2}}\right),$$

so that, observing that $\Phi(-\gamma) = -\Phi(\gamma)$, we have

$$(105) \int_{\underline{s}}^{\bar{s}} e^{-\frac{s^2}{2}} \cdot ds = \sqrt{\frac{\pi}{2}} \cdot \left[\Phi\left(\frac{\bar{s}}{\sqrt{2}}\right) - \Phi\left(\frac{\underline{s}}{\sqrt{2}}\right)\right]$$

$$= \sqrt{\frac{\pi}{2}} \cdot \left[\Phi\left(\frac{\bar{s}}{\sqrt{2}}\right) + \Phi\left(\frac{-\underline{s}}{\sqrt{2}}\right)\right],$$

where $\bar{s}$ and $-\underline{s}$ are both positive.

XII. Probable Values.

We have defined the *probable value* of the observed quantity to be that value that is just as likely to be exceeded as not: if the *probable residual* ξ_p and the *probable relative residual* s_p be taken to correspond to this probable value z_p, ξ_p and s_p will be determined by the equation

$$(106) \qquad \int_{\underline{\xi}}^{\xi_p} \psi(\xi) \cdot d\xi = \int_{\underline{s}}^{s_p} f(s) \cdot ds = \tfrac{1}{2}$$

and z_p by

$$(107) \qquad z_p = z_0 + \xi_p,$$

in accordance with (3), so that the arithmetical mean is (in general) not the probable value even of a quantity directly observed (as it is assumed to be in the method of Gauss).

XIII. The Ordinary Case.

In the ordinary theory it is assumed that

$$(108) \qquad P_k = 0 \quad \text{for} \quad 3 \leqq k,$$

so that the only P regarded as not negligible is $P_0 = 1$, which being substituted in (77) with the approximate value $C_0 = \frac{1}{\sqrt{2\pi}}$, by (101), gives, by (51) and (63),

$$\theta(s) = C_0 \, P_0 \, S_0 = \frac{1}{\sqrt{2\pi}};$$

$\theta(s) = 0$ has, then, no real finite root, so that

$$\underline{s} = -\infty, \ \bar{s} = +\infty$$

and (80) gives again the same value of C_0 as (101), which is, therefore, the most exact value possible on the assumption made. We have further, by (78) and (27),

$$(109) \qquad f(s) = \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}}, \ \psi(\xi) = \frac{1}{\mu\sqrt{2\pi}} e^{-\frac{\xi^2}{2\mu^2}};$$

while (60) gives

$$\rho_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} s^k \cdot e^{-\frac{s^2}{2}} \cdot ds,$$

that is

$$(110) \qquad \rho_{2m+1} = 0 \quad \text{for} \quad 0 \leqq m$$

and

$$(111) \qquad \rho_{2m} = 2^m \left(m - \tfrac{1}{2}\right)^{(m)} \quad \text{or} \quad p_{2m} = 2^m \left(m - \tfrac{1}{2}\right)^{(m)} \cdot \mu^{2m}$$

for $0 \leqq m$, so that, far from being negligible for the greater values of m, ρ_{2m} increases beyond all limit as m increases.

XIV. Two Other Particular Cases.

For the purpose of illustrating the variety of forms the general theory may take in particular cases, and especially its possible deviation from the ordinary theory, we have plotted the curve represented by $y = \frac{1}{\mu} f(s)$ (the *probability-curve*) for $\alpha = 0$, $\beta = 1$ and for $\alpha = 2$, $\beta = 0$, using the approximate values of C_0, C_3, and C_4 given by (101), for such different values of ρ_3 and ρ_4 (in the two cases, respectively) as to show all the

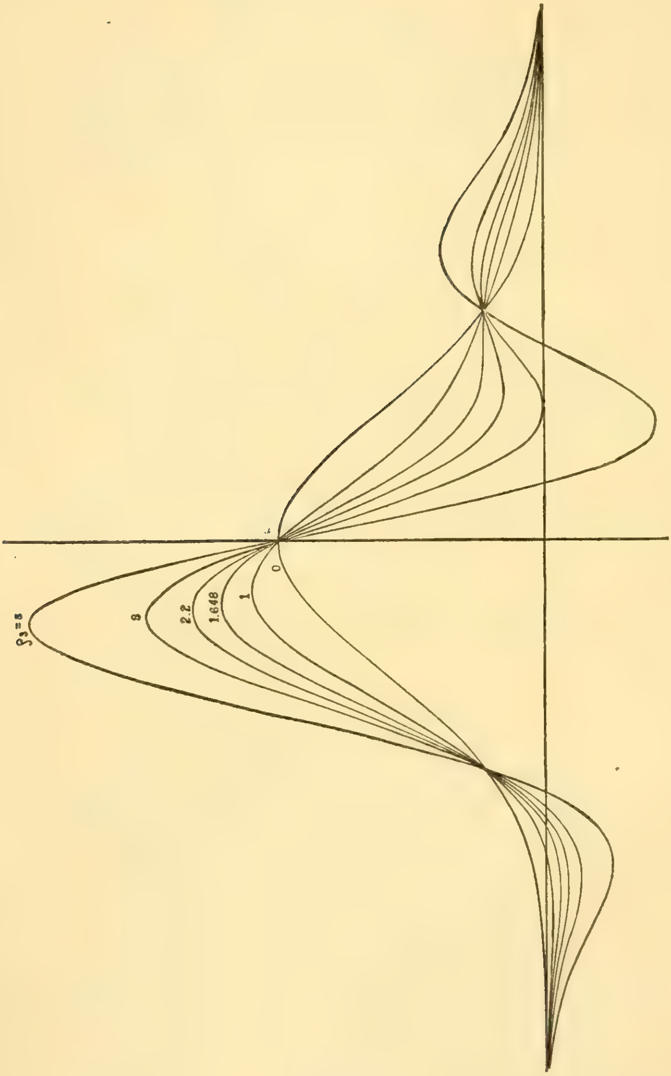

FIGURE 1.

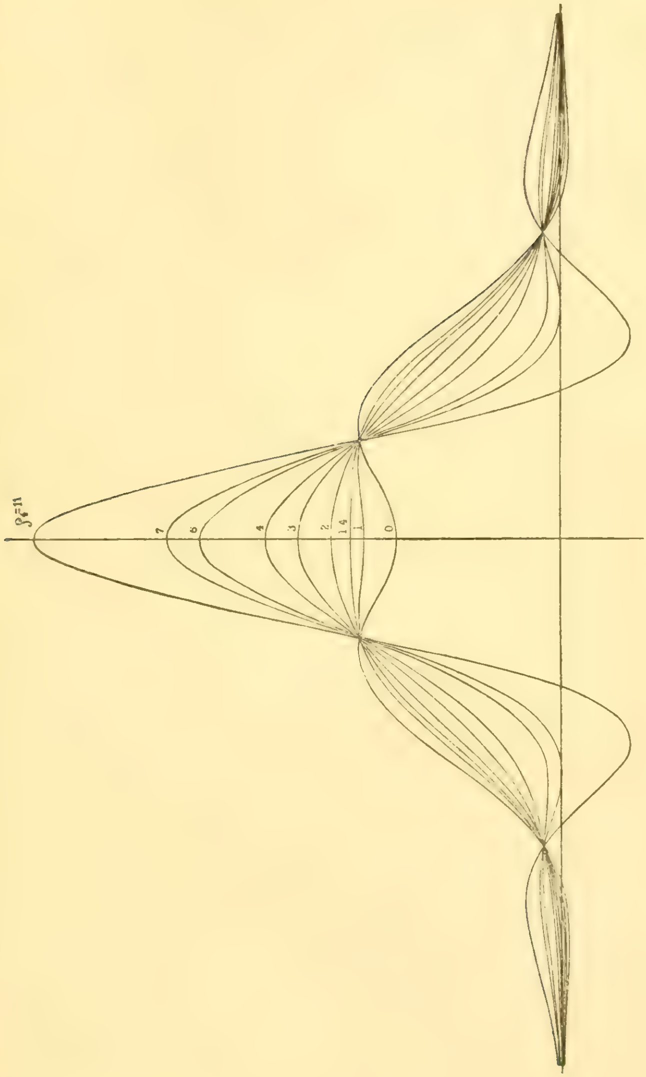

FIGURE 2.

essentially different forms for these pairs of values of α and β. Figure 1 shows the locus of the equation

$$(112) \qquad y = \frac{1}{\mu \sqrt{2\pi}} \left[1 - \tfrac{1}{2} \rho_3 \left(s - \tfrac{1}{3} s^3\right)\right] e^{-\frac{s^2}{2}}$$

for $\rho_3 = 0, 1, 1.648, 2.2, 3, 5$, and Figure 2 the locus of the equation

$$(113) \qquad y = \frac{1}{\mu \sqrt{2\pi}} \left[1 - \tfrac{3}{8} \left(1 - \tfrac{1}{3} \rho_4\right) \left(1 - 2 s^2 + \tfrac{1}{3} s^4\right)\right] e^{-\frac{s^2}{2}}$$

for $\rho_4 = 0, 1, 1.4, 2, 3, 4, 6, 7, 11$. The centre of each figure corresponds to $s = 0$ and the two extremes to $s = \pm 4$; the horizontal line is the axis of abscissæ. It will be seen that the ordinates of all the curves are practically 0 for values of s whose absolute magnitudes exceed 4. The curve for which $\rho_3 = 0$ in Figure 1 and that for which $\rho_4 = 3$ in Figure 2 coincide with the ordinary probability-curve. The curve for which $\rho_3 = 3$ in Figure 1 and that for which $\rho_4 = 7$ in Figure 2 touch the axis of abscissae; these values are, by the footnote on page 173, the extreme values of ρ_3 and ρ_4, respectively; each figure contains one curve that corresponds to a value beyond these limits, which shows the discontinuity of $\underline{s}$ or $\bar{s}$ as the curve passes through the form that has contact with the axis. While ρ_4 is essentially positive, as a mean even power of the relative residual, ρ_3 may be negative; if the sign of the value of ρ_3 that corresponds to either curve in Figure 1 is changed, the curve will be reversed, right and left. In the cases represented in Figure 1, the maximum relative frequency and the probable value of the observed quantity correspond to negative values of s; in the cases represented in Figure 2, the maximum relative frequency corresponds to $s = 0$ for some values of ρ_4, while there are two maxima for the smaller values of ρ_4, the probable value of the observed quantity corresponding always to $s = 0$.

Because $s = 1$ corresponds to $\xi = \mu$, that is to $\xi^2 = p_2$, where p_2 is the mean square of the residual, it is evident that at least one of the limits $\underline{s}$ and $\bar{s}$ must be considerably greater than 1 in absolute magnitude, in any case; but the curves plotted show that very little of accuracy is gained by considering values of s beyond $\pm$ 4 in the cases represented by them.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — LVIII.

SEXUAL REPRODUCTION IN THE MUCORINEAE.

By Albert Francis Blakeslee.

With Four Plates.

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By Albert Francis Blakeslee.

Presented by R. Thaxter, May 11, 1904. Received May 11, 1904.

PREFACE.

For several years past the writer has been engaged in a study of the Mucorineae, which he at first intended should comprise a general survey of the morphology and development of the group, and include at least an attempt to set in order some of the confusion in which the systematic study of these plants has been involved. Having accumulated a considerable amount of material for the purpose, the greater part of which has been kept under cultivation in pure cultures, he was led about a year since to abandon his original plan and restrict his field of investigation to a single phase of the subject in hand, and to confine his attention to the conditions influencing, or associated with, the production of the so-called "sexual spores," or zygospores, which are characteristic of these fungi. Although during the past thirty or more years this subject has received much attention from botanists, and has been the object of numerous researches, the results obtained in the present instance appear to be quite novel, and, in the writer's opinion, present for the first time a rational basis by means of which the phenomena of sexual reproduction in the group may be explained.

That the present paper, which is practically based on hardly a year's work, embodies a necessarily inadequate treatment of the subject need hardly be pointed out; but although the necessities of the case have rendered this incomplete presentation unavoidable the writer proposes to continue his investigations, and trusts that he may be able in a future communication to remedy in some measure the deficiencies of the present one.

It is a pleasurable duty to acknowledge at this place indebtedness to those who have kindly given assistance in one way or another: to Dr. W. G. Farlow for the use of his herbarium and for several of the papers herein cited; to Prof. E. C. Jeffrey for photographing certain of the plates; but above all, to Prof. R. Thaxter, who, besides being the one under whom this research has been prosecuted and to whom a pupil's thanks are due for advice and encouragement, has placed at the writer's disposal his mounted specimens and tube cultures of forms of the Mucorineae which have been accumulated during his extensive investigations among the lower fungi.

Thanks are due to the following for material: Prof. George Klebs; Prof. H. M. Ward, and Dr. R. H. Biffen; Prof. O. Brefeld, and Dr. R. Falck; Prof. D. H. Campbell; Prof. F. Cavara; Prof. Ed. Fischer; Prof. P. Vuillemin; Mr. Robert M. Grey; Mr. C. S. Leavenworth; Mr. J. B. Rorer; Mr. J. J. Wolfe; and to numerous others both in America and Europe for similar favors, especially in connection with the Rhizopus table (p. 263).

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PART I.

INTRODUCTION.

Among the Mucorineae the usual form of reproduction is by means of nonsexual spores in sporangia. Although the sexual method of reproduction by zygospores has, through the researches of individual investigators, now been found in nearly all the genera, it is still in a majority of the species entirely unknown, and our knowledge of its occurrence in about four-fifths the cases rests on the unconfirmed reports of single individuals. The only common species which could be depended on to produce its zygospores has been *Sporodinia grandis*. Ever since de Bary's ('64) investigation of this form and of *Rhizopus nigricans* ('66) the causes inducing the formation of zygospores have been a subject of much speculation and considerable study among mycologists. The elaborate papers of Klebs ('98) and Falck ('01) have given us much conflicting information on the problem in Sporodinia, but attempts to apply the results of these investigations, or of the earlier researches of van Tieghem and others, to the Mucorineae in general, with a view to obtaining zygospores, have proved ineffective. Since the evidence on this subject, in a form like Sporodinia for example, which has been most thoroughly examined, is not only conflicting, but is evidently inapplicable to a majority of existing forms, it has been the aim of the present paper to consider the conditions which lead to the production of zygospores in the group as a whole rather than in any single species.

In 1901, at Professor Thaxter's suggestion and under his direction, the writer began the collection and preservation in pure cultures of forms of the Mucorineae, but especially of the genus Mucor, having in mind, as has already been mentioned, a subsequent monograph of this confused genus. In the course of this preliminary work zygospores of different species were several times found, but the forms differed greatly in their behavior. Some of them, among which Sporodinia may be taken as a type, could be readily induced to produce zygospores on a suitable substratum when a pure transfer was made by means of spores taken from a single sporangium; others, of which Rhizopus may be taken as a type, would never form zygospores from a pure transfer, but only when a mass of spores from a zygosporic culture was used for the inoculation. Of the first class, all have been isolated and kept running in tube cultures without abatement in their zygosporic activity, but with such species of the second as could not be separated from the abundance of other fungi and bacteria and preserved with their zygospores on the dried substratum, it was impossible to continue the formation of zygospores. In the attempt to grow a form of the Rhizopus type on nutrient agar in a more convenient condition for sectioning, a separation culture was made from the midst of the zygosporic region, and zygospores were found (May 2, '03) to appear at the junction of certain of the mycelial colonies. By following up this suggestion it was demonstrated that this species consists of two strains or races which, when grown apart, produce only sporangia, but which form zygospores when hyphae from the physiologically different mycelia are allowed to come in contact. If we designate one strain by the sign (+) we can conveniently use the (—) sign to represent the other without committing ourselves as to the sexual relation which the strains may bear to each other. A (+) strain, for example, will never produce zygospores on any medium if sown alone, nor if sown with some other (+) strain however different in origin; but if the complementary (—) strain be sown at the same time either mixed with the (+) or so disposed that their mycelia can come in contact, zygospores are formed by the union of gametes produced from the (+) and (—) mycelia respectively. The condition is essentially similar to that in dioecious plants and animals, and although in this case the two complementary individuals which are needed for sexual reproduction are not in general so conspicuously differentiated morphologically as in the higher forms, such a morphological difference is often distinctly visible, and, as hereafter noted, would undoubtedly be considered by systematists generally as an amply sufficient basis for their specific separation. Inasmuch, however, as conjugation is possible only through the interaction of two differing thalli, we can express this fact by calling all species the sexual relations of which correspond to the Rhizopus type, *heterothallic.* In marked contrast to the conditions just described, Sporodinia and the other members of the group of which it is the type, invariably reproduce sexually under suitable conditions when grown from a single spore. The zygospores thus originate from a single mycelium, and are comparable to hermaphrodites among the higher plants. Such forms may therefore be called *homothallic.*

In several of the heterothallic forms experimented with, certain races have been found which apparently cannot be induced to respond to the (+) and (—) strains, the existence of which has been demonstrated in such species. Moreover it has been shown that in *Mucor Mucedo* the power of conjugating possessed by the two sexual strains may be at will entirely inhibited by cultivation under unfavorable conditions. These "neutral" strains will be further discussed in connection with the individual forms considered in Part II.

It may be here stated that the writer has at present under cultivation over a dozen forms, including both types, which will conjugate whenever the proper conditions are furnished. Of the heterothallic group five species of the genus Mucor are represented, including *M. Mucedo*, besides *Rhizopus nigricans*, *Phycomyces nitens*, *Absidia caerulea*, and an undescribed species of a new genus. Of the homothallic are *Sporodinia grandis*, *Zygorhynchus Moelleri*, *Dicranophora sp.*, and two undescribed species of Mucor. The undetermined Mucors will be provisionally indicated by Roman numerals, Mucors I and II being homothallic, Mucors III to VI heterothallic, while Mucor N represents the undescribed new genus.

It is the purpose of the writer to give in Part I a brief historical review of the theories that have hitherto been advanced in regard to the causes influencing zygospore formation, followed by a citation as complete as the writer has been able to make it, of all the species of which this form of reproduction has been reported. In Part II the original research on the subject of this paper will be arranged under the different species investigated.

In this connection it may be advisable to define certain terms, some of which have been used variously by different authors, and to give a short account of the morphology of conjugation. In Rhizopus or in Sporodinia, for example, wherever two adjacent (+) and (−) hyphae touch, the stimulus of contact may cause outgrowths to form; which, by remaining mutually adherent, and increasing in size, gradually push apart the hyphae from which they have arisen. These outgrowths — the gametes of some authors (e. g. Léger, '96) — may better be termed *progametes*, since each becomes divided by a septum into a proximal portion, the suspensor, and a terminal cell, the gamete proper, and it is by the union of these gametes that the zygote is formed. This terminology brings the process into accord with other plants and with animals. Here, instead of dividing and developing into an embryo, the zygote becomes transformed into a single-celled, thick-walled, resting zygospore. It sometimes happens that, owing to unfavorable conditions, the gametes cut off fail to unite and develop independently into thick-walled azygospores. The formation of these spores may be considered abnormal, but in certain species where conjugation is not known to occur, so-called azygospores have been found to be produced regularly in great abundance. These reproductive bodies have been considered in connection with the zygospores in the literature cited, but it is impossible to state at the present time what their significance can be.

Progametes may either arise from undifferentiated aerial hyphae, as in Rhizopus, or may originate as in *M. Mucedo*, from special branches, which, rising from the mycelium, are mutually attracted and develop their progametes at the point of contact. It will thus be convenient to distinguish two types of more or less distinctly differentiated fertile hyphae; namely, sporangiophores bearing nonsexual spores in sporangia, and *zygophores*, which give rise to progametes and zygospores, although this distinction is not always well marked and both types of spores may originate from the same hypha. The successive steps in the process of conjugation as it occurs in *Mucor Mucedo* is represented in Plate II, Figures 25–35.

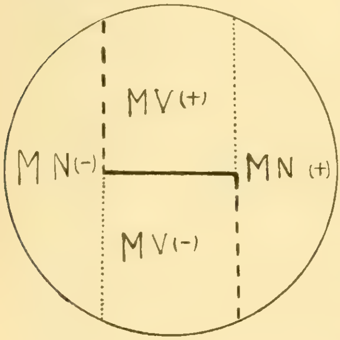
With the exception of de Bary's ('**66**) early description of Rhizopus, which is not in accordance with his later statements ('**84**, p. 159), and Falck's ('**01**, p. 241) description of Sporodinia, which, however, he was unable to confirm by observation, writers on the Mucorineae, in describing the process of conjugation, have stated that club-shaped progametes develop from opposite sides of hyphae not already in contact, and, increasing in size, grow through mutual attraction to meet each other at their swollen extremities. This account, illustrated with figures, has been generally current in text-books which mention the subject. However, in the species in which the process has been observed by the writer, including both homo- and hetero-thallic forms, the progametes are from the very first always normally adherent and arise as the result of the stimulus of contact between the (+) and (−) zygophoric hyphae which in some cases at least have been shown to be mutually attractive, and may therefore be termed *zygotactic*.

Although there is in general little if any differentiation in the conjugating apparatus which might possibly indicate a sexual differentiation, yet two genera of the homothallic group — Zygorhynchus and Dicranophora — are *heterogamic* in that their gametes show a certain constant inequality in size which corresponds to even a greater difference in size and form of their suspensors and zygophores. All other species of this group known are apparently isogamous.

In the heterothallic forms, where we should perhaps most expect to find a structural difference in the conjugating cells, the gametes are morphologically equivalent, so far as has been determined. Differences in the size of the gametes or of the suspensors, as, for example, in Rhizopus, and priority of one suspensor over the other in developing outgrowths, as in Phycomyces, although considered by some authors as indicative of a sexual differentiation, are inconstant, and will be subsequently shown to have no sexual significance. It is true in nearly every heterothallic species investigated that the strains show certain morphological differences which correspond to their physiological differentiation as sexual strains. In general the difference is one in degree rather than in form, and is indicated in several ways. *Mucor Mucedo* may be taken as an illustration of the simplest type of such a differentiation from the fact that, although the two strains are not distinguished by any apparent morphological difference, the (—) strain has been shown to lose its sexual capacity sooner than the (+) strain. This difference is further indicated by the fact that, while successive mycelial transfers from young cultures of the (+) strain retain their sexual capacity through indefinite generations, this, with the power of vegetative development, is rapidly lost by the (—) strain when subjected to a similar treatment. In Phycomyces the differentiation is more distinct in that the sporangiophores are more numerous and less delicate, and develop later in one strain than in the other (on proper nutrients this is more marked than in the culture photographed, Plate IV, Figure 52), while in several species in which a still greater differentiation is exhibited, when the strains are contrasted one will regularly show a lower, less luxuriant, sporangial growth, and often a decided color contrast. The most striking instance of this kind is seen in the species photographed (Plate IV, Figure 58), where the difference in luxuriance of sporangial growth, added to a corresponding difference in the size of the spores (Plate I, Figure 23, 24), might easily lead one in investigating these strains separately to consider them as individual species.

It is thus evident from even a cursory examination of the known heterothallic species that their sexual strains are with few exceptions characterized by more or less marked differences, especially in relation to their relative luxuriance of growth, and it was owing to this peculiarity that it was thought appropriate to apply the terms (+) and (—) to the more and less luxuriant strains respectively. The assumption that their differentiation to (+) and (—) strains was indicative of a corresponding sexual difference, and that the strains thus designated should be regarded respectively as male and female, or vice versa, although naturally suggested by the observed condition, did not at first appear to be warranted. Evidence, however, is not lacking which would seem to justify such a conclusion. Plate IV, Figure 57, for example, shows the strains of Mucor v, forming zygospores at their median line of contact. On either side of these conjugating strains were sown corresponding strains of Mucor x (+) at right, (—) at left. Where the growth of the (+) strain of one species comes in contact with the (—) of the other, a white line will be observed which a microscopic examination has shown is composed of incipient stages of conjugation wherein abundant progametes, and less frequently gametes, have been formed. It is an important fact that only when strains having unlike signs come in contact do these attempts at hybridization occur.

The accompanying diagram will perhaps render clearer the condition shown in the culture photographed. The continuous line between Mv(+) and Mv(—) indicates the zygospores formed by the two sexual strains of this species. The broken lines between Mv(+) and Mn(—), and between Mv(—) and Mn(+), indicate the lines formed by the attempts at hybridization between those strains of the two species which correspond to different signs. The dotted lines indicate the contacts of those strains of the two species corresponding to like signs. No distinct line is observed here in the photograph, and a microscopic examination shows that no attempts at hybridization have occurred.



By making contrasts between the (+) and (—) strains of those heterothallic species which have been producing zygospores in the laboratory it has been possible to arrange in a series all strains which had been marked (+) on account of their vegetative characters, and in another series all those marked (—) on the same basis. Those in the (+) series are found to be capable of forming imperfect hybrids with those of the (—) series, but no attempt at conjugation can ever be obtained between strains with like signs, either in the same or in different species.

It is further possible to place in the proper sexual columns the strains of those species (e. g. Rhizopus) in which a vegetative differentiation has not yet been determined (cf. table, p. 305), by means of the tendency already referred to, which leads to attempts at hybridization when (+) and (—) strains of different species, and even genera, are made to grow in contact with one another. Such an attempt at hybridization between Absidia and Rhizopus, for example (Plate I, Figure 18), shows in the most advanced cases the characteristic appendages from the Absidia suspensor, and the hyphae of these forms as in many other species differ sufficiently to be distinguished even without tracing them to their sporangia. Perfect hybrids have never been found, and though for convenience the term hybridization will be used hereafter in the present paper, it should be remembered that these attempted imperfect conjugations have never been observed to go further than, at most, a cutting off of the firmly united gametes.

In other organisms, where the sexes are segregated on separate individuals, the female is generally characterized by a greater vegetative development, but the assumption that the same condition prevails here, and (+) and (—) strains are equivalent to female and male, respectively, would from our present knowledge hardly seem justifiable. It is hoped, however, that further investigation may demonstrate the correctness of such a conclusion.

A further indication of the strictly sexual nature of these processes in general is found in the interaction between homothallic forms which may be assumed to be bisexual [(+) *and* (—)], and (+) and (—) strains of heterothallic types. Plate IV, Figure 56, for example is photographed from a culture in which the (+) and (—) strains of a heterothallic species, Mucor V, have been sown on opposite sides of a homothallic species, Mucor I. Had a (+) or (—) strain of a heterothallic species been sown in place of Mucor I, conjugation would have taken place on one side only, namely, on that side where (+) and (—) strains were opposed. In the present instance, however, the middle strain having both (+) and (—) qualities, conjugates with the (+) heterothallic strain on the one hand, and with the (—) strain on the other, as is indicated by the evident white lines of demarcation between them. The response here demonstrated between a homothallic form and *both* (+) and (—) heterothallic strains which have been shown to be unisexual, can thus be explained only on the assumption that the homothallic Mucor I possesses both sex qualities.

It may be mentioned in this connection also that a number of species the zygospores of which are unknown have been made to produce imperfect hybrids with the (+) and (—) strains under cultivation. Thus individual strains of Syncephalastrum, a new species of Chaetocladium, and a number of other forms, have been found to be (—) from the fact that they interact only with (+) strains, while a culture of *Cunninghamella echinulata* Thaxter (*C. Africana* Matruchot), which was formerly classed as a Hyphomycete, has been shown to be (+). From our present knowledge one is justified in assuming that a culture which responds sexually to only one of the test strains must belong to the heterothallic group, and its zygospores have not been seen simply because it has not been found in contact with its complementary strain. On this basis we can add to the heterothallic forms already known representatives of the genera Syncephalastrum, Circinella and Chaetocladium, and this will extend heterothallism to all the families of the Mucorineae recognized by A. Fischer ('92) except the Mortierellaceae. No attempt has yet been made to determine the sexual condition in this family.

From the fact that the writer has demonstrated the possibility of running out the sexual activity in *Mucor Mucedo*, and the apparent impossibility of obtaining zygospores from contrasts made between certain races of *Rhizopus nigricans* and (+) and (—) strains of this same species, it is further evident that neutral strains, or at least strains in which the sexual activity is dormant, exist in nature. The character of the substratum, and also the nature of the forms contrasted, are important factors to be considered in hybridizing. Phycomyces, for example, will hybridize with *Mucor Mucedo* only on concentrated nutrients, but will not hybridize with certain other forms under the same conditions. Consequently a failure to obtain imperfect hybrids by contrasting a given culture with the opposite strains of a test form proves nothing positive in regard to the thallic condition of the culture under investigation.

HISTORICAL REVIEW OF THE THEORIES RELATING TO ZYGOSPORIC REPRODUCTION IN THE MUCORINEAE.

With the foregoing brief review of the more important matters treated in the present research, and before considering them in detail, it has seemed desirable to present a short summary of the more prominent views which have been held by previous investigators regarding the conditions influencing the formation of zygospores in the Mucorineae, and the nature of the processes involved, to which is appended as complete a statement of the reported occurrence of these bodies as it has been possible for the writer to compile.

In Ehrenberg's (['20]'29) work on Syzygites, which contains the first mention of the occurrence of zygospores among the Mucorineae, he recognizes the conjugation as comparable with the act in such algal forms as Spirogyra and as of a probable sexual nature. Subsequent investigators, however, who have advanced theories to account for the appearance of zygospores, either deny that there is any sexuality in the process of their formation, or consider such a possibility as without bearing on the problem. Such explanations as have been offered are thus based in general on the assumption that zygospore formation is primarily dependent on external conditions in connection with which they have been developed.

We are indebted to de Bary ('**66**) for the first discussion of this question, in his investigation of the common heterothallic form *Rhizopus nigricans*. Zygospores were found to form abundantly in tightly closed cultures, but less abundantly and only in the lower parts of cultures open to the air. "The fungus," according to this author, "takes in oxygen and gives off carbon dioxide; therefore in a closed tube, and below the network of hyphae in a gross culture the oxygen content of the air must be reduced. This circumstance, in that it retards the process of oxydation in the fungus, is apparently influential in the formation of zygospores, since the composition of the substratum, the water-supply, and the temperature, were the same in the open as in the closed cultures." In direct opposition to this conclusion, it may here be mentioned that Bainier ('**83**^{b}, p. 342) gives the exclusive presence of oxygen as a cause previously assigned for the production of zygospores, but the writer has been unable to discover who is responsible for this theory.

By following de Bary's suggestion, van Tieghem ('**75**) obtained the zygospores of Rhizopus, and appeared to confirm his theory by his results with other forms. In his Troisième Mémoire (p. 324) he concludes: "That which determines the formation of zygospores upon a mycelium still provided with a sufficient quantity of protoplasm is the impoverishment of the nutritive medium in one or more of its elements, an impoverishment which places in danger even the existence of the plant." The total nutrient he divides into (1) air, or its oxygen at the pressure of one-fifth of an atmosphere; (2) water; (3) the soluble substances, or food proper. Reduction in oxygen is the cause assigned for the occurrence of zygospores in Sporodinia and Rhizopus; desiccation for the occurrence of those found by Cornu in *Absidia septata;* while for the appearance of the zygospores of Piptocephalis and Sporodinia obtained by Brefeld on bread soaked with beer wort, the cause is supposed to be the unfavorable condition of the food supply of these naturally parasitic plants. Although van Tieghem adds that he gives the above generalization with great reserve, since it is necessary to determine in each case the special nature of the impoverishment which is effective, yet his last word on the subject ('**91**, p. 1076) is a reiteration of his impoverishment theory with the mention of cold as an additional factor.

From his experience in obtaining zygospores of *Absidia septata*, Sporodinia and Spinellus, Cornu ('**76**) concludes, "It is when the substratum becomes more and more dry that the zygospores appear."

Bainier ('82, '83^b, '84, '89, '03) has been especially successful in obtaining zygospores, but though he concludes that the substratum has much to do with their production, he is at variance with previous authors in that he says ('83^b, p. 344) " when the plant finds the nutriment inferior it produces sporangia; when the food supply is abundant, zygospores appear." He looks upon the zygospores as so much stored-up nutriment, and consequently implying a very vigorous condition in the plant producing them, and demanding that the thallus be able to extend itself in length and depth. His point of view, with the theory resulting from it, is thus radically different from that of van Tieghem, who looked more on the function than on the formation of this condensed mass of nourishment, and consequently considered that the formation of a resting spore must be brought about by the appearance of those unfavorable conditions against which it was designed to protect the plant. As regards moisture, Bainier thinks there can be nothing said in general, the species differing in their preference for more or less dryness. The season of the year and the temperature seem to be with him factors of some importance, for in his notes on the zygospores of different species the months in which they are supposed to form is generally stated, and in his latest work ('03), where zygospores are described for a considerable number of species, the special condition mentioned is the low temperature of a cellar where his cultures were kept during a warm period of summer. A majority of Bainier's zygospores were obtained in van Tieghem cell cultures in a concentrated decoction of plum and prune, which was sterilized by boiling and mixed with ten to twenty per cent of alcohol to prevent fermentation. This investigator further states that freedom from parasites is a condition to be satisfied before zygospores can be expected, in marked contrast to the opinion expressed by Zopf ('88 and '92) in connection with his study of Pilobolus, where the suppression of sporangia through the attacks of a parasite is regarded as responsible for the appearance of zygospores (cf. p. 238 of present paper).

Brefeld's ('75, '81, '00) position has been one of agnosticism as regards the causes for the appearance of zygospores, and the valuable part of his conclusions is contained in a series of denials substantiated by the results of his cultures. The character of the nutriment alone he holds to be of slight significance, since on the same substratum from differently derived spores of the same fungus, e. g. Piptocephalis, he obtained on the one hand zygospores, and on the other only nonsexual spores. Changing the substratum and the oxygen content of the air did not alter the results. The age of the mycelium is likewise not considered influential, since he has cultivated a mycelium for four weeks on an unlimited substratum with only nonsexual reproduction. The sexual reproduction is not necessarily induced after a series of nonsexual generations, since he has cultivated Rhizopus, *Mucor Mucedo*, and *M. racemosus* for four years up to nearly one hundred nonsexual generations. With Sporodinia and Spinellus no culture fails to show zygospores; with *Mortierella Rostafinskii*, Piptocephalis, and Chaetocladium, zygospores form after a few generations of nonsexual reproduction; while with Rhizopus and *M. Mucedo* a longer time elapses before the appearance of zygospores. There is thus, according to Brefeld, a gradual withdrawal of the sexual in favor of the nonsexual reproduction, till in such forms as *M. racemosus* and Pilobolus this has resulted almost or entirely in the disappearance of sexuality. Brefeld's ('00 and '01) final conclusion is that in most Zygomycetes the zygospore formation is dependent on unknown inner causes, and it is only by accident that it may be found.

While the authors thus far cited have based their theories and conclusions chiefly or wholly on what they could learn from the chance occurrence of zygospores in their cultures, and scarcely ever resorted to experiments, more recent investigators have endeavored to subject their processes to a more exact experimental research. Klebs ('98) was the first to investigate in detail the effect of varying external conditions on the resulting form of reproduction of any of the Mucorineae, and in his paper on Sporodinia we have a careful account of the factors influencing sporangium and zygospore formation in this homothallic form. Van Tieghem's idea that lack of oxygen is the cause for zygospore formation is disproved, and it is shown that all the influences which hinder sporangium formation, as a reduction in the amount or in the concentration of the nutrient, hinder much more the formation of zygospores. Most of the carbohydrates tested with gelatine and the acid salts of organic acids favor zygospores, while the nitrogen compounds are unfavorable. The immediate cause of sporangium formation is held to be transpiration from the hyphae, and though impoverishment of nutriment has a similar effect it is impossible to obtain zygospores in relatively dry air. The stimulus for production of zygospores, however, is conceived to be the decreased transpiration due to increased relative humidity in the air. The limits of concentration within which zygospores are produced vary greatly, but for grape sugar in gelatine, lie between one per cent and fifty per cent. The water content of the substratum, temperature, and light, are effective only as they influence the relative humidity and consequent transpiration. The formation of azygospores is said to be induced in a culture beginning to form zygospores by a change to any one of a number of unfavorable conditions.

In marked contrast to the results of Klebs, Falck ('01) finds that, within normal limits, relative humidity of the air and transpiration have no influence in determining the form of fructification. He denies Klebs' assumption that the hyphae bearing sporangia, and those bearing zygospores, are morphologically equivalent, and shows that the zygospores do not result from the meeting of adjacent outgrowths, but that these outgrowths arise through the stimulation of contact of neighboring hyphae, and are never separate. Falck concludes that with sufficient nutriment the concentration is the special condition effective for zygospore formation. Starting with a normal nutrient solution by the addition of increasing amounts of grape sugar, peptone, glycerine, or mineral salts, a gradual change can be induced from a condition in which sporangia alone are produced, to one in which zygosporic reproduction predominates. Concentration of a normal solution will bring on zygospore production, while if a solution already producing zygospores is sufficiently diluted, sporangia only are formed. When carbohydrates predominate in the nutrient, the harvest depends on the amount of peptone, while the reverse is true when peptone predominates. In the former case the zygospores are black and relatively poor in protein, while in the latter they are brown and the protein per cent is higher.

The publication of the results obtained by Falck led Klebs to test again the effects of relative humidity and transpiration, and in a second paper ('02) he confirms his former conclusion that the appearance of zygospores or of sporangia in different cases depends chiefly on the amount of transpiration, and that the concentration is of secondary importance. He suggests that the opposite results of Falck were due to his working with a different race. That such physiologically different strains of Sporodinia do exist is shown by Falck's (l. c., p. 300) discovery of a form which produced zygospores even in dilute solutions, and would seem therefore to be more like the strain used by Klebs.

It is evident that the conclusions above enumerated are diverse and conflicting to a marked degree. While one author demands excess of moisture or of oxygen as a crucial factor, another postulates conditions exactly the reverse. The seeker for zygospores may choose between an abundant food supply or a substratum poor in nutriment. The presence of parasites is held by one writer to produce, by another to inhibit, zygospore formation. As has already been suggested, the lack of agreement among the results of different investigators is, in the writer's opinion, due to the fact that their experimentation has been based on the erroneous assumption that external conditions were the controlling factors in the problem, whereas they are in reality either wholly without influence or of quite secondary importance.

In regard to the significance of the phenomena of conjugation it may be said that although from the time of Ehrenberg ('**20**), the union of gametes has been generally held to represent a primitive sexual process, there have been some who see in this fusion only a vegetative process. Vuillemin ('**87**) at some length maintains that in his *Zygorhynchus heterogamus* the inequality of the gametes, which he homologizes with sporangia, is due to differences of alimentation and is of no sexual significance. This union he compares to the anastomosis between mycelial cells commonly seen in the clamp connections of the Basidiomycetes, and from the absence in the formation of zygospores of characters which are supposed to be essential to a sexual process he casts doubt on the presence of sexuality in the whole group. Brefeld, though opposing the majority of mycologists in denying sexuality to the Ascomycetes, speaks throughout of the formation of zygospores as a sexual process. Van Tieghem even goes so far as to find an indication of sexual differentiation in the size of the gametes in Rhizopus and Pilaira ('**75**, p. 81), and in the fact that the forked spines of the zygospores of Phycomyces ('**73**, p. 295) are produced from one suspensor before they are from the other. Klebs ('**98**), in his paper on Sporodinia, although recognizing sexuality in the formation of zygospores, does not discuss the subject, and investigates the production of zygospores on the same basis as he would that of any other reproductive form of a polymorphic fungus. Falck ('**01**, p. 302) merely mentions in a footnote the word "sexuality," and then says that the process of conjugation can be also explained on the basis of mechanical forces, and that the azygospores can as well be sexual.

In regard to the cytology of the zygospore there is little decisive knowledge. The zygote is at first multinucleate, but, according to Léger ('**96**), with the accumulation of oil in the centre the nuclei entirely disappear, and at opposite poles of the zygospore appear two groups of small granular bodies which are called "Sphères embryogènes." Those in each group fuse together, and the two thick-walled bodies thus formed are the "Sphères embryonnaires" which are to be found in mature zygospores of all the Mucorineae. At germination the "Sphères embryonnaires" increase in size, lose their walls, and fuse with each other. The nuclei which now reappear undergo one mitotic division and pass into the germ tube. The fusion in the formation of the "Sphères embryonnaires" is looked upon as the only sexual part of the process. On this basis the azygospores are to be considered equally sexual, since they differ from the zygospores only in containing a single "Sphère embryonnaire." These observations of Léger have not been confirmed by Istvanffi ('**95**) nor by Gruber ('**01**), who worked carefully on the zygospores of the same form, *Sporodinia grandis*, and Dangeard ('**96**, p. 255), in collaboration with whom Léger began his investigation, disputes his pupil's later results. The writer also, from an investigation as yet incompleted, of sections of the zygospores of a number of different species, is satisfied that they contain no structures corresponding to "Sphères embryonnaires."

Although the exact nuclear history in the process of conjugation is a matter of great interest, a knowledge of it is not necessary in order to establish the sexuality of the act. Nuclear fusion is considered to be the criterion of a sexual process, yet, as is seen from the nuclear fusion in the ascus and basidiospores, one must have recourse in addition to physiological considerations for an interpretation of the sexuality involved. The results set forth in the present paper show beyond question that sexuality is present in at least a very large percentage of the Mucorineae, and cytological investigation can merely extend our knowledge of the details of a process, the true character of which seems obvious without its aid. It will be of especial interest to determine the cytological conditions in connection with the segregation of the sexes occurring in the formation of zygospores or later, but no systematic experiments with the germination of zygospores have been made during the present investigation. The writer hopes, however, in the near future to be able to present more satisfactory data on the subject.

CITATION OF SPECIES FROM WHICH ZYGOSPORES HAVE BEEN REPORTED.

In the preparation of the ensuing references, recourse has been had to Just's Jahresbericht for the years 1876 to 1902 inclusive, and to the Botanische Centralblatt for 1903 and for the present year up to May 1. For some of the later species Saccardo's Sylloge has been trusted to mention the occurrence of zygospores when they have been reported in the original description, but the compilation of Berlese and de Toni ('**88**) in Volume VII is of little value in this respect, and fails to mention even the zygospores of *Rhizopus nigricans*. Although the writer has endeavored to accumulate in the following pages all citations which include references to the occurrence of zygospores among the Mucorineae, the literature cited is, no doubt, not wholly complete. It may, however, be assumed to comprise the great majority of citations of this nature.

An attempt to determine in this literature the thallic conditions of the various species enumerated would be manifestly impossible except in connection with those species in which a homothallic condition was made manifest from the figures or descriptions. A figure showing the progametes arising ultimately from the same branch would, if correct, be decisive. Unfortunately those who have done most work on this group of fungi and from whom we should expect the most accuracy have erred in describing and figuring a detail which at the time was not considered of importance. Thus, of known heterothallic forms, de Bary ('66, Figure 2, Plate VII) figures Rhizopus, and van Tieghem ('73, Figure 4, Plate XX) and Bainier ('82, Figure 15) both figure Phycomyces as homothallic; and if Piptocephalis proves to be heterothallic, as is extremely probable from Brefeld's ('72) account of his cultures, this author has figured a similar inaccuracy. That the zygophores can be traced to more or less distant parts of the mycelial growth, though to a certain extent suggestive, is by no means conclusive evidence, since we know that while in homothallic forms the zygophores generally arise comparatively close together, yet in some forms, like Mucors I and II, the zygophores may arise some little distance apart. In the literature appended, an examination of those species for which a heterothallic condition has now been made out will show that it is unsafe to place too great dependence on the accuracy of the figures and descriptions of morphological characters as published.

But aside from the morphological data, the action of the fungus in cultures may become circumstantial evidence of considerable value. If cultures from a single spore or from a single sporangium always fail to produce zygospores, while in cultures from a mass of sporangia zygospores appear, the evidence even without morphological facts points almost certainly to a segregation of sexual characters on separate mycelia. In most of the cases cited, zygospores were found by chance, and no further cultures were made, so that the evidence from the action in cultures is lacking. It has not infrequently happened, however, that zygospores have been found after a sowing on some unsterilized medium which is a common source of the species investigated, and subsequent attempts to continue the growth of zygospores by transfers of spores has resulted in failure. An accidental mixture of sexual strains is the only available explanation of the failure to obtain zygospores in one culture of a given species when a second culture similar in all respects to the first produces them in abundance. A failure to obtain zygospores by making a gross transfer of sporangia from a zygosporic culture may be due to the fact that one of the strains has short sporangiophores, and thus may escape below the taller growth of the other. To say that all species are heterothallic in which the production of zygospores from spore sowings has not been a constant phenomenon under similar culture conditions, would at present be too sweeping a statement, yet such is undoubtedly the condition in the forms with which the writer is familiar. From actual knowledge and from indirect evidence, the heterothallic condition seems to be the more common among the Mucorineae, and if we are to lay the burden of proof on either side we should be led to consider a species heterothallic until it has been proved otherwise.

Of the 66 forms cited below, the occurrence of zygospores or azygospores in 51 cases has been reported but once, and in the remaining species, a list of which is appended, the number following each form indicates by how many authors the zygospores have been found and reported, — *Mucor Mucedo*, 6; *M. racemosus*, 4; *M. fragilis*, 2; *M. erectus*, 3; *Zygorhynchus Moelleri*, 2; *Phycomyces nitens*, 2; *Spinellus fusiger*, many; *Sporodina grandis*, many; *Rhizopus nigricans*, 5; *Absidia caerulea*, 2; *Pilaira anomala*, 2; *Chaetocladium Brefeldii*, 4; *Piptocephalis Freseniana*, 2; *Syncephalis nodosa*, 3; *Syncephalis Cornu*, 4.

Since the purpose of the present paper is in no sense systematic, no attempt is made to include a special study of the nomenclature and systematic position of the species cited. In so far as possible A. Fischer's ('**92**) admirable and familiar classification of the group has been followed.

Mucor Mucedo Linné.

"Following a sowing of Helicostylum on dung, zygospores appeared in abundance after twelve days in the recesses where the brownish mycelium was protected against access of air and light." This note of van Tieghem's ('**72**, p. 1000) is the first reported occurrence of the zygospores of *M. Mucedo*.

Brefeld ('**72**) states that zygospores never appeared on his slide cultures, though their occurrence was not infrequent in spontaneous cultures of horse dung where he first found them. In speaking of the zygophores he says: "It is fairly certain, however, that they do not arise from two branches of the same hypha as in Sporodinia, since in the youngest condition observed the copulating hyphae could be followed for a considerable distance."

According to Bainier ('83^{b}, p. 342), if in March and April dried horse dung be examined from a beaten road, it is not rare to find zygospores in the interior, and if laboratory cultures of fresh horse dung arranged in a thin layer be sown with this same *M. Mucedo*, one easily obtains a large quantity of zygospores during this period. Zygospores were obtained many times in January on Bainier's alcoholic decoction of prunes. In his Étude, Bainier ('82) speaks of the zygospores occurring in the dung when the spores of the species are abundant in the horse's food.

Léger ('96, p. 59) obtained the zygospores on horse dung when the substratum had begun to dry; Vuillemin ('82^{b}), used zygospores of this form for the study of the zygosporic membrane; and Spegazzini ('91) mentions zygospores in his description of this species from Argentine. The occurrence of zygospores of *M. Mucedo* has not been infrequent on dung cultures in the Harvard Laboratory during the past fifteen years.

The species is heterothallic.

Mucor racemosus Fresenius.

Bainier ('82) found zygospores of this species on moist bread, horse dung, and plaster wet with a solution of glucose, and in a later paper (83^{b}, p. 347) he says: "If *Mucor racemosus* be grown in cell cultures in alcoholic decoction of plums during December, January, and February, one infallibly obtains an exaggerated production of zygospores. . . The air was often renewed in the cultures, hence increase or decrease of oxygen can be only an accessory factor." He further remarks that "Since the plant grew on a liquid and in a nearly saturated atmosphere, dryness cannot be a general cause of zygospore production. In solution of peptone or extract of malt zygospores are rare, while in solution of glucose only chlamydospores and sporangia occur." In the plates accompanying this paper (Plate XVII, Figure 6; Plate XVIII, Figure 9) as many as three zygospores are figured arranged in a scalariform fashion one above the other between two adjacent hyphae. No figures showing a homothallic condition are given, and probably such a condition does not exist. Under this species Bainier includes six forms which differ in branching, size of sporangia, and relation to nutrients. One of these gave zygospores only exceptionally. Being of the opinion that polymorphism may exist in this species Bainier considers these as varieties. It is for this reason, perhaps, that the zygospores which he figures as belonging to *M. racemosus* do not correspond and show at least three distinct types (Bainier, '82, Plate I, Figure 11; '83^{a}, Plate V, Figure

4; **'83**^{b}, Plate XVII, Figure 6; Plate XVIII, Figure 9; **'84**, Plate VIII, Figure 1).

Léger (**'96**, p. 60) also encountered these zygospores in his researches, but never obtained a sufficient number for study, although he followed Bainier's method during the months said by him to be most favorable for the purpose.

Saito (**'04**) mentions this species as common on plate cultures exposed to the air. No description is given, but in the three zygospores figured (Plate II, Figure 7) zygophores are represented as belonging to separate hyphae.

Syzygites ampelinus Hildebrand.

Mucor racemosus sec. Fischer.

From a sowing of his *Mucor Vitis* on black bread Hildebrand (**'67**) obtained zygospores below the sporangial growth. "The origin of the conjugating branches differs from that in Sporodinia, since they never arise in close proximity from the same hypha. They belong without exception to different branch systems."

Fischer (**'92**) places this form under *M. racemosus*, together with *Mucor Vitis*, with which it is probably connected.

Mucor racemosus, var. **brunnea** Morini.

According to Morini (**'96**) the zygospores of this form may be obtained by allowing the nutrient to become gradually exhausted, or by inducing a gradual desiccation of the substratum, though not infrequently zygospores are formed on a substratum rich in nutriment. In all cases the sexual phase supposes full vitality on the part of the fungus, hence the substratum should be conveniently arranged, and in sufficient quantity, so that the mycelium may spread freely. In a suitable temperature, principally from January to April, zygospores form in horse dung, moist bread, and solution of peptone. In horse dung they are most abundant, and form after about ten days. In peptone solution the production of sporangia is abundant, while that of zygospores is scarce. The want of a frequent renewal of air seemed to hinder the development of zygospores, though it favored oidium formation.

Mucor tenuis Bainier.

In this species, described by Bainier (**'83**^{b}, p. 353), no zygospores were found, but in decoction of plums during December, January, February, and March, azygospores were produced on erect filaments.

Mucor spinosus van Tieghem.

Bainier ('**84**, p. 204) obtained the zygospores on alcoholic decoction of pears and plums during the month of August. They are not produced, he says, during the winter. The zygospores are figured (Plate VII, Figure 6), forming in a scalariform fashion between two filaments, one of which bears terminally a columella.

Mucor circinelloides van Tieghem.

Bainier ('**84**) found zygospores of this species forming in the moist lower portion of a horse dung culture which had probably been sterilized according to Bainier's method with carbon bisulphide. They do not form in decoction of malt nor of prune. The single zygospore figured fails to show the hyphal connections.

Mucor erectus Bainier.

According to Bainier ('**84**), cultures in prune juice in winter and spring constantly give very numerous zygospores. Double azygospores are as abundant as zygospores. One of the zygospores is figured (Plate VIII, Figure 6), formed from the base of a sporangiophore.

Schröter ('**86**^{b}, p. 204) reports that the zygospores of this species were found by Eidam, and Fischer ('**92**, p. 197) has apparently also observed them.

Mucor fragilis Bainier.

Bainier ('**84**) states that the zygospores of this species are obtained with greatest ease during the winter and spring in decoction of prunes, and that the culture becomes literally black with them at the end of eight days.

Vuillemin ('**04**^{b}) has used the zygospores of this species in his investigation of the zygosporic membranes.

Mucor mollis Bainier.

Although the zygospores of this species are described and figured by Bainier ('**84**), their method of formation is not determinable.

Mucor tristis Bainier.
Mucor modestus Bainier.

Bainier ('**84**) mentions obtaining zygospores of these two species, but descriptions and figures are lacking in both cases.

Mucor neglectus Vuillemin.

In this species Vuillemin ('87) found no zygospores. His figures, however, represent azygospores formed at the ends of recurved branches.

Thamnidium mucoroides Zukal.

This species was found by Zukal ('90) on a moist culture of alligator dung. Not infrequently four to five zygospores are formed in a scalariform fashion between two filaments. The zygospores are formed in the substratum and in the air, but sporangia are not developed from the same hyphae. "The simultaneous occurrence of sporangia and zygospores shows that the alternation of sexual and nonsexual generations has been practically obliterated, and is only indicated by the fact that from zygospores are developed only sporangia, or a sporangium producing mycelium." This is not a Thamnidium, and A. Fischer ('92) rightly places it among the cymose Mucors.

Mucor rubescens Léger.

The zygospores of this species, which was discovered by Léger ('96, p. 69) on beer wort, are said to occur only in the interior of the substratum.

Mucor geophilus Oudemans.

This form of Oudemans and Koning ('01, p. 13) was obtained from a separation culture of forest humus. The zygospores as figured appear to be merely large chlamydospores.

Mucor I Winkler.

This form, experimented on by Winkler ('02), was cultivated from forest earth. "It is a cymo-mucor, very similar to *Mucor alternans*, except that it forms zygospores abundantly on wort gelatine."

Mucor alpinus Hansen.

Mucor neglectus Hansen (not Vuillemin, nor Bainier).

These two species were obtained by Hansen ('02) from soil, the first from the Alps, the second from the Harz Mountains. His *M. neglectus* differs from *M. alpinus* in that the zygospores are produced two days earlier than the sporangia, and the maximum temperature for their development is higher than for that of the sporangia, while the reverse is true in both respects of *M. alpinus*. Numerous cultures were apparently made in investigating the capacity of these forms to produce yeast cells, and no mention is made of any inconstancy in the formation of zygospores under like culture conditions. Although no morphological description nor figures are given of these forms to which Hansen has given *nomina nuda*, their action in cultures and the fact that they were obtained from the soil which we know is a source of homothallic Mucors, renders a homothallic condition probable.

Mucor flavus Bainier.

This species was found by Bainier ('03, p. 157–9) on decomposing agarics and generally with zygospores. "Upon whatever substance one cultivates this form one easily obtains, towards the end of fall, a large number of zygospores." They were thus obtained on bread, horse dung, agarics, flaxseed flour and in cell cultures on decoction of horse dung and of prunes. "When the plant decides to give zygospores they are produced in great abundance." They form on sporangial filaments, as in *M. racemosus*, and as many as four may be seen one above the other in scalariform fashion.

Mucor communis Bainier. Mucor prolificus Bainier.
Mucor limpidus Bainier. Mucor vulgarus Bainier.

In these four unfigured species of Bainier ('03) the zygospores are formed in a scalariform fashion on sporangiophores. Zygospores were found during summer in alcoholic prune juice in cell cultures which had been kept in a cool place.

Mucor neglectus Bainier (not Vuillemin, nor Hansen).
Mucor vicinus Bainier.

In these two forms azygospores only formed under the conditions given by Bainier ('03) in the species mentioned above.

Zygorhynchus heterogamus Vuillemin ('03a).

MUCOR HETEROGAMUS Vuillemin ('86a).

This species was found by Vuillemin ('87) on a spontaneous bread culture. The unequal gametes are cut off from branches of the same erect filament from which also sporangia may develop.

Zygorhynchus Moelleri Vuillemin.

This species was found by A. Möller ('03) from cultures of the mycorrhiza of pine and oak. In 1884 this species came up in the

Harvard Laboratory on a bread culture which had been set for Rhizopus; in 1889 Professor Thaxter in New Haven found the same species growing from a single spore in a van Tieghem cell culture of sterilized peach decoction into which an inoculation had been made from roots infected with peach yellows, and therefore the form was probably of soil origin, and the material which is at present under cultivation was brought by Professor Thaxter from New Haven, where it had been found by Prof. W. C. Sturgis in a separation culture from earth. The writer has examined preparations of all this material, and also of that mentioned by Coker ('03) as occurring spontaneously on a bread culture in the University of North Carolina, and finds them to agree with Vuillemin's ('03^a) description of *Z. Moelleri*. The species differs from *Z. heterogamus* chiefly in the smaller size of the zygospores, and in the size and shape of the spores. The zygospore formation is essentially the same.

The genus is obviously homothallic.

Circinella umbellata van Tieghem and Le Monnier.

Zygospores were found by Bainier ('03) during a warm period of summer in a culture of fresh liquorice root placed on sphagnum in a cool cellar. They arise on erect filaments upon which sporangia have not been found. Plate VII, Figure 10, of Bainier's paper shows a zygospore the suspensors of which connect with the branches of the same hypha, suggesting the condition in Sporodinia. If the figure is correct in this respect the species belongs to the homothallic group. It will have, however, the distinction of being the only known homothallic form in which the sporangial stage is very common, while the zygospores are extremely rare. The observation needs confirmation. [This is especially true in view of the fact that since writing the above the writer has found that a laboratory culture of this species will form imperfect hybrids with a (+) strain, but is inactive toward a (—) strain. The probability therefore of a heterothallic condition is very great.]

Circinella nigra Bainier.

This species of Bainier ('03) was found by him on horse dung. The zygospores are similar to those of *C. umbellata*. and were found under the same conditions. Plate VII, Figure 9, does not show enough of the hyphae to indicate whether the progametes originate as in *C. umbellata* or not.

Phycomyces nitens Kunze.

Van Tieghem and Le Monnier ('**73**) were the first to report the zygospores of this species. There is no evidence in the text which would decide the thallic condition of this species, but Plate XX, Figure 4, shows zygophores arising from the same hypha, and it is on the strength of this figure that de Bary ('**84**, p. 159) classifies Phycomyces with Sporodinia as having zygospores, the progametes of which arise from hyphae organically closely connected, in distinction from Rhizopus and Piptocephalis. Van Tieghem's first cultures of Phycomyces were from cochineal from a certain lacquer factory. These died during his vacation, and the cochineal from the same factory had become sterile, but later he obtained cultures from horse dung, and again from cochineal. In obtaining the zygospores it is probable that a transfer from the horse dung was sown on cochineal which already contained the spores of another conjugative strain, since no mention is made of sterilizing the cochineal. Cell cultures, presumably from a single spore or from a few spores from a single sporangium, never gave zygospores; nor were gross cultures of fruits successful, probably either because they were sterilized, or if not, because they failed to contain spores of an opposing strain.

Bainier ('**82**) has many times obtained the zygospores upon horse dung mixed with flaxseed flour or soaked with oil. But he adds that often his experiments have not given the expected results. The same author subsequently remarks ('**83**^{b}, p. 343), that if during February and March a layer of fresh horse dung 5 to 6 cm. thick in a rather large crystallizing dish is sown with Phycomyces, one obtains the zygospores in abundance after ten to fifteen days or more, according to the richness of the sowing. He has many times repeated this experiment, and has always succeeded during the months indicated. Phycomyces with its zygospores has been distributed by Bainier in Rumeguère, Fungi Gallici 4645; see note in Rev. Myc., x. p. 188.

Although Phycomyces is not uncommon in laboratory cultures, the zygospores have not been reported from this country. Professor Thaxter found them, however, in 1898 in a culture of rabbit dung from Daytona, Florida, and the writer has obtained them synthetically from the two sexual strains (cf. Plate IV).

The species is typically heterothallic.

Phycomyces microsporus van Tieghem.

A single zygospore of this form was found by van Tieghem ('**75**) on horse dung, and from the sporangium to which it gave rise further cultures were made, but without the subsequent appearance of zygospores. The species, if indeed it is a distinct species, differs from *P. nitens* only in the smaller size throughout. It is possible that this is but the less luxuriant sexual strain to which the germination of the zygospore had given rise.

Phycomyces Pirottianus Morini.

This species was found by Morini ('96) on horse dung in Sicily. The zygospores were rarely obtained, and form only on the surface of the substratum. The progametes are derived from branches which develop each from a separate mycelial hypha.

Spinellus fusiger (Link) van Tieghem.

The brothers Tulasne ('66) were the first to report the zygospores of this species, having found them on *Collybia fusipes.*

Van Tieghem ('75), who also observed them, was able to grow the species only on certain agarics. "The two arched branches which form the zygospores," he says, "come sometimes from the same branch."

Bainier ('82) found the zygospores on the gills of *Collybia fusipes*, and in the laboratory was able to obtain them from cultures on the same host. The zygophores are said to arise sometimes from the same hypha, and the figures distinctly show this condition.

This species appears to be not rare in Europe, and zygospores have been found in connection with the sporangia by all except the earlier investigators of the group. It is not necessary to cite the entire literature on the subject. Writers agree in the constant occurrence of zygospores with the sporangia, and their descriptions and figures point to a homothallic condition. It is of interest to note that the related species *Spinellus macrocarpus*, which in this country has been found by Dr. Farlow on *Mycena Sp.* in Vermont, and by Professor Thaxter on the same host at Kittery Point, Maine, has been kept running for some time in the laboratory, and has never been observed to produce zygospores. The form may well be heterothallic, although the hybridization test has not been applied.

In this country *S. fusiger* has been reported but twice, — Hark. and Moore, Cat. Pac. Fung. 31: 1880, and M. A. Curtis, Bot. N. Car. 153: 1867 (collected by Ravenel). Dr. Farlow has in his herbarium the fungus collected by H. S. Horseford from Charlotte, Vermont, 1886, and collected by himself from the following localities: Shelburne, N. H., September, '97; Campobello, N. B., September, '98; Campobello, N. B., July, '02; Keene Valley, Adirondacks, N. Y., September, '02. Professor Thaxter has the species collected by him from Burbank, E. Tennessee, 1887, and Mr. A. H. Moore collected it on Mount Washington, September, '02.

The writer has examined the material from all the localities mentioned, except that of Harkness and Moore, and is able to substantiate the evidence of the thallic condition shown in the literature. In all the cases the zygospores are abundant between the gills of the host, and the progametes arise at times from branches of the same hypha.

The species is thus homothallic.

Spinellus sphaerosporus van Tieghem.

This form, which has been observed only by van Tieghem ('75, p. 75), is similar to *S. fusiger* in the formation of its zygospores.

Spinellus chalybeus (Dozy and Molkenboer) Vuill.

Vuillemin ('04) separates this species from *S. fusiger*, with which it had been united by Fischer ('92). A tendency toward heterogamy is described, which is indicated by the fact that although the progametes are equal in size, one is of more delicate texture, and after the zygospore is formed, fails to develop a swollen suspensor as does the other. Neither the description nor the figures give us positive evidence as to the thallic condition. None of the five zygospores figured, however, shows the zygophores originating from the same hypha, as is commonly the case in *S. fusiger*.

Spinellus gigasporus Cooke and Massee.

This was found by Cooke and Massee ('89) on decaying agarics from Victoria. The zygospores are borne on slender flexuous zygophoric filaments. The meagre description tells nothing of the thallic condition. Since, however, it is placed in this genus we may assume that the species in question differs little in this respect from *S. fusiger*.

Sporodinia grandis Link.

This is our most common homothallic species, and has been found by all investigators of the group. The descriptions or figures of Ehrenberg (['20] '29), Corda ('39), Bonorden ('51), de Bary ('64), van Tieghem ('75), Bainier ('82), Léger ('96), Klebs ('98), Falck ('01), and others agree in making the progametes commonly originate from branches of the same erect filament. Here, as in Spinellus, the connection of the uspensors is not obscure, and in consequence the figures of any author, even were they not confirmed by so many others, would be more trustworthy than in most cases. From a few experiments on this form van Tieghem ('75) thought that he had established the principle that lack of oxygen is the controlling factor in the production of the zygospores. The papers of Klebs and Falck have already been referred to (p. 218). The species is very common in this country, and cultures have been kept running in the laboratory for several years.

The species is homothallic.

Rhizopus nigricans Ehrenberg.

In de Bary's ('66) paper on *Rhizopus nigricans* is the first mention of the zygospores of a heterothallic form. They were discovered in this species by A. Janowitsch in the Botanical Laboratory at Freiburg. De Bary found zygospore formation occurring during warm weather in May, June, and July, in cultures on fleshy fruits and on bread. On other substrata and in winter only sporangia were observed. In Plate VII, Figure 2, we have the apparently impossible case represented of two conjugating zygophores arising from the same hypha.

Van Tieghem ('75) obtained the zygospores by employing de Bary's suffocation theory. A cylindrical vase, previously washed with boiling water, is filled one-half or two-thirds full with fresh bread crumbs. Upon this are scattered a few drops of boiled water into which *a* sporangium ("un sporange") has been transferred, and the vase is closed. After a dozen days, when the mycelium is sufficiently suffocated by the first vegetation, zygospores appear in the interior of the bread, and especially between the bread and the walls. Our present knowledge of this species indicates that spores from more than a single sporangium must have gained access to the culture, since the form is typically heterothallic.

Eidam ('83) found zygospores of this species in winter on "Erdnusskuchen."

Léger ('96, p. 74) tried van Tieghem's method for obtaining zygospores, but found only a small number in his cultures.

It is of interest to know that de Wevre ('92), in working on this species, attempted to get the zygospores by following the method given by de Bary and van Tieghem, and in addition subjected the fungus to all the unfavorable conditions he could devise, but without obtaining other than sporangial spores. He concludes that either the form with which he worked had lost its sexual capacity or else was a different variety from that which produces zygospores.

The zygospores of this, one of the commonest of our moulds, though infrequently reported, can hardly be rare. They are generally covered by the sporangial growth, and to one unfamiliar with their appearance are distinguishable only with the aid of a microscope. In the fall of 1892 Professor Thaxter found zygospores on a spontaneous bread culture which had been started for the class in Cryptogamic Botany. By transferring from the zygosporic region, the production of zygospores was increased, and the young sporangia appeared unusual in that they showed a distinct orange-yellow color. This fact suggested that this was a special race or strain, and that zygospore formation might be connected with such strains in which for some reason the sexual activity had become apparent. The material was preserved dried on bread from year to year, and could be depended on to produce zygospores when pieces of the old culture were transferred to fresh bread, until the spring of 1901, when the sexual activity appeared to have run out. As the "Harvard strain" this was carried to Barnard College by Prof. H. M. Richards, and to the University of Chicago by Prof. B. M. Davis, and has been used for class work by several laboratories in the country. Professor Richards has told the writer that his scion of the strain ran out at about the same time with the material in the Harvard Laboratory, and the same is true of the material taken to Chicago.

Coker ('03) reports finding zygospores of this species, and has kindly sent the writer material which came originally from a spontaneous bread culture in the University of North Carolina, Chapel Hill, N. Car.

Prof. D. H. Campbell has kindly sent the writer zygospore material which was part of a spontaneous growth on bread exposed in his laboratory at Leland Stanford University, Cal. He writes that he almost always gets zygospores in this way. They were first obtained in 1892 from a spontaneous growth on a piece of squash, and every year since he has had them in abundance.

Zygospores were found in sporangial material from a spontaneous infection of sweet potato kindly sent the writer this January by Prof. F. L. Stevens, of Raleigh, N. Car.

Mr. L. A. Scott, of Cambridge, called the writer's attention to a growth of mould on a mixture of miscellaneous vegetables which had been left together for some days in a jar at the Harvard Botanic Gardens. An examination showed that Rhizopus was the principal fungus, and upon a soft spot in an Irish potato a mass of zygospores was found. This potato was otherwise unaffected, and zygospores could not be found elsewhere among the growth.

After the running out of the original Harvard strain, isolated cases of zygospores were occasionally found in cultures of starchy material in the laboratory, and cultures on paste of the shells of almond nuts have always produced them in abundance.

The species is heterothallic.

Rhizopus necans Massee.

This form is described by Massee ('97) as causing disease of lily bulbs brought from Japan. "Several large spiny zygospores were found in the matted mycelium present in bulbs in the last stage of decay, and presumably belong to our fungus." The single zygospore figured shows no mycelium connections except the halves of the suspensors nearest the zygospore.

Rhizopus Artocarpi Raciborski.

Raciborski ('00) found this species in Java infecting the male inflorescence of breadfruit (*Artocarpus incisa*). The zygospores did not occur on the host plant, but appeared on an agar culture a week old which was contaminated with a Hyphomycete. No figures are given, and it is not impossible that in the laboratory culture where zygospores appeared, their formation was due to an accidental infection with a complementary strain.

Absidia capillata van Tieghem.

The zygospores of this species of van Tieghem were found in the interior of the horse dung and on the lower face of the substratum in contact with the bottom of the plate containing it. The description and figures (Figures 32 and 35) represent zygophores originating from the same branch in close proximity to the future zygospore, and there is no mention of cultures in which zygospores were not found. Such a condition would force us to consider the species homothallic.

Absidia septata van Tieghem.

Cornu found zygospores of this form which van Tieghem ('77, p. 360) refers to *A. septata*, on the lower side of a cork hermetically sealing a flask in which grapevine roots were preserved. "In both cases (*A. capillata* and *A. septata*)," says van Tieghem, "zygospores were produced in an atmosphere poor in oxygen when the vegetation had become sufficiently checked and the formation of sporangiophores completely suppressed by this poverty of oxygen." The description of *A. capillata* as regards the origin of the zygophores is made to apply to *A. septata*, and Figures 42–45 and 47 would place this species also in the homothallic group. We know with certainty that one species of Absidia (*A. caerulea*) is heterothallic, but this is not a necessary indication that van Tieghem's description of the two species just reviewed is inaccurate, for we have a similar condition in the genus Mucor, where, although the species are predominantly heterothallic, yet homothallic forms are known to exist. It is unfortunate that these two species have not been found by other investigators.

Absidia caerulea Bainier.

MUCOR SACCARDOI Oudemans.
PROABSIDIA SACCARDOI (Oud.) Vuillemin.

According to Bainier ('89) the zygospores are obtained during the warm season, whenever the substance upon which the fungus develops is laid on a more or less thick layer of a moist absorbent material. Thus "zygospores are always obtained in abundance" when the Absidia is cultivated on bread which is laid on sphagnum, "excelsior," carrot slices, or wood bark.

Oudemans and Koning ('01, p. 13) found this species in February on a separation culture of forest humus made on nutrient gelatine. Zygospores and azygospores are figured. Professor Oudemans has pronounced a specimen of the writer's zygosporic Absidia sent him to be identical with his *Mucor Saccardoi*, and M. Bainier has likewise identified the same fungus with his *Absidia caerulea*. He writes that it is common in the environs of Paris on substances that decay in the woods, and that it is remarkable for the ease with which the zygospores may be obtained. The writer has often found the species on decaying agarics, chestnut burrs, acorns, etc., and together with its zygospores on a culture of rabbit dung. So far as is known it was first found in the United States by Professor Thaxter at New Haven, in 1889, in mixed cultures, and has been several times found by him from various localities in New England, where it appears to be common.

The species is heterothallic.

Pseudo-Absidia vulgaris Bainier.

According to Bainier ('03), "zygospores are sometimes found in horse dung cultures in the lower part of the substratum." The zygospore figured gives no information in regard to the suspensor connections.

Absidia scabra Cocconi.

This form was found on horse dung by Cocconi ('00). When the nutriment in the dung becomes impoverished by the asexual generations, the sexual generation occurs. In the formation of a zygospore "two superficial branches, each of which belongs to a distinct mycelial-hypha, direct themselves toward each other till they meet."

Professor Thaxter has found zygospores of an undescribed species of Absidia on a culture of refuse from a termite's nest from West Africa. In tube cultures made at the time from sporangia, no zygospores developed. In preparations examined of this material the zygophores remain separate as far as they can be followed.

Mycocladus verticillatus Beauverie.

This fungus was found by Beauverie ('00) growing in abundance on a damp wall with many other moulds. The zygospores were found in a culture of the sterile form of *Botrytis cinerea* where the two species developed together with intrication of their filaments. The culture was five months old, and the zygospores were extremely numerous. Beauverie suggests that their abundance may have been because of the support which filaments of the Botrytis furnished. Nothing is said of the character of the substratum except that it had become exhausted by a luxuriant vegetation and had almost completely dried. The single figure of a zygospore has the suspensors incompletely drawn. In the meagre description given us, we have little clue to the thallic condition, except the fact that the zygospores were obtained apparently in but this single culture, although the fungus was subjected to varying conditions in Beauverie's physiological investigations. If this zygosporic culture were the first culture made — a not improbable supposition from its age and admixture with Botrytis — it can easily be seen, upon an assumption of a heterothallic condition, why zygospores should occur here and not later.

Thamnidium elegans Link.

During May and June Bainier ('84) obtained the zygospores of this well-known species in pure cultures in great abundance. They are borne upon erect filaments, which also produce the dichotomies, and a scalariform series is figured (Plate X, Figure 7) between two separate hyphae which are each terminated by a columella.

Dicranophora fulva Schröter.

Schröter ('86) is the only one who has found or described this species which is reported growing on *Paxillus involutus*. The zygospores are formed on the mycelium, which develops abundantly between the lamellae of the host and arise from the copulation of two erect unlike branches. One of them is very thick and sack-like, arising from a mycelial hypha by a narrow stalk, and its upper third is distinguished by a cross wall for the formation of the zygospore. The second branch is hardly thicker than the hypha from which it originates, and from it a short cylindrical piece is cut off. "The unlike character of the copulating branches, especially in size and thickness, points distinctly to a sexual difference in the branches and suggests an antheridium and an oogonium." Figure 113 of Schröter ('97), in Engler and Prantl, leaves no doubt as to the heterogamic character of this species, but the figure is not sufficient to show the connections of the suspensors. The closely related American form subsequently referred to, however, is homothallic.

Choanephora Cunninghamiana Currey.

The species was found by Cunningham ('80) on the flowers of Hibiscus in India. The following statements occur in connection with the description of the formation of zygospores. "This happens comparatively rarely, and I have as yet been unable to determine what the precise conditions are under which it occurs. . . . In all cases in which I have been able to determine the point accurately, the opposed organs have been derived from two distinct mycelial filaments. In many cases, however, the relations of the processes and filaments are very much obscured . . . and in some the appearances seem to indicate that contact occasionally occurred between processes arising from the same filament."

The evidence thus indicates a heterothallic condition.

Choanephora Simsoni Cunningham.

The species was found by Cunningham ('95) on *Ipomoea rubro-caerulea*. In both this and *C. Cunninghamiana* the nutrition was found to regulate the fructification, zygospores being obtained when the nutriment was below the normal. "There is nothing to indicate what is the essential determinant of the appearance of zygospore fructification, for in two cultures side by side from the same specimen of nutrient material zygospores are absent in the one and almost entirely replace the sporangia in the other. . . . In some instances the conjugating processes take origin from the same mycelial filament, in others from distant ones." Three out of the seven figures of this species are distinctly homothallic in character, and thus oppose the cultural evidence which alone would strongly suggest a heterothallic condition.

Pilaira anomala (Cesati) Schröter.

Van Tieghem ('75) found the zygospores only in two cell cultures, in one of which three spores were sown and in the other five. According to the account given the zygospores form from "two flexuous branches arising from the same lateral branch or from different lateral branches which cross." The zygospores occurred in a cell under favorable conditions for observation, and van Tieghem's description of the origin of the zygophores would make the species homothallic. The fact that in all other cell and gross cultures he never obtained zygospores, added to the fact that where they were found more than one spore was sown, renders the thallic condition a matter of uncertainty.

Brefeld ('81) found zygospores only once, and on horse dung, where the sporangia were luxuriantly developed. Not more than about fifty zygospores were found, and in cultures from the sporangial spores there were no more zygospores produced. The same was true of sporangial sowings from germinated zygospores. The figures are negative as regards the origin of the zygophores with the possible exception of Figure 27, which seems to make the suspensors connect with the same hypha.

Pilobolus crystallinus (Wiggers) Tode.

Zopf ('88) attempted to induce the formation of zygospores by varying the substratum, and used for this purpose both solid and liquid media, but obtained negative results. Sterilized ("ausgekochter") horse dung was placed in a crystallizing dish and inoculated with pure spore material. The Pilobolus at first developed normally, but later was attacked by *Pleotrachelus fulgens* and an undetermined Syncephalis. Zygospores here appeared in abundance, and Zopf concludes that they were caused by the suppression of the formation of sporangia resulting from the attacks of these parasites. In proof of this, the following experiments were made. The spores from sporangia caught pure were sown on sterilized horse dung and gave rise to sporangia alone. Another similar culture was infected with parasites by strewing on it fragments of dung from the original diseased culture. Sporangial formation was checked and zygospores appeared in abundance. It is stated (p. 356): "In Plate

6, Figure 9, the suspensors arise immediately near each other from the same mycelial branch; in other cases they are the ends of longer or shorter hyphae whose insertions lie at a more or less considerable distance from each other and belong to altogether different systems of branching."

Although parasites caused the zygospore formation in this special instance, Zopf thinks it must be possible to cause them by artificial means without the parasites. He expresses his desire to discover the special culture conditions necessary, but as nothing has been published on the subject since, it may be assumed his investigations were fruitless.

Were it not for Figure 9, we should consider the evidence of a heterothallic condition very strong. The other twelve figures of zygospores, however, illustrate rather the condition mentioned in which the progametes belong to different systems. From the presence of Pleotrachelus and Syncephalis in Zopf's zygosporic culture it is obvious that he did not have a pure culture of Pilobolus, and it is conceivable that an opposing strain entered by the same road with the two parasites. His experiments prove nothing, since in the second culture he did not introduce the parasites pure, but on fragments of the very dung that was producing zygospores. If the form is heterothallic he transferred with the parasites the mycelium of the two opposing strains. The account is strongly suggestive of the writer's early work with Mucor III. (p. 286), the zygospores of which first appeared in a tube culture infected with bacteria.

Pilobolus Kleinii van Tieghem.

In one of Zopf's ('92) cultures of *P. Kleinii* on sheep dung the formation of sporangia suddenly ceased, and all the sporangial primordia were found converted into galls infected by the parasitic organism *Pleotrachelus fulgens.* Since in dung cultures of the same Pilobolus where the parasite was absent zygospores were not produced, Zopf concludes that as in *P. crystallinus* the suppression of sporangia by Pleotrachelus is the cause of the zygospore formation. The figures are indecisive as regards the origin of progametes. That the cause assigned by Zopf is not the true explanation is evinced by the fact that Professor Thaxter has found the zygospores of this species on sheep dung but without the parasite. In preparations of this material which the writer has examined, the hyphae in connection with the suspensors remain separate as far as it is possible to follow them.

Pilobolus nanus van Tieghem.

The thick-walled tuberculate "spores durables" which van Tieghem ('**76**, p. 341, Plate X, Figure 22) describes as developing terminally on short curved stalks from the mycelium, are considered by Fischer ('**92**, p. 268) to be azygospores.

Mortierella Rostafinskii Brefeld.

According to Brefeld ('**81**) cultures of this species may be run through a continuous series, perhaps to ten or twelve generations, with exclusively non-sexual reproduction, when zygospores largely take the place of sporangia. In further cultures made with the mycelium or with sporangial spores, he obtained zygospores continuously and in great abundance. There is nothing said which would indicate whether these sporangial transfers were made from a single sporangium or from several, in which latter case sexual strains, if such existed, might have been mixed. From the intricate network of hyphae that surrounds the young zygospore, it was not possible to determine whether the conjugating branches were formed from the same or from two different stolons. The fact that zygospores would not form on slide cultures while they were found on the free walls of his culture dishes is highly suggestive of a heterothallic condition when we remember that it was Brefeld's practice (l. c., p. 11 and 12) in making slide cultures, to use spores from a single sporangium. The zygospores themselves, when placed in a moist atmosphere, did not germinate, but the enveloping hyphae gave rise to sporangia or to new zygospores. The latter condition is not inconsistent with a supposition that the enveloping as well as the conjugating hyphae are from different mycelia.

Mortierella nigrescens van Tieghem.

Van Tieghem ('**76**) found this species several times in October, 1875, on different agarics, boleti, and on Lycoperdon, and cultivated it on *Agaricus campestris* and truffles. In June, 1876, the fungus was again found and cultivated on *Agaricus campestris* where the zygospores were obtained. "The small tubercle composed at maturity of an envelope of many thicknesses of crowded branches enclosing the zygospore is inserted by a short pedicel upon a filament of the mycelium and sometimes at the point of anastomosis of several different filaments." The morphological observations and figures apparently indicate that the progametes arise in a tuft from a single hypha. It seems strange that if this is a homothallic species, van Tieghem should have been unable to get the zygospores on *Agaricus campestris* from his several findings of the fungus in the autumn, when he succeeded in obtaining them in the spring on the same substratum.

Chaetocladium Jonesii Fresenius.

According to Brefeld ('81) the zygospores appear in serial cultures after a short period of cultivation. Neither the description nor the figures indicate the hyphal connections of the suspensors.

Chaetocladium Brefeldii van Tieghem and Le Monnier.

Only once from among a large number of spontaneous and artificial cultures which Brefeld ('72) carried on continuously for a long time did the zygospores of this fungus appear, and then on a spontaneous horse dung culture. The Mucor was scant, but the Chaetocladium was very luxuriant, producing side by side conidiophores and yellow zygospores, the genetic connections between which could be easily made out. Brefeld distinctly states that he was unable to discover zygophoric branches arising near each other from a single hypha, although they were found adjacent in great numbers and in continuity with conidiophores. Though attempts were often repeated, zygospores were never obtained from the sowings of spores produced from germinating zygospores.

Pale and dark zygospores of this species were also obtained by Bainier ('84), the color corresponding in either case to temperature differences in the cultures. Nothing is said about the conditions accompanying zygospore formation, and the figures add no information on this point. It is possible, as A. Fischer ('92, p. 287) suggests, that Bainier may have had the zygospores of two species mixed in his cultures.

Schroeter ('86) reports that this species is rather frequently found in Breslau with zygospores, and Léger ('96) also mentions their occurrence.

In 1902 the writer found zygospores of this species on horse dung sent from Berlin. They had developed before reaching the laboratory, and it was then impossible to induce further formation. In the spring of 1903 Professor Thaxter obtained zygospores from sowings on sterilized horse dung. The Chaetocladium came from a tube culture, and the Mucor host was taken from a gross culture. This does not render impossible a heterothallic condition, for a complementary strain may have been introduced with the host, and even the tube culture may have contained two strains which were prevented from forming zygospores by unfavorable nutrient conditions.

Syzygites echinocarpus Hildebrand.

This form was found by Hildebrand ('**67**) on an open moist black bread culture of Arthrobotrys and another mucor. The zygophoric branches arise either both from a lateral branch which dichotomizes near its origin, forming two branchlets which bend toward each other to conjugate; or secondly, the two zygophoric branches arise close together directly from a hypha; or thirdly, they arise from different threads of the mycelium. The first and the third conditions seldom occur. Of twelve figures of zygospores, five show an undoubted homothallic condition. Hildebrand has here and in his *S. ampelinus* made a special study of the origin of the zygophoric branches, and if his account is correct the species is homothallic.

De Bary ('**84**, p. 163) thinks this may belong to Chaetocladium, and A. Fischer ('**92**, p. 287) places it under *Chaetocladium Brefeldii.* The figures of these zygospores, however, differ from those of Brefeld, and the thallic condition which is here described has been determined in neither *C. Brefeldii* nor *C. Jonesii.* It would seem unwise, therefore, to connect this form with any known species in the absence of further information concerning it.

Piptocephalis Freseniana de Bary.

The zygospores were discovered by Brefeld ('**72**) while investigating the parasitic relations of this plant. He does not state whether they were first found in slide or in gross cultures. "It is highly probable that the parasite was incited to sexual reproduction through the especially luxuriant and undisturbed nourishment on the host. The more abundantly it occurred the more in the background did the nonsexual form remain, and this was especially the case if the development received a normal and not too strong impulse through a moderate temperature." Brefeld figures (Plate VI, Figure 19 i) a single spore giving rise to an intricate mycelium which bears both zygospores and conidiophores. The zygophores of one of his zygospores, moreover, can be distinctly traced to the same branch. If this is a correct representation, therefore, the zygospores in *Piptocephalis Freseniana* originate from branches of the same mycelium. Since the importance of this point was not recognized at the time, we can hardly expect in all of Brefeld's elaborate drawings an absolute accuracy as to the finest ramifications, especially in view of the fact that he himself tells us that the condition was much more complex than represented and that in general the mycelial hyphae are so narrow (1–2μ diam.) that connections can be made out with great difficulty and only by use of stains. In speaking of the zygophoric hyphae he remarks: "In spite of the intricate twisting of the hyphae they can in every case be followed in different directions and belong to different threads of the mycelium." This fact, added to Brefeld's inconstant results with sporangial sowings ('**81**, p. 75), renders a heterothallic condition probable.

Spegazzini ('**91**) mentions zygospores in his redescription of this species under the name of *P. arrhiza.*

Pitocephalis Tieghemiana Matruchot.

Matruchot ('**99**) found this form growing on *Rhizopus nigricans* which had developed upon germinating seeds of pea, bean, etc. This is its only reported occurrence, and neither the description nor the figures give us any evidence as to the thallic condition.

In 1896 Professor Thaxter found a Piptocephalis with zygospores growing in a bread culture from Mammoth Cave, Ky. In the position of suspensor attachments the form resembles *P. Freseniana*, but the zygospores are quite distinct from those figured by Brefeld. Another form found by him in the same year on a Mucor-infected Myxomycete from Kittery Point, Me., has zygospores whose suspensors are attached at considerable distance from each other, and thus resemble those of *P. Tieghemiana*, but the species differs somewhat from Matruchot's description. The writer has examined preparations of these two forms and finds the hyphae connected with the suspensors to remain separate as far as they can be followed.

Syncephalis nodosa van Tieghem.

In Figures 17 and 19, Bainier ('**82**) shows the conjugative branches arising from the same hypha, and the secondary zygospores are said to be formed from zygophoric branches arising from the same base.

Thaxter ('**97**) reports the species as being very common in this country, and remarks that one seldom fails to obtain the zygospores in abundance whenever it grows on a copious substratum of other mucors. The figures are negative as regards the origin of the zygophoric hyphae.

Vuillemin (**86**^{b}) used the zygospores of this species in his study of the zygosporic membranes. The writer has several times found this species with zygospores, and it has frequently been found in the laboratory with the same form of reproduction since 1887. He has been unable to confirm the statement of Bainier, and so far as the zygosporic hyphae can be traced they remain separate. The difficulties in cultivating this obligate parasite has deferred the determination of its thallic character. The very common occurrence of zygospores would seem perhaps to indicate a homothallic condition.

Syncephalis Cornu van Tieghem and Le Monnier.

Van Tieghem ('75) found the zygospores of this species which had infected a gross mixed culture of *Mucor plasmaticus* and *Pilaira anomala*. "In formation of a zygospore a slender mycelial branch swells at its summit and dichotomizes irregularly much as when it prepares to produce a sporangial tube. While the other branches of this 'palmure' remain short and form expansions like fingers of a glove, two of them grow more in length, place themselves parallel to each other for a very short distance, and generally in contact in their lower part." The text and Figure 88 indicate a homothallic condition. However in Figures 89 to 92, although the zygophores are not traced to their origins, enough is given to show that they arise at least at some distance from the "palmure" mentioned above.

Bainier ('82) describes the zygospores of this species under the name of *S. curvata*. "Zygospores form in the same fashion as in *S. Cornu*, and the description of van Tieghem may apply here." The zygophoric branches are made to arise from the same base or from two distinct filaments of the mycelium, and these two conditions are shown in Figures 9 and 10 respectively.

Léger ('96) reports zygospores, but adds nothing of interest to van Tieghem's account.

Thaxter ('97) describes the mature zygospores which were found on mouse dung, but the origin of the zygophoric hyphae is not shown by the figure.

In February of the present year the writer found abundant zygospores which correspond to figures of this species in a rat dung culture where *S. cordata* and *S. depressa* had also developed their sporangial fructifications. So far as the zygophoric branches were traced they remained separate.

Syncephalis reflexa van Tieghem.

Thaxter ('97) found zygospores of this species on a culture of mouse dung. The figures do not show the origin of the zygophoric hyphae.

Dispira Americana Thaxter.

Professor Thaxter informs me that the zygospore-like bodies found by him in connection with his cultures of Dispira ('95) were probably accidentally associated with it, and that they are doubtless referable to the genus Parasitella recently published by Bainier ('03).

Massartia Javanica Wildeman.

This fungus was found by Wildeman ('97) in the mucus about terrestial algae which had been collected in Java from the bark of a tree and sent him as preserved material. Certain globose double stalked cells are classed as zygospores, and stages of development are mentioned and figures are given, but no other form of reproduction was discovered. These cells, if zygospores, are more suggestive of Piptocephalis or Syncephalis than of any other known forms among the Mucorineae, but the foundation of a new genus on such imperfect data seems undesirable. Although in Saccardo's Sylloge (vol. xiv, p. 437) this is put under the Chytridiaceae, it can best be left among doubtful genera, where Wildeman himself places it. The hyphae connected with the cells in question are separate, as far as shown.

PART II.

In Part I the investigations and theories of previous authors on the subject of the present paper have been summarized, and the results of the writer's own researches have been briefly outlined. It now remains to present in Part II the body of detailed experiments which form the basis of the conclusions already mentioned. It will be convenient to treat the individual forms examined under the thallic groups to which they belong, to which will be added a consideration of the phenomena of hybridization.

HETEROTHALLIC FORMS.

The Mucorineae, as has already been stated in the Introduction, may be divided into two main groups. In the homothallic forms the mycelia are bisexual and the zygospores produced are consequently formed through the interaction of branches of the same mycelium. In the heterothallic group, however, to be considered in this section, the mycelia are unisexual and the zygospores produced are formed therefore through the interaction of branches of two mycelia which are necessarily different in character. Every species which possesses this type of zygosporic reproduction is capable of being separated into two opposite strains, which may be cultivated separately to an indefinite number of generations without the formation of zygospores, but which will, when allowed to grow in contact on proper nutrients, produce them by means of gametes derived from their respective mycelia. If these strains are sowed side by side, a distinct line of zygospores may be produced when the respective mycelia come in contact. On account of its enormous jet black zygospores thickly beset with forked spines, and the luxurance with which they are produced in a broad line many layers thick, when the two opposite strains of the species are properly contrasted, Phycomyces offers the most striking example of zygospore formation in a heterothallic species.

While in some forms, in addition to the inherent difference between the strains which becomes apparent by the formation of zygospores when they are allowed to come in contact, there is a more or less marked differentiation in vegetative luxuriance, in others the differentiation is less marked. To distinguish the two strains thus differentiated in the heterothallic group, it has been found convenient to use the signs (+) and (—). As has already been mentioned in the Introduction, hybridization will occur only between those strains which have unlike signs, and these designations, it may be mentioned, were first used in distinguishing strains where a vegetative differentiation was apparent. For this reason it has become evident, notwithstanding certain theories to the contrary, that the process of conjugation is sexual in character, and that the (+) and (—) strains represent the two sexes repectively. In several species certain strains have been found which, as far as they have been tested, fail to respond to (+) and (—) strains, the character of which had already been determined in these species, and for this reason they have been called " neutral."

Rhizopus nigricans.

This heterothallic species is the most widely distributed form among the Mucorineae, and shares with Penicillium the doubtful distinction of being the most common fungus weed in laboratory cultures. As has been already mentioned (p. 233), the " Harvard strain," supposed to be a race in which the faculty of zygospore formation was specially developed, has been in use in the principal botanical laboratories in this country, and that the peculiarity of its zygosporic superiority to ordinary Rhizopus has been the subject of some investigation goes without saying. That heretofore, however, the question has baffled all inquirers is due to the difficulties inherent in the manipulation of this species, and it was not until some time after the real thallic condition was strongly suspected that the writer was able to separate out the two sexual strains.

Preliminary Tests of "Zygosporic Strains."

After the running out of the original Harvard strain the first appearance of more than isolated cases of zygospores in the laboratory was in a paste culture of nut shells which will be called culture "A" to distinguish it from later zygosporic cultures. The zygospores were rather abundant, mixed with other moulds and with bacteria, but by transferring a mass of the zygospores to sterilized bread, and from this again to a third culture, it was possible to obtain eventually a culture containing only Rhizopus. On the supposition that a new zygosporic strain had appeared, pure transfers were made from separate sporangia, but only sporangial growth resulted. As many as thirty pure transfers were made on such nutrients as experience had shown to be productive of zygospores whenever a mass of the zygospores themselves were used for the inoculation, and in only one case — a bread culture — did zygospores appear. It seemed therefore probable that the ordinary Rhizopus, which was not an uncommon weed in the laboratory, had become mixed with the culture, and perhaps from its greater luxuriance had crowded out the zygosporic strain.

To avoid the contamination of these "weed" spores, small masses of the young zygospores with the mycelium adherent were washed and teased out in sterilized water till a microscopic examination showed that they were free from spores, and a few of these zygosporic hyphae were then used in the inoculation of a number of stender dish cultures. In one of these, zygospores were almost exclusively present in the middle of the dish with sporangia only toward the sides, and while the growth was still young the process was repeated, but the cultures were covered with caps of filter paper to insure increased moisture and placed in a chamber kept damp by a bottom layer of wet filter paper. All produced a great abundance of zygospores, which grew up into the paper caps, and before the sporangia had matured, zygospores with attached hyphae were washed, and having been found free from spores were transferred to van Tieghem cell cultures. Although they were finally infected with bacteria and no new zygospores formed, a miscroscopic examination showed that the few small sporangia which developed came from the hyphae that had been transferred and not from the spores. It seemed thus unquestionable that the sporangia in these last cultures must be produced from the same hyphae that were connected with the zygospores, yet of five sporangial transfers from these only one gave rise to zygospores, although the conditions of nutrient, temperature, and moisture, were identical. This tube, moreover, furnished no clue to a solution of the problem, since ten pure transfers from it to a variety of favorable substrata failed to produce anything but sporangia. On the original assumption that zygospore formation was characteristic of certain strains this fact seemed to indicate that the sexual character was not distributed throughout the mycelium, but was handed on through the spores of only a part of the sporangia. All attempts, however, to locate these favored sporangia by taking transfers from the lower part of the cultures near the zygospores were as fruitless as were transfers from sporangia higher up. Yet whenever a mass of the sporangia from a zygosporic culture even without the zygospores was transferred, zygospores invariably developed.

TABLE I.

Substratum.	Results.
Egg (yolk)	sporangia; zygospores scattered.
Cocoanut	sporangia; zygospores abundant.
Potato	infected with bacteria.
Apple	sporangia scanty; one zygospore.
Banana	sporangia; no zygospores.
Orange	sporangia; no zygospores.
Horse dung	sporangia; zygospores scattered.
Milk in sponge	sporangia; zygospores abundant.
Urine in sponge	no growth.
Cocoanut milk in sponge	sporangia scanty; zygospores scattered.
Milk in test-tube	sporangial growth 1.5 cm. high; no zygospores.
Urine in test-tube	no growth.
Cocoanut milk in test-tube	sporangial growth 2.5 cm.; zygospores abundant.
Decoction of banana in test-tube	sporangial growth 0.5 cm.; no zygospores.
Decoction of prune in test-tube	sporangial growth 0.5 cm.; no zygospores.

From Petri dish separation cultures which were made to determine whether zygospores could develop from a single spore, separate mycelial "colonies" were transferred to suitable nutrients in tube cultures and in all cases only sporangia resulted, although scattered zygospores occurred in some of the Petri dishes after the various colonies had become indiscriminately mingled. Our present knowledge shows that in those few cases in which transfers from a single sporangium developed zygospores, a contamination of sporangial spores from an opposing strain must in some way have taken place.

During this period of investigation, only a few series of comparative cultures were carried on, since it became evident at the very outset, as was shown by the experiments already mentioned, that the influence of external conditions was a wholly inadequate explanation of the presence or absence of zygospore formation in this species. Such tests as were then made are summarized in the following table. The natural substances mentioned were sterilized in the autoclave at about 125° C., and, after inoculation with a few drops of sterlized water which contained a mixture of spores from a zygosporic culture, were kept covered in a drawer in the laboratory, with the results indicated in Table I.

In the following stender dish cultures shown in Table II, pieces of sponge and masses of sphagnum were cleansed, sterilized, and soaked with the different liquids tested, the inoculations being made from a mixture in sterilized water of spores from a zygosporic culture. It will be seen that the formation of zygospores occurs when such dilute nutrients are used as are available either in a one per cent solution of grape sugar on sponge, or in damp sterilized Sphagnum. On sponge, peptone alone in varying per cents is unable to support the formation of zygospores. Perhaps owing to the dilute solutions used, the sponge cultures did not produce a luxuriant vegetation. In general, however, it is noticeable that the carbohydrate, grape sugar, in contrast with the same per cents of the nitrogen compound, peptone, affords a nutrient comparatively favorable to the production of zygospores.

TABLE II.

Substratum.	Results.
Sponge with distilled water	no growth.
" " dilute prune decoction	sporangia; few zygospores.
Sphagnum with distilled water	sporangia; few zygospores.
" " dilute prune decoction	sporangia; zygospores somewhat abundant.
Sponge with 0.5% grape sugar	few sporangia; no zygospores.
" " 1.0% " "	sporangia; very few zygospores in cavities of sponge.
" " 2.0% " "	sporangia; no zygospores.
" " 5.0% " "	sporangia; few zygospores.
" " 7.0% " "	sporangia few; no zygospores.
" " 10% " "	sporangia; few zygospores.
" " 0.5% Peptone	sporangia scanty; no zygospores.
" " 1.0% "	sporangia scanty; no zygospores.
" " 4.0% "	sporangia scanty; no zygospores.

In Table III nutrients in stender dishes were sterilized as before in the autoclave and inoculated with a mixture in sterilized water of spores from a zygosporic culture. Those marked *Normal* were placed with their covers on in a locker in the laboratory; those marked *Very Moist* were placed under a bell-jar on a plate lined with wet filter paper, and in addition caps of damp filter paper were fitted over the uncovered stenders; those marked *Dry* had covers removed and were left in the locker in a crystallizing dish loosely covered with a glass plate; and those marked *Very Dry* were uncovered and tightly closed in a glass jar alongside with four stenders filled with calcium chloride. The series showed, more distinctly than is indicated in the table, that increase of moisture increases, and decrease of moisture decreases, the relative abundance of zygospores on a given substratum. On favorable nutrients in very moist air the sporangia were comparatively late in developing,

TABLE III.

Condition of Surrounding Air.

Substratum.	Very Moist.	Normal.	Dry.	Very Dry.
Carrot	sporangia few; zygospores abundant.	sporangia; zygospores.		
Parsnip	white felted growth; sporangia rare; zygospores.	white felted growth; few sporangia; no zygospores.	white felted growth; sporangia dense; no zygospores.	white felted growth; sporangia; no zygospores.
Beet	few sporangia; zygospores.	sporangia; zygospores.		
Sweet potato	few sporangia; zygospores abundant.	sporangia; zygospores.	sporangia; zygospores beneath.	sporangia low and thick; no zygospores.
Banana	sporangia low; zygospores.	scant growth; no sporangia; no zygospores.		
Turnip	sporangia; zygospores.	sporangia; zygospores.	sporangia; zygospores beneath.	
Bread soaked with dilute prune decoction	very few sporangia; zygospores abundant.	sporangia few; zygospores.	sporangia; zygospores beneath.	sporangia; few zygospores in cavities of bread.
Bread soaked with water	few sporangia; zygospores abundant.	sporangia few; zygospores.	sporangia; zygospores beneath.	few sporangia; no zygospores.

and appeared only in the upper parts of the paper cap, leaving below a loose growth of pure zygospores. With a small amount of moisture the zygospores were completely hidden below the well-developed sporangial growth, and in very dry air the formation of zygospores was entirely inhibited, except in the cavities of the bread soaked with prune decoction, where the amount of moisture necessary for their formation had been retained. These conclusions regarding the effect of relative moisture of the surrounding air upon the production of zygospores in Rhizopus have been confirmed by the experience in a large number of cultures both before and since the "sexual strains" referred to later on were separated.

The early study of zygosporic cultures of Rhizopus, some of the experiments in which have been embodied in the above tables, has indicated that the fundamental factor in the production of zygospores is quite independent of the external conditions to which the fungus was subjected. External conditions are seen to have a secondary influence, however. Moisture in the surrounding air is a condition favorable to the formation of zygospores and dryness is unfavorable; while, of the nutrients tested, carbohydrates are more favorable than nitrogenous compounds. In a general way, therefore, the effect of external factors is found to be in agreement with the results obtained by Klebs ('98) with the homothallic form Sporodinia. In the following pages the conditions which are believed by the writer to be of primary importance in the production of zygospores in this species will be considered.

Resolution of Rhizopus into (+) *and* (—) *Strains.*

In June, 1903, an attempt was made to demonstrate the existence of (+) and (—) strains in Rhizopus by opposing a half-dozen pure transfers from a zygosporic culture but without success, while in separation cultures from the same source the zygospores were inconstant in occurrence, and when present were scattered; and on account of the stoloniferous habit of growth of the fungus were prevented from showing even a suggestion of an intermediate line such as has been above referred to as occurring in such forms as *Mucor Mucedo*. On November 20 there were in the laboratory three van Tieghem cell cultures made from zygosporic hyphae, one of which was producing zygospores; three stender dish cultures which had been covered with moist filter paper caps and contained zygospores in abundance; and one gross zygosporic culture on unsterilized bread. From these sources forty-six pure sporangial transfers were made to paste and to potato agar each at such a distance from the others that zygospores might be expected to appear where some of the mycelial colonies came into contact. Four crystallizing dishes with paste, containing each six transfers, and two stenders with potato agar containing respectively two and three transfers were placed in the warm oven at 26° to 28° C., while three crystallizing dishes with paste containing four transfers each and two stenders with potato agar containing respectively two and three transfers were left at the room temperature. When these cultures matured there was no evidence of zygospores, even though a careful microscopic examination was made of the low-lying hyphae at the areas of contact of all the colonies. On the assumption that Rhizopus belonged to the heterothallic type it seemed strange that out of the forty-six contrasts between pure transfers from zygosporic cultures, in addition to the contrasts previously made, not a single zygospore should have been found. Nevertheless, although the frequent yellow appearance of the young sporangia was suggestive of a varietal difference in the zygosporic strains, the action of the fungus in the cultures above described, as well as the fact that the zygophoric hyphae could never be traced to the same filament, convinced the writer that the species could not be other than heterothallic.

By running a large number of van Tieghem cell cultures into which a small number of zygospores connected with the aerial mycelium had been transferred, it was hoped that the hyphae connected with the two sides of the zygospore would continue their growth and that by dissecting these out under a microscope it might be possible to obtain cultures which one could be sure had the same origin as the two respective gametes, and further that by a contrast on the proper nutrients of the two cultures thus obtained it might be possible to demonstrate the presence or absence of a heterothallic condition. Unfortunately when the hyphae were sufficiently few in number to make out their connections with the suspensors, they failed to grow, and though when larger masses of hyphae were used in the inoculation zygospores were generally developed, it was not possible to trace their mycelial connections.

Finally, in one cell containing a single young zygospore, it was found that the suspensors themselves had germinated, and by transferring the mycelial growths which had thus arisen, two cultures derived from mycelia of different origin, both of which were concerned in the formation of the original zygospore, were obtained. These strains, which for reasons to be mentioned later have been called (+) and (—) respectively, were derived, as we have seen, from the germinations of opposite suspensors. They were inoculated December 1, side by side in a van

Tieghem cell culture of potato agar acidulated with orange-juice, — a nutrient which has the advantage of preventing the growth of bacteria and at the same time favoring the formation of zygospores in this species. Two similar cell cultures were made containing only the (+) and (—) strains respectively, and in these sporangia alone were formed. In the culture, however, in which the (+) and (—) strains were contrasted, from fifty to one hundred zygospores developed, and in several cases the two suspensors could be traced with absolute certainty to the two different (+) and (—) mycelia.

The heterothallic condition of Rhizopus was thus established, and from these two original suspensors a series of (+) and (—) tube cultures have been run to the twenty-seventh generations, and form a certain means of obtaining zygospores at will, whenever spores from the (+) and the (—) strains are sown, either in a mixed condition or side by side on any suitable substratum.

The two strains having been at last obtained, it seemed a matter of some interest to endeavor to discover an experimental explanation of the early failures to obtain zygospores by contrasting sporangial transfers in the paste cultures above described, some twenty-eight of which still remained in the laboratory in five crystallizing dishes. From each dish a mass sporangial transfer was opposed in separate stenders against the (+) and (—) strains respectively obtained in A. The five contrasts of groups of uniform strains against the A(—) strain gave zygospores, while those against the A(+) strain produced only sporangia, thus proving that the twenty-eight transfers were all (+) in character. It is further probable that the same was true in regard to the remaining eighteen of the forty-six original transfers, and that they were (+) in character. The absence of zygospores from the mutual contrasts of these eighteen transfers shows that all in the same culture dish must have been either (+) or (—), and in view of the uniform (+) condition of the twenty-eight transfers tested, it seems unlikely that a (—) condition was present in every case.

By the experiments performed with culture A just discussed, it had been demonstrated that zygospores were produced only through the interaction of the hyphae of the diverse strains A(+) and A(—). Only the two (+) and (—) strains from this one culture were then known, and though such a condition was hardly to be expected, it was perfectly possible that other strains might be found which, on account of having grown under dissimilar conditions, would be sufficiently different in physiological character to form zygospores with both the (+) and (—)

strains which had been determined in the zygosporic culture A. In other words, although the presumption was in favor of considering the (+) and (—) strains sexual in character, it might be found that such was not the case, and that in other zygosporic cultures the opposite strains, if present, were different in nature from those obtained in A. In order to test the prevalence and nature of these strains in the formation of zygospores, a detailed analysis has been made of the following seven cultures, B to H, which were found producing zygospores in widely separated parts of this country. But before further considering these cultures, a brief account of the general procedure used in making them may not be out of place.

It has been found that a temperature of 26° to 28° C. accelerates the growth and favors the production of zygospores in Rhizopus, and therefore in all cases where a different treatment has not been stated the cultures have been run in the warm oven at the temperature above mentioned. Excessive desiccation has been prevented by keeping a dish of water on the same shelf with the cultures. Plain flour paste thickened over a steamer and sterilized in the autoclave has been the nutrient most frequently employed, and small stender dishes, 3.5 cm. in diameter, and 2 cm. tall, have been constantly used, since the zygospores develop in them in greater abundance than in the larger dishes, which are useful for the culture of other forms. The advantage of the smaller dishes may be due to their closely fitting ground-glass covers, which better prevent a loss of moisture. In making contrasts, spore material from the cultures to be tested is inoculated on opposite sides of stenders of flour-paste. Generally within forty-eight hours, when the cultures are run in the warm oven, young zygospores can be readily seen with a hand lens forming a more or less distinct and often yellowish line, clearly indicating the demarcation between (+) and (—) strains wherever they come in contact. In all cases, however, whenever negative results have occurred, the stenders have been re-examined on the fourth day or later.

Culture B was obtained from nut shells from Mrs. J. W. Cushing, Brookline, Mass., and after having been freed from other moulds and bacteria was kept running on bread. A piece of this material which was found to produce abundant zygospores was transferred to nutrient agar December 15, and the day following terminal branches of the mycelium were dissected out in a pure condition from different sides of the culture and transferred to flour paste in stenders. Seven cultures were in this way obtained, and their treatment is indicated in Table IV.

Since when B_7 or B[illegible] were contrasted with B_2, zygospores resulted, as indicated in the table, it is apparent that they belong to strains which are opposite to B_2, and by taking into account the fertile as well as the infertile contrasts, it will be possible to arrange the cultures B_1 to B_6 on one side in a column as belonging to the same strain in opposition to B_7 and B_8. By contrasting representatives from the two different strains of this culture B, with the two strains A(+) and A(−), it is seen that the culture under discussion forms its zygospores through the interaction of two strains which have the same value as those previously found in A, and can accordingly be placed under the proper signs in the same column with them.

TABLE IV. CULTURE B.

Dec. 14. Zygospore mass transferred to orange potato agar.
Dec. 15. Pure mycelial transfers B_1 to B_8 to paste stenders.

Contrasts (Mycelial). Dec. 16.	Results of Contrasts. Dec. 19.	Contrasts (Sporangial). Dec. 18.	Results of Contrasts. Dec. 21.	Strain Equivalents. A (−)	A (+)
$B_1 \times B_4$	no zygs.	$B_2 \times A(+)$	no zygs.	B_7	B_1
$B_1 \times B_8$	zygs.	$B_2 \times A(-)$	zygs.	B_8	B_2
$B_2 \times B_3$	no zygs.	$B_7 \times A(+)$	zygs.		B_3
$B_2 \times B_5$	no zygs.	$B_7 \times A(-)$	no zygs.		B_4
$B_2 \times B_7$	zygs.				B_5
$B_2 \times B_8$	zygs.	$B_2 = A(+)$			B_6
$B_5 \times B_6$	no zygs.	$B_7 = A(-)$			

In the above table, B_1 to B_8 are cultures from transfers made in separating these strains of nut culture B. In the columns marked *Contrasts* is found their disposition, the mark (×) indicating that the cultures between which it is placed were inoculated side by side on flour paste. In the columns marked *Results of Contrasts* is indicated whether zygospores were present or absent as a result of these contrasts. In the columns A(+) and A(−) under *Strain Equivalents* are arranged the cultures from Nut Culture B according to their strain character as determined by (+) and (−) strains of A.

Culture C was obtained on paste from nut shells from Mr. V. B. Swett, Newton, Mass., and for some reason offered greater difficulties to a separation of the strains. As seen from Table V, the two attempts, by using pure transfers from the edge of the mycelium, gave only the single (+) strain. Later, a large number of young zygospores were dissected out and laid separately on nutrient agar in Petri dishes. The following day the suspensors on one side of many of the zygospores, and in three instances those on both sides, had germinated, and made it possible to obtain pure mycelial transfers from them. It should be here noted that in cases where one of the two suspensors failed to germinate, it was invariably the one belonging to the (—) strain. Including the dissection from the edge of the mycelium, eighteen transfers in this culture were (+) and only three were (—); in culture B, six mycelial transfers were (+) and two (—); while in culture A, probably all of forty-six sporangial transfers were (+) in addition to the transfers from the two opposite suspensors that finally resulted in the separation of the original single (+) and (—) strains. This makes from the three nut cultures a total of

TABLE V. Culture C.

Dec. 8, zygospore mass transferred to potáto agar.
Dec. 9, pure mycelial transfers to paste stenders C_1 to C_5.

Contrasts (Sporangial). Dec. 12.	Results. Dec. 15.
$C_1 \times C_5$	no zygs.
$C_1 \times C_2$	no zygs.
$C_1 \times C_3$	no zygs.
$C_3 \times C_4$	no zygs.
$C_3 \times C_4$	very few z.
$C_2 \times C_5$	no zygs.
$C_1 \times A(+)$	no zygs.
$C_1 \times A(-)$	zygs.

C_1 to $C_5 = A\ (+)$

Strain Equivalents.

A(—)	A(+)
C_{16}	C_1
CX_{17}	C_2
CX_{18}	C_3
	C_4
	C_5
	C_6
	C_7
	C_{10}
	C_{12}
	C_{14}
	C_{15}
	CX_{16}
	C_{17}
	C_{18}
	C_{19}
	C_{20}
	C_{21}
	C_{22}

Dec 14, zygospore mass transferred to potato agar.
Dec. 15, pure mycelial transfers to paste stenders C_6 to C_{15}.

Contrasts (Mycelial). Dec. 16.	Results. Dec. 18.
$C_7 \times C_{10}$	no zygs.
$C_7 \times C_{12}$	no zygs.
$C_7 \times C_{15}$	no zygs.
$C_{10} \times C_{14}$	no zygs.
$C_{12} \times C_{15}$	no zygs.
$C_6 \times C_7$	no zygs.
$C_7 \times A(+)$	no zygs.
$C_7 \times A(-)$	zygs.

$$\left.\begin{array}{l} C_6 \\ C_7 \\ C_{10} \\ C_{12} \\ C_{14} \\ C_{15} \end{array}\right\} = A(+)$$

transfers C_8, C_9, C_{11}, C_{13} failed to grow.

Dec. 21, young zygospores laid separate on agar in Petri dish.
Dec. 22, pure mycelial transfers made from germinated suspensors into test-tubes C_{16} to C_{22}.

Contrasts (Mycelial). Dec. 23.	Results. Dec. 30.
$C_{16} \times CX_{16}$	zygs.
$C_{16} \times CX_{17}$	no zygs.
$C_{16} \times C_{19}$	zygs.
$C_{16} \times CX_{18}$	no zygs.
$C_{16} \times C_{21}$	zygs.
$CX_{16} \times C_{22}$	no zygs.
$C_{17} \times CX_{17}$	zygs.
$C_{17} \times C_{19}$	no zygs.
$C_{17} \times C_{22}$	no zygs.
$C_{18} \times CX_{18}$	zygs.
$C_{18} \times C_{19}$	no zygs.
$C_{20} \times C_{21}$	no zygs.
$C_{21} \times C_{22}$	no zygs.
$C_{16} \times A(+)$	zygs.
$C_{16} \times A(-)$	no zygs.

$C_{16} = A(-)$

CX_{16}, CX_{17}, CX_{18} were transfers from the opposite suspensors to C_{16}, C_{17}, C_{18} respectively. Other transfers were from the germinations of single suspensors.

seventy-one (+) and six (−), and of these latter, four came from the suspensors of individual zygospores in cases where both had germinated.

A number of spontaneous infections of Rhizopus in laboratory cultures were tested at this time and found to be (+) in character, a fact which made it seem possible that the (−) strain was exotic, and, being carried on the nut shells, either by handling or by infection in the paste cultures, became mingled with the (+) strain apparently predominant in the laboratory. Through the kind assistance of Mr. V. B. Swett it is possible to state that in all probability the two strains are present side by side on the nuts as bought. Several spontaneous cultures of bread and of a number of different vegetables were started by him at the writer's suggestion at the same time with a culture of nut shells on moist bread. The failure of Rhizopus to develop on the other spontaneous cultures indicated the comparative rarity of spores of this fungus in the air of the apartments and rendered probable that its luxuriant growth on the nut bread culture came entirely from spores carried on the nut shells. The zygospores which here appeared were the starting point of *Culture D*, which was separated by taking transfers from two zygospores both suspensors of which had germinated so that two transfers belonged to each strain. The strains obtained produced zygospores when contrasted with the proper strains of *Culture A*.

Culture E was obtained as already mentioned (p. 233) from an infection of a mixture of vegetables at the Botanic Gardens. Pure sporangial transfers were made from a carrot, an onion and a sweet potato, and in addition young zygospores from cultures obtained from this mixed growth were laid on nutrient agar and mycelial transfers E_1 to E_6 were taken from germinations of their suspensors. E_5 was apparently an impure transfer as seen by its giving zygospores with both A(+) and A(−). That zygospores did not show in the tube culture made from

TABLE VI. CULTURE E.

Contrasts. Jan. 9.	Results. Jan. 12.	Contrasts. Jan. 10.	Results. Jan. 12.	Strain Equivalents.	
				A(−)	A(+)
Sweet potato × A(+)	no zygs.	$E_1 \times E_2$	no zygs.	Carrot	Onion
Sweet potato × A(−)	zygs.	$E_1 \times$ A(−)	zygs.		Sweet potato
Onion × A(+)	no zygs.	$E_2 \times E_4$	no zygs.		E_1
" × A(−)	zygs.	$E_3 \times E_4$	no zygs.		E_2
Carrot × A(+)	zygs.	$E_5 \times E_1$	zygs.		E_3
" × A(−)	no zygs.	$E_5 \times E_6$	zygs.		E_4
		$E_5 \times$ A(+)	zygs.		[E_5 mixed]
		$E_5 \times$ E(−)	zygs.		

the original transfer is due to the fact that the potato agar used as nutrient is not a favorable subtratum for their production.

The disposition of these transfers may be seen in Table VI.

TABLE VII. Culture F.

Dec. 22, zygospore material received from Prof. W. C. Coker, Chapel Hill, N. C., and used in inoculating orange potato agar.
Dec. 24, young zygospores laid separate on agar in Petri dish.
Dec. 25, pure mycelial transfers made from germinated suspensors into test-tubes.

Contrasts (Mycelial). Dec. 26.	Results. Dec. 30.	Strain Equivalents. A(−)	Strain Equivalents. A(+)
F I × f_1	zygs.	F I	f_1
F II × f_2	zygs.	f_2	F II
F III × f_3	zygs.	F III	f_3
F II × f_1	no zygs.	F IV	f_6
F IV × f_3	zygs.	F V	f_7
F III × f_2	no zygs.	f_8	F_9
F V × f_7	zygs.		
F V × f_8	no zygs.		
F II × f_9	no zygs.		
f_2 × F_9	zygs.		
F III × A(+)	zygs.		
F III × A(−)	no zygs.		
F V × f_6	zygs.		
F IV × f_6	zygs.		
F II × A(+)	no zygs.		
F II × A(−)	zygs.		

TABLE VIII. Culture G.

Dec. 27, zygospore material received from Prof. D. H. Campbell, Stanford University, Cal., and used in inoculating orange potato agar.
Dec. 29, young zygospores laid separate on agar in Petri dish.
Dec. 30, pure mycelial transfers made from germinated suspensors into test-tubes.

Contrasts (Mycelial). Dec. 31.	Results. Jan. 2.	Strain Equivalents. A(−)	Strain Equivalents. A(+)
G_1 × G_3	zygs.	G_1	g_2
G_1 × G II	no zygs.	G II	G_3
G_1 × G VI	no zygs.	G IV	
g_2 × G II	zygs.	G V	
g_2 × G IV	zygs.	G VI	
G_3 × G V	zygs.	G_7 (germination from a young zygote).	
G II × G IV	no zygs.		
G_7 × g_2	zygs.		
G_7 × G II	no zygs.		
G_7 × A(−)	no zygs.		
G IV × A(+)	zygs.		
G IV × A(−)	no zygs.		

In all the nut cultures, while occasionally one suspensor was larger than the other, the difference was not marked, but in cultures F and G that on one side was regularly swollen to such an extent that its diameter often equalled or exceeded that of the zygospore itself. In Tables VII and VIII a transfer from the germination of a swollen suspensor is represented by a capital letter with the addition of a Roman numeral, as e. g. F II, and a transfer from an unswollen suspensor by a small letter, as, e. g., f_2. Where an Arabic numeral is used with a capital, the suspensors were equal in size. Transfers with the same numerical subscript came from opposite suspensors of the same zygospore. It will be seen that in the two cultures together, eight out of nine of the swollen suspensors taken were from a (—) mycelium and five out of seven of the smaller suspensors belonged to a (+) mycelium. Not enough tests were made to show whether so large a majority of the swollen suspensors are normally (—) as the figures would indicate; but it is significant that since this greater enlargement occurs also on the (+) side the swelling of the suspensor cannot be considered as an indication of sexual differentiation. G_7 was taken from the germination of a young zygote whose walls had just begun to darken, and whose suspensors had shrivelled. The zygospores of Rhizopus, so far as is known, have never been germinated, and in the present instance the word "zygospore" is not certainly appropriate, since whatever ripening process goes on in the zygote may not have been completed in so short a time (two days from inoculation). In culture F, the two strains are equally represented, while in G the (—) strain predominates six to two.

It should be noted that in culture H, Table IX, the (—) strain outnumbered the (+) in the ratio of nineteen to one, while in culture C the (+) strain outnumbered the (—) in the ratio of eighteen to three; and it is further noticeable that the one (+) strain of the former, and the three (—) of the latter were all obtained from germinating suspensors. This fact is of importance since it indicates that in this species the (+) and (—) strains are not distinguished from one another by any difference in the luxuriance of their vegetative development, but that in different cases one or the other may be predominant in that respect.

The (+) and (—) strains in these eight cultures (A to H) having been separated and their character determined by means of contrasts with (+) and (—) strains, chiefly of culture A, it was thought well to confirm the results by using as a standard the strains obtained from the last culture H. Accordingly, April 4, the (+) and (—) strains from each of the seven cultures A to G were contrasted respectively with the (—) and (+)

strains from culture H, and in no case did a growth fail to show zygospores at the line of contact. The constant presence, in all these cultures which have been found spontaneously producing zygospores, of two opposite strains, the individual character of each of which is the same

TABLE IX. Culture H.

(Zygospore material obtained from Prof. F. L. Stevens, Raleigh, N. C.)

Transfers obtained from germinations of single suspensors.

Contrasts. Feb. 12.	Results. Feb. 16.
$H_1 \times H_2$	no zygs.
$H_2 \times H_3$	no zygs.
$H_3 \times H_4$	no zygs.
$H_4 \times H_5$	no zygs.
$H_5 \times H_6$	no zygs.
$H_6 \times H_7$	no zygs.
$H_7 \times H_8$	no zygs.
$H_1 \times A(+)$	zygs.
$H_1 \times A(-)$	no zygs.
$H_7 \times A(+)$	zygs.
$H_7 \times A(-)$	no zygs.

H_1 to $H_8 = A(-)$

Transfers obtained from sporangia from a culture producing zygospores.

Contrasts. Feb. 19.	Results. Feb. 22.
$H_9 \times H_{10}$	no zygs.
$H_9 \times H_{11}$	no zygs.
$H_9 \times H_{12}$	no zygs.
$H_9 \times H_{13}$	no zygs.
$H_{11} \times H_{13}$	no zygs.
$H_9 \times H_7$	no zygs.
$H_{13} \times H_1$	no zygs.

H_9 to $H_{13} = H_7 = A(-)$

Transfers H_{18} and H_{19} were obtained from sporangia; H_{15} to H_{17} came from germinations of single suspensors; HX_{17}, HY_{17}, and HZ_{17} were from the suspensors of a single zygospore.

Contrasts. March 12.	Results. March 16.
$H_{15} \times H_{16}$	no zygs.
$H_{16} \times HX_{17}$	zygs.
$HY_{17} \times HX_{17}$	zygs.
$HY_{17} \times HZ_{17}$	no zygs.
$HZ_{17} \times H_{18}$	no zygs.
$H_{18} \times H_{19}$	no zygs.
$HY_{17} \times F(+)$	zygs.
$HY_{17} \times F(-)$	no zygs.
$H_{19} \times F(-)$	no zygs.
$H_{15} \times F(-)$	no zygs.
March 16.	March 20.
$HX_{17} \times F(+)$	no zygs.
$HX_{17} \times F(-)$	zygs.

Strain Equivalents.

A(−) F(−)	A(+) F(+)
H_1	HX_{17}
H_2	
H_3	
H_4	
H_5	
H_6	
H_7	
H_8	
H_9	
H_{10}	
H_{11}	
H_{12}	
H_{13}	
(H_{14} mixed)	
H_{15}	
H_{16}	
HY_{17}	
HZ_{17}	
H_{18}	
H_{19}	

throughout the series, and the coöperation of which is necessary for the process of conjugation, indicates that it is sexual in nature, and that the (+) and (—) strains represent the two opposite sexes.

Investigations of the "Harvard Strain."

A brief history of the "Harvard strain" has been given under the citations in Part I (p. 233), where it was mentioned that the strain, after running for nearly ten years in the laboratories of a number of different institutions, had ceased to produce zygospores. Since, as we now know, this "strain" was merely a chance admixture of (+) and (—) strains which had been preserved in its zygosporic activity by transfers of portions of the substratum containing their spores, it is a matter of interest to discover the effect on the sexual strains of this ten years association, and to learn in what way the culture has lost its power of producing zygospores.

Although since 1901 zygospores have not been obtained in the laboratory from cultures of the "Harvard strain," the material has been preserved on pieces of dry bread. From such an old culture several transfers were made to stenders of flour paste which, by failing to produce zygospores, showed that not more than one of the sexual strains was present. This, by several tests with (+) strains, has been found to be (—) in character, and the same is true of the Harvard material carried to Dartmouth College in 1901 by Prof. G. R. Lyman, which was kindly sent to the writer for examination. In both these instances the zygospores produced by contrasts with (+) strains were perfectly normal.

Dr. Otis W. Caldwell of Charleston, Ill., has kindly sent the writer material which traces descent to the "Harvard strain" through the laboratory of the University of Chicago, and writes that he has been unable to obtain zygospores from it since 1902. A culture from this source fails to produce zygospores when contrasted with (+) strains. With (—) strains, however, although a reaction is evident, accompanied by the formation of numerous progametes, and the characteristic yellowish tinge appears in the zygosporic apparatus, no perfect zygospores are formed. One of the progametes generally remains small, and fails to distinguish its gamete while the other becomes abnormally swollen, often discolored, and more or less tuberculate throughout with a constriction where a septum for the separation of its gamete might be expected. These imperfect conjugations are suggestive of the conditions associated with hybridization, and are similar to what has been observed of one other culture at least of the Rhizopus material which has been sent to the laboratory.

The fact that in the Chicago culture the strain remaining has (+) characters, while in the culture in the Harvard laboratory the strain remaining has (—) qualities, shows that the running out of these scions of the original Harvard zygosporic culture is probably due to accidental circumstances accompanying the gross transfers of their spores or to contamination from outside strains. The method of perpetuating these zygosporic strains in gross cultures for class work is such that there is no assurance that the culture at the end of the series has any genetic connection with the one at the beginning. Comparatively little can be learned, therefore, from a study of what remains from the "Harvard strain."

Distribution and Thallic Character of Strains in Nature.

The existence of (+) and (—) strains in Rhizopus having been definitely established, it seemed desirable, owing to the ease with which this very common form may be obtained from distant sources, to test its thallic character by a detailed examination of as many and as varied cultures as could be procured, and with this object in view material was collected from various sources and tested in detail, with the results hereafter enumerated.

Associations of (+) and (—) strains of this species have been found producing zygospores in Germany, and France, and have been obtained by the writer from diverse parts of this country, and investigated, with the results set forth in the previous pages. Accordingly questions now arise in regard to the distribution of the individual strains. Requests have been sent to a number of botanists in different parts of this and other countries for spore material from their localities, together with suggestions for obtaining the fungus on spontaneous cultures of bread. Thanks to the kindness of the writer's correspondents, he is able to represent by means of Table X the results obtained by the cultivation of material from a wide range of territory. In cases where more than a single contribution was received from a given locality the material was of different origin, and in the table numerals indicate the order in which the tests were made.

Nearly all the material was received in a dried condition, generally on bread, and inoculations were made into acidulated agar tubes in the writer's lodging where danger of contamination from the laboratory spores was minimized. When the cultures had been freed from the frequent accompaniment of Penicillium and bacteria, contrasts were made on paste in small stenders which were kept in the warm oven at 26°–28° C. It may be said, however, in this connection that, although infections of Rhizopus are frequent in the laboratory in cultures at all exposed to the air, yet out of between 1500 and 2000 pure tube cultures of different mucors, which the writer has made during the present year in the laboratory where cultures of this species were constantly being opened to the air, only three tubes have shown contaminations by Rhizopus. The chances, therefore, that an admixture of laboratory strains with the material received had possibly occurred may be disregarded in view of the precautions taken.

TABLE X.

Arrangement of Strains according to Sexual Character.

(—)	Neutral.	(+)
Jamaica Plain, Mass. (1)	Brookline, Mass. (2)	Brookline, Mass. (1)
Bread	Bread	Sweet potato
Jamaica Plain, Mass. (2)	Brookline, Mass. (3)	Worcester, Mass.
Bread	Bread	Bread
New Haven, Conn. (2)	Winthrop, Mass.	New Haven, Conn. (1)
Bread	Bread	Baked beans
Lansing, Mich. (1)	Middletown, Conn.	Washington, D. C.
Bread	Bread	Bread
Lansing, Mich. (2)	Granville, Ohio	Delaware, Ohio
Sweet potato	Bread	Bread
Austin, Texas	Breslau, Germany	Madison, Wis.
Bread	———	Bread
Nassau, Bahamas	Halle, Germany	
Bread	———	
Cambridge, Eng.	Honolulu, H. I.	
———	Plant roots	
Berne, Switzerland	Margarita, Venezuela (1)	
Bread	Nuts	
Nancy, France	Margarita, Venezuela (2)	
Quince	Nuts	
Catania, Sicily	Caracas, Venezuela	
Pistachio nuts	Dog dung	
	Port au Prince, Haiti	
	Dung	

Repetitions were made when there was a suspicion that absence of zygospores might be due to bacterial infection or to other conditions unfavorably affecting the growth, and those cultures in which zygospores were not produced under the circumstances mentioned were placed in the "neutral" column without making any further attempt to obtain positive results by varying the substratum. A number of the cultures in this column showed, before being tested, a certain difference in gross appearance from those in the other columns. This is true of the cultures obtained from the flowers, roots, nuts, and dung, there listed from tropical regions. With these exceptions the majority of the cultures are located in one or other of the sexual columns. Since no critical examination of these forms has been made, the possibility that more than one species is represented among them should not be overlooked. Experiments to determine the actual condition in these neutral strains have not as yet been made.

A few of the (+) and (—) strains constantly form perfect zygospores with the corresponding test strain, but in very small amounts. The strain sent by Professor Cavara from Sicily produces with the (+) strain a distinct line of contact wherein the zygophoric hyphae, young sporangia, and progametes show the characteristic yellow coloration, yet the process of conjugation as in the hybridization contrasts does not go further than the formation of gametes. It is possible that in the wholly neutral strains a similar reduction of the (+) and (—) quality has extended still further to the point of extinction, but it is not certain that their apparent neutrality represents a complete loss of all sexual character. The sexual nature may still remain, and the strains be (+) and (—), although for some reason unable to respond to a sexual stimulus.

The table is not yet sufficiently complete to enable one to determine the relative distribution in nature of the (+) and (—) strains, but enough data are furnished to show that both are widely distributed. It is hoped that a further accumulation of facts will throw more light on the occurrence of these strains in nature.

The following persons have kindly sent, for use in compiling the table, material from the places indicated, and to them the thanks of the writer are due: Miss T. L. Blakeslee, Delaware, Ohio; Prof. O. Brefeld and Dr. R. Falck, Breslau, Germany; Mr. F. I. Brown and Miss L. G. Adams, Brookline, Mass.; Prof. E. A. Burt, Middlebury, Vt.; Mr. G. D. Bussey, Winthrop, Mass.; Prof. O. W. Caldwell, Charleston, Ill.; Prof. D. H. Campbell, Stanford University, Cal.; Prof. F. Cavara, Catania, Sicily; Prof. H. W. Conn, Middletown, Conn.; Prof. F. W. Coker, Chapel Hill, N. C.; Dr. E. B. Copeland, Manila, P. I.; Dr. G. P. Clinton, New Haven, Conn.; Mr. J. W. Curd, Austin, Texas; Prof. J. B. Dandeno, Lansing, Mich.; Prof. Ed. Fischer, Berne, Switzerland; Prof. C. F. Hodge, Worcester, Mass.; Mr. J. R. Johnston and Mr.

L. W. Riddle, Jamaica Plain, Mass.; Prof. Geo. Klebs, Halle, Germany; Prof. C. S. Leavenworth, Shanghai, China; Prof. G. R. Lyman, Hanover, N. H.; Prof. L. Matruchot, Paris, France; Prof. A. C. Moore, Columbia, S. C.; Dr. E. W. Olive, Madison, Wis.; Mr. J. B. Rorer, Washington, D. C., and Nassau, Bahamas; Prof. F. L. Stevens, Raleigh, N. C.; Mr. M. E. Stickney, Granville, Ohio; Prof. P. Vuillemin, Nancy, France; and the writer has himself obtained material from Margarita and Caracas, Venezuela; and Port au Prince, Haiti.

Serial Cultures.

In investigating the (+) and (−) strains of this species a number of serial cultures were conducted to observe the effect of continued cultivation on the sexual activity. December 2, a sporangial series of both strains from culture A was started in tubes of potato agar kept in a culture drawer and the transfers continued up to the thirteenth generations. A similar series was begun at the same time and carried on in the warm oven at 26°–28° C. up to the twenty-seventh generations. From cultures at the room temperature sporangial transfers could be made every third or fourth day, while from those in the warm oven transfers could be made a day earlier. In making transfers a mass of the mature sporangia was generally used, though occasional pure transfers from single sporangia were made in each series. In later tubes from 3 to 4 per cent of grape sugar was added to the nutrient, and increased both the rapidity and luxuriance of the growth. From tubes $A(+)_3$ and $A(-)_3$, the third generations of A(+) and A(−) strains, a mycelial series was started December 11. Each morning a piece of agar with the attached mycelium was transferred with a platinum spatula into a new tube. In this way the growth of the mycelium was uninterruptedly prolonged for over two months and prevented from producing any sporangia whatever until February 19, when the seventy-first tubes were reached and the series discontinued. The last tubes of each of the three series were tested and the growths in them were apparently unaffected in their sexual character or activity, nor had one strain, as in the similar experiment with *Mucor Mucedo* subsequently described, shown any tendency to lose its vitality.

Morphology and Physiology of Conjugation.

In a discussion of the morphology of zygospore formation in this species it will not be out of place to quote from de Bary's ('66) paper on Rhizopus. According to his account zygospores form on low irregularly branched hyphae which start from the mycelium like stolons whose branches intertwine and develop zygospores at the places of contact and crossing. Here one hypha produces a perpendicular outgrowth which is duplicated by the second at the point of contact with the first. The outgrowths are at first equal in size and have a diameter not greater than that of the hypha which bears them. The gametes cut off are generally unequal in size, one being as long as broad, the other only half as long, and there is generally a marked difference in the size of the mature suspensors, the larger corresponding to the smaller gamete. The zygospores occur entirely separate or in great numbers close together, and are often the only form of reproduction developed from the hyphae that produce them. At times the same hyphae may bear single sporangiophores near the origin of the zygospores.

By growing the two strains opposed in a van Tieghem cell on the proper substratum, it is not difficult to confirm most of these observations of de Bary. In addition to the vigorous stolons which may function as conjugative hyphae, more or less branched slender filaments arise from the mycelia of both strains and may ultimately give rise to sporangia or remain sterile. Where these, however, come in contact with similar growths from the opposite strain, progametes are produced which by their development push apart the fertile filaments. A stolon arising from a node may form progametes if it comes in contact with a filament of the opposite strain, and may later give rise to another node or produce sporangia in close proximity with the zygospores.

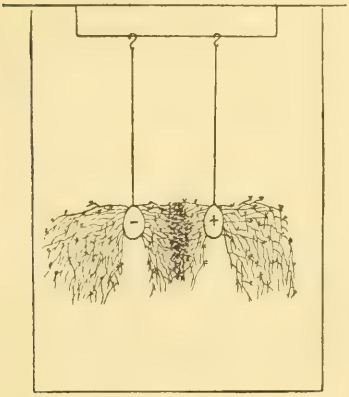
When certain (+) and (—) strains are contrasted the young zygospores and the hyphae connected with them are more or less filled with globules of yellow oily material which gives a striking color effect to the line formed by the young zygospores, while between other (+) and (—) strains of this species on the same substratum the young zygospores are colorless. The color when most marked may invade the sporangiophores, and such a culture shows that the young sporangia where zygospores are being formed are bright yellow in color, while those in other parts of the culture are a pearly white at the same stage of development. It has not been determined whether the yellow sporangia are common to both strains, but such a condition is probable, since where the color is present the oily material may occur in the hyphae connected with both suspensors. It will be remembered that the peculiar appearance of these young sporangia on certain nutrients when the "Harvard strain" was first found and when it subsequently reappeared, suggested the idea that zygospore production, in this as well as in other species, might be a character associated with special varieties or races. The oily material is no doubt a concentrated form of nutriment developed in connection with the storing up of food in the zygospores, but why it should occur only when certain (+) and (—) strains are opposed has not been investigated. The solution doubtless lies in a study of the differences in the individual (+) and (—) strains.

From the spreading habit of this species and from the fact that certain conditions of moisture and nutriment must be satisfied before zygospores will form, it is rather difficult to follow the growth of the zygophoric hyphae. Mutual attraction of these hyphae has been demonstrated in certain species of the genus Mucor; but though the same condition may well hold here, this assumption is not necessary in order to account for the observations which have been made. Whether this be the result of accident or of some attractive force, a contact of hyphae from opposite strains is the immediate stimulus for the formation of progametes, and apparently this contact incites an increased branching in the filaments affected, as is indicated by the fact that zygospores are found in the greatest abundance on intricately branched and interwoven hyphae, especially in the lower parts of the culture where sporangia do not commonly occur.

Moisture in the surrounding air has previously been shown to have a decided influence on the formation of zygospores, and it is probably for this reason that zygospores form preferably between the bottom and sides of the culture dish and the substratum, and especially between layers of filter paper placed over the nutrient for this purpose. The sporangia, on the contrary, are seldom produced in similar places, but rather are formed in the upper parts of the culture. It is not unusual in stender dish cultures to find that the sporangia form chiefly in a thick ring under the edge of the cover, and in test-tube cultures the sporangia are massed in the upper part of the tube while the zygospores are confined to the lower part. In one test-tube culture a crack was found to have occurred at the base of the tube and its location was marked by a distinct line of sporangia which, aside from those in the upper part of the tube, were confined to the limits of the crack. All these facts just mentioned point to something in the outside air which acts as a stimulus to the production of sporangia. Dryness is the factor which suggests itself, and which is considered by Klebs ('98) to be effective in Sporodinia, but test experiments have not been made for Rhizopus.

If the immediate stimulus to the formation of progametes lies in the contact of hyphae from different strains, it probably becomes operative through the osmotic activity of the hyphal contents. If the surfaces of two membranes in contact become dry there can be no mutual osmotic influence between the fluids contained within them, and it is for this reason, perhaps, that progametes fail to form in a relatively dry air. There are no doubt other causes active, however, an investigation of which the writer hopes to undertake later. The evident advantages of small stenders with close-fitting covers, over large ones with covers that are loose as well as soft over hard agar in connection with zygospore cultures, is probably also due to their influence upon the moisture content of the air.

In order to test the condition necessary for transmission of influence and to determine whether it acts through air or through the solid and moist medium of cultures the following experiment was devised. From the cover of a battery jar, the air in which was rendered moist by a lining of wet filter paper, two bags of cheese cloth containing the substratum (bread soaked in dilute orange juice) were suspended by threads so that they hung free with their inner faces 1¼ inches apart, as is shown in the accompanying diagram. The bags were inoculated with opposite strains, and the apparatus was placed in the dark. In one week zygospores had formed in abundance where the aerial growth radiating from the bags of bread had come in contact. In this experiment any influence upon the origin or direction of growth of the zygophoric hyphae which might have been exerted through the solid culture medium, or which was due to a contact of the masses of vegetative mycelia, was eliminated, and any such influence, if it existed, must have been confined to the aerial branches. If there is an orienting of the zygophoric hyphae, which cannot in any case be a well marked phenomenon in this species, the directive influence must lie outside the substratum and in the hyphae affected.



Differentiation between (+) and (—) Strains.

In view of the fact that in a majority of the heterothallic forms investigated a morphological differentiation is, as has been previously mentioned, more or less well marked between (+) and (—) strains, it was hoped that a similar condition might be found in Rhizopus. The unequal swelling of the suspensors has been already referred to as a variable circumstance independent of the (+) or (—) character of the strains contrasted, yet swollen suspensors are almost constantly present in contrasts between certain (+) and (—) strains, while they are as constantly absent in contrasts between others. In addition, conditions of the substratum unfavorable to zygospore formation seem to be a hindrance to suspensor enlargement. Both suspensors may remain small or both may enlarge, and the explanation which naturally suggests itself is that the swollen suspensor is connected with the hypha which happens to contain an excess of protoplasm. Whenever a number of zygospores have been carefully dissected out, the swollen suspensors have always been observed to have the same hyphal connections and are often seen to arise from the larger hypha. In contrast to the opposite filament which, together with the smaller suspensor connected with it, is generally soon empty, the larger suspensor seems to furnish most of the nourishment to the growing zygospore, and it is not unusual to find a young zygospore associated with a suspensor greater than itself and acting as a reservoir of dense protoplasm which it gives up as the zygospore matures. That, however, the difference in size does not correspond to an inherent difference in the (+) and (—) strains as such is shown by its inconstant occurrence as well as by such results as were obtained by culture F (p. 259), in which germinations of both larger and smaller suspensors gave rise to (+) and (—) strains indiscriminately.

Although van Tieghem ('75) was unable to confirm de Bary's view, that there is a relationship between the size of the gametes and that of the suspensors, he considers the inequality in the size of the gametes an indication of a sexual differentiation. The difference in vigor of the two sides is often indicated by an inequality in the progametes; and in cultures where unequal swellings of the suspensors is marked, the difference in their size is generally evident as soon as the gametes are cut off. Though in the material examined the smaller gamete was in a majority of cases adjacent to the enlarged suspensor, the reverse condition has been observed. From Plate I, Figure 15, it will be seen that the size of the gamete can be of no sexual significance in this species. The connected suspensors and their gametes belong obviously to the same strain, and consequently in the two cases where the gametes are unequal the smaller gametes are seen to belong to the opposite strains. This figure is a camera drawing from a preparation taken from a culture where the suspensors were approximately equal in size.

It has now been shown that inequality in the size of the gametes and suspensors is of no sexual significance, and a microscopic examination of both strains shows no appreciable difference in the size and markings of the spores nor in any other of the characters observed. Moreover the gross appearance of their growth in test-tubes and in gross cultures will not enable one to distinguish the strains, and experiments have not been carried on as yet to determine if they possess any secondary physiological characters, in addition to their primary sexual differences.

When the strains of culture A were first separated it was thought that the strain now marked (+) on account of its hybridizing action with other species was the one which showed a greater vigor of growth. In a single bread culture where the (+) and (—) strains were opposed in two parallel lines and zygospores formed at the contact of the opposite growths, it was found that the (+) strain had grown up against the cover, while the (—) strain showed a growth distinctly lower. It seemed possible to explain the exclusively (+) character of all the sporangial transfers taken in separating the strains of culture A by supposing that when sown together there was a constant separation of sporangial growth, the (+) sporangia being in the upper part of the culture where they would be more readily taken for transfers. Further to test this supposition, pure transfers were made from five different sporangia taken from different parts of the upper growth of a bread culture where the spore material used in the inoculation of the two strains had been mixed in approximately equal amounts. All of these transfers, when tested, turned out to be (+) in agreement with the supposition mentioned.

The difficulties which were encountered in obtaining other than the (+) strain in separating cultures A and C (p. 255) pointed to a similar conclusion, as well as the fact that it was definitely shown by a number of tests of spontaneous infections of Rhizopus that (+) spores were the more abundant in the air of the laboratory. Out of nine such infections tested the first five were (+), one was (—), and three belonged apparently to the group of neutral strains previously mentioned (p. 263). Nevertheless, it should be mentioned that in a Petri dish culture which had been accidentally infected from the air of the laboratory it was found that two mycelial colonies had thus originated at opposite edges of the dish and showed that they belonged to different strains by forming zygospores when they came in contact.

In marked contrast to the condition above described, culture H (p. 259), in which the (—) transfers predominated in the ratio of nineteen to one, seemed to indicate here that the relative conditions were reversed. Accordingly to test the matter further and to determine what if any difference in growth exists between the (+) and (—) strains, on March 23 the (+) and (—) strains of the cultures A to H were mutually contrasted in crystallizing dishes of flour paste, and in addition contrasts were made between (+) and (—) strains obtained from Worcester, Mass., and Nancy, France. All of these contrasts produced zygospores, but although in some the sporangial growth was slightly higher on one side of the dish, this difference had no relation to the places of inoculation, and in none of the cultures could any difference be observed in the growth of the two strains. The only difference, therefore, by which at present the strains of Rhizopus can be distinguished lies in their sexual action when grown in contact.

MUCOR MUCEDO.

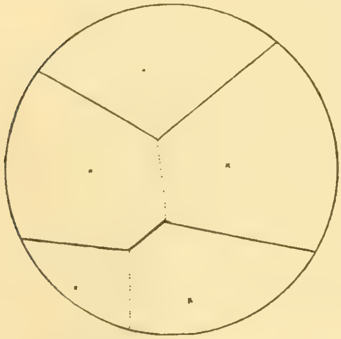
During the past few years the writer has been conducting numerous gross cultures of coprophilous fungi in several of which zygospores have been found which from their general appearance and from the sporangial growth with which they were associated have been referred to that subdivision of the genus Mucor of which *M. Mucedo* may be taken as the type. Although it seems certain that the forms thus associated represent more than a single species, the one which forms the basis of the present discussion appears to agree with Brefeld's ('72) description, and therefore may be assumed to be the typical *M. Mucedo*. It has been generally impossible, when zygospores have been found, to continue their formation in tube cultures by either sporangial or mycelial transfers.

In January, 1903, however, zygospores of this type form were found growing spontaneously on a certain gross dung culture in so comparatively pure a condition that it was possible by making mycelial transfers to hard agar to free the growth from the associated bacteria and to obtain a pure culture of this Mucor with an abundant production of its zygospores. The same difficulty that has already been mentioned of the early cultures of Rhizopus was here experienced in inducing zygospore formation from a sporangial inoculation. If a mass of the mycelium producing zygospores was transferred to nutrient agar, zygospores formed in a dense mass around the point of inoculation, but their numbers soon diminished as the mycelium widened its growth. Transfers from single sporangia never produced zygospores and zygosporic cultures were therefore kept running by mass transfers of the mycelium. Repeated attempts to produce zygospores from mixed sporangial transfers demonstrated the fact, however, that while mixed sporangial transfers taken from the edge of a zygosporic culture were unsuccessful the reverse was true if similar transfers were taken from the low growth in the centre of such a culture.

Resolution of (+) *and* (—) *Strains.*

In order to discover whether the zygosporic activity could be transmitted through the mycelium arising from any single spore, a separation culture was made of a mixed sporangial transfer from this favorable region in the centre of a culture which was producing zygospores in abundance. This culture was made in three Petri dishes containing a corresponding number of dilutions of the original transfer; and on May 2 (1903) an examination of the dish containing the last dilution in which but five colonies were present, showed the presence of certain remarkable conditions which, by furnishing the first suggestion in regard to what appear to be the fundamental causes affecting zygospore production, have formed a basis for the greater portion of the present investigation.

The conditions referred to are graphically illustrated in the accompanying diagram, in which the crosses indicate the centres of the mycelial colonies which had arisen from five single spores. At the regions of contact between certain of these colonies a luxuriant formation of zygospores was observed, as is indicated by continuous lines in the diagram, while between others in the areas indicated by dots no zygospores were formed. It was soon demonstrated that in every case the branches between which the zygospores were borne could be traced to the individual mycelia on either side of these zygosporic lines just mentioned. The apparent explanation of this unexpected phenomenon was that in this species, or at least in the cultures from which this separation was made, the zygospores are developed through the interaction of individual mycelia which are different in nature.



To determine whether there were more than two distinct strains of these different mycelia, other separations were made from various zygosporic cultures then in the laboratory, and pure cultures up to eighteen in number were secured from the opposite sides of the several lines of zygospores which resulted. By testing these nine pairs of pure cultures, it was then determined that the mycelium derived from either member of these pairs would take part in the formation of zygospores when grown in contact with the mycelium of a certain member of any other pair. By contrasting all the cultures with one another, it was further found possible to arrange them all in two series in such a manner that those in the opposite series would produce a line of zygospores when grown together, while those in the same series would produce no such line. It may be added that since this time zygospores of this species have been procured from a number of different sources and when the strains producing them have been separated they have always arranged themselves in one or the other of the two series to which, as has already been explained in the Introduction, the terms (+) and (—) have been applied.

The appearence of the zygosporic lines which result when the mycelia of (+) and (—) strains of this species are allowed to grow in contact is illustrated in the culture photographed (Plate IV, Figure 55), where inoculations of the two opposite strains were made as indicated by the (+) and (—) signs. Between the mycelia distinguished by unlike signs a black line marks the position of zygospore formation. The comparatively meagre development of zygospores in the middle of this culture is caused by the less depth of nutrient at this place due to a convexity in the bottom of the Petri dish used for the culture.

The presumption that the formation of zygospores is a truly sexual process, and that in the two strains of this species we have represented the two sexes is substantiated by the results obtained by hybridization to be discussed in a later section. The fact that in this and certain other species the sexes are separated in different thalli led to the use of the term heterothallic to distinguish them from the homothallic forms in which it must be assumed that both sexes are associated in the same thallus.

Morphology and Physiology of Conjugation.

Mucor Mucedo, from the fact that its separate strains have been longest under cultivation, and from the comparative ease with which its growth can be directly observed, has furnished most of the information which has been obtained in connection with a somewhat detailed examination of the process of conjugation. Many points of interest, however, remain untouched, a further examination of which has been deferred for the present.

Cell and slide cultures, which are so advantageous for other forms, are unfavorable to the production of zygospores. In the majority of the observations made, the (+) and (−) strains were contrasted on nutrient agar in Petri dishes, and their growth was examined with the Zeiss A objective. The thin sheets of glass which have been used as covers need to be frequently removed and cleared from the moisture which condenses on their under side from the evaporation of the substratum. It is for this reason perhaps that the best results have been obtained during damp weather, when the drying effect of such an exposure to the air of the room has been lessened.

In the species under consideration, no difference in the growth of the mycelia of the two strains has been observed which can be used to distinguish them. From both arise scattered more or less branched aerial filaments which from the very first are slender and apparently remain sterile, being readily distinguished from the comparatively stout hyphae destined to become sporangiophores, by their more delicate habit, and by the fact that they are not heliotropic. An exaggerated production of these filaments which may occur on certain nutrients often gives a felted appearance to the mycelial growth. Erect filaments of this nature occur throughout the whole mycelial area, but where the mycelia of the opposite strains come together hyphae are produced which, while they are intermediate in size between the sterile filaments just mentioned and the stout young sporangiophores, agree with the former in not being heliotropic.

Direct observation of these hyphae seems to indicate that a mutual attraction, which may be termed *zygotactic*, is exercised between the zygophoric hyphae belonging to opposite mycelia, and they may be seen gradually to approach each other, but in the minority of cases is their contact exactly terminal (Plate II, Figure 30c). In some instances the point of contact may be slightly back of their tips (Figure 32d), or one zygophore may be laterally met by the end of the other (Figures 30b and 33), and in other cases it may even happen that both zygophores touch laterally. At the point where the opposite zygophores come in contact, a swollen progamete is rapidly developed from each hypha, and, according to the position in which the zygophores have met, appears either lateral or terminal. The progametes, however, in all cases are, from the very first, mutually adherent, and by their enlargement push apart the zygophoric hyphae from which they have originated. These latter do not seem to undergo any alteration other than a slight curvature in some instances, occasioned by the lengthening of the progametes, and further function merely as tubes for the transmission, by way of the suspensors, of nourishment used in the formation of the zygospores.

As has been already mentioned in the Introduction, the account current in text-books states that club-shaped outgrowths (progametes) arise from adjacent sides of hyphae not already in contact and by mutual attraction come to meet at their swollen extremities. In no form which the writer has under cultivation have the gametes been found to be cut off from portions of the zygophoric hyphae which were present before they became approximated. It is only from those structures (progametes), which have developed from the zygophores as a result of their contact, that gametes are delimited. The condition has been correctly interpreted by Falck ('01) from an observation of young stages, and the erroneous opinion just referred to is no doubt due to the difficulty usually associated with any attempt to follow the process by direct observation. Investigators have in general been dependent on a comparison of young stages as a means of reconstructing the process, and have regarded as typical the abnormal conditions frequently encountered in which the apposed progametes have either been torn apart in the dissection of the preparation examined, or, in cases where the zygospores are arranged in a scalariform fashion, have been separated by the more rapid development of a zygospore between the same hyphae.

After the progametes have attained a considerable size there are formed in turn, generally not at the same time, septa which cut off more or less equal terminal cells — the gametes (Figure 29a) — and by the subsequent dissolution of the intervening wall, which proceeds from the centre outwards (Figure 34), an open communication between the contents of the two cells is afforded. The zygote thus formed by an increase in size, a rounding out of its contour, a darkening and denticulation of its outer wall and the formation of a thick layered hyaline one within, assumes the character of the mature zygospore (Figure 35).

In the main the description just given will probably apply equally well to the majority of the heterothallic forms. *M. Mucedo* is peculiar, however, in that its zygophores are sharply differentiated from the sporangiophores and rarely if ever bear sporangia. They are seldom even branched, and a scalariform arrangement of the zygospores, which is characteristic of certain members of the genus Mucor, has never been observed.

The stages in the process of conjugation may be illustrated by the series shown on Plate II, Figures 25 to 32, which consist of camera drawings taken at the intervals noted from a Petri dish culture in the region where a zygosporic line was commencing to form. In the conjugations (a) and (c) the zygophores met terminally; in (b) a zygophoric branch arose from the (—) mycelium to come in contact with the side of a (+) zygophore which had grown beyond it; and in (d) and (e), the early stages for which are not represented, the ends of the zygophores, when they had grown slightly past each other, were attracted laterally so as to meet somewhat behind their terminations. Two minutes before contact occurred at (d) and while the hyphae were separated by a distance equal to about a third of their width, very slight protrusions were observed on the sides mutually facing (Figure 31), seemingly as if the forces which were drawing the filaments laterally had effected a bulging of the delicate walls at the growing points. Figure 33 is a horizontal view taken from a young zygosporic line, and demonstrates even more strikingly than the vertical view already described, the mutual attraction which the zygophores exert.

The stimulus for the formation of the progametes is thus very evidently the contact between the sexually opposite zygophores, but what is the nature of the stimuli which result in the formation of zygophores and subsequently cause them to approach one another is by no means clear. It has been demonstrated that zygospores would be produced in the case of Rhizopus (p. 270) when only the aerial hyphae of opposite strains were allowed to grow in contact, and therefore that the stimuli to the origin or direction of growth of the conjugative hyphae were communicated solely through the air. From the habit of the species a similar experiment with *M. Mucedo* is much more difficult, and the results which have as yet been obtained are inconclusive.

External Conditions.

No systematic attempt has been made to determine the effects of varying external conditions on the formation of zygospores in this species. In the numerous cultures which have been made, however, it has been observed in general that, as with the other forms investigated, unfavorable conditions check the production of zygospores before that of sporangia. Thus in cell and slide cultures, perhaps because of the thinness of the nutrient layer, and in cultures affected with bacteria, zygospores are not copiously developed. By growing the strains opposed at a temperature of 26° 28° C., it is also possible to prevent conjugation entirely. Moreover, if potato agar, upon which zygospores readily form, be acidulated with orange juice and used as the substratum, no conjugations can be obtained. In general it may be said that in this, as in other cases, various external conditions have a secondary and variable effect in influencing zygospore formation.

Serial Cultures.

In investigating the persistence of the (+) and (—) characters in the individual strains, the writer has begun a number of serial cultures, and while they are for the most part incomplete and far from satisfactory, the results already obtained are of sufficient interest, it is believed, to warrant a presentation of them even in their present fragmentary condition. In describing the series which were carried on as pure cultures in test-tubes, it will be convenient to represent the series by letters of the alphabet, and the individual cultures in each by the letter of the series followed by the sign of its sexual strain and a numerical subscript indicating the generation which the culture represents.

It has been found that zygospores do not form in the warm oven at 26°–28° C., and in order to test the effect of this unfavorable temperature on the (+) and (—) strains a warm oven series was started February 20, in the tubes $A(+)_1$, and $A(-)_1$. On March 21, the (+) and (—) strains from this series were mutually contrasted. Between $A(+)_2$ and $A(-)_2$, a normal zygosporic line was developed; between $A(+)_3$ and $A(-)_3$, a few zygospores only were found in the centre of their line of contact; while from the contrast between the remaining strains up to A_8, the last generations, no zygospores resulted. It was uncertain from this experiment whether this loss in the power of conjugation, which had been evidently induced by the unfavorable temperature, was equally shared by the two opposite strains, and in order to decide the question contrasts were made from all the (+) and (—) tubes in the series against normally active (—) and (+) strains. Transfers from tubes $A(+)_2$ to $A(+)_5$ were able to take part in the formation of a decreasing number of zygospores, but contrasts against transfers from tubes $A(+)_6$ to $A(+)_8$ produced no zygospores. With transfers made from the (—) tubes it was possible to obtain zygospores only in the case of tube $A(-)_2$. Moreover, when this latter tube was contrasted between $A(+)_3$ and $A(+)_4$, poorly developed white tufted lines occurred at the areas of contact and were found to be made up of arrested stages of conjugation suggestive of the imperfect hybrids discussed in a later section. The line on the side of $A(+)_4$ was slightly better developed and showed its superiority over the other in possessing a single perfect zygospore. A similar contact of $A(+)_2$ between $A(-)_3$ and $A(-)_4$ resulted in neither zygospores nor arrested stages of development and showed, as the contrasts with the normal strains had also done, that these two later generations of the (—) strain had lost their sexual character before those of the (+) strain. It is apparent that the final disappearance of the (—) characters occurred at or before the time of sporangial formation in tube $A(-)_3$, while the disappearance of the (+) characters occurred at or before their formation in tube $A(+)_6$, or in other words the attenuation of the (—) strain took place three generations before that of the (+) strain under like conditions of cultivation. It remains to be seen whether, along with the disappearance of their power of conjugation the (+) and (—) characters of the two strains have been entirely lost beyond the possibility of being regained by a continuation of the series under more favorable conditions.

The temperature of about 80° F. at which the above series was conducted is not at all unusual during summer months, and an attenuation of a double series of (+) and (—) strains which was carried on during the spring and autumn may be accounted for by the unfavorable effects which the warm weather occurring at that time might have exercised on their growths. That the mere number of sporangial generations, however, through which the strains are conducted has little influence on their sexual power is shown by the unchanged sexual nature in a double sporangial series of the (+) and (—) strains carried on during the winter, the one to the tenth, the other to the eighteenth generations.

In order to discover to what extent the growth of the fungus could be continued by mycelial transfers alone, and to observe the effect of such treatment on the (+) and (—) strains, transfers were made, December 11, into tubes $B(+)_1$ and $B(-)_1$ and formed the starting points for a mycelial series of tube cultures such as was begun on the same date for Rhizopus and has already been described (p. 265). Each morning a piece of the nutrient agar with the mycelium adherent was transferred to a new tube, and the series was continued in this manner, with no perceptible difference in the growths of the two strains, until about the twentieth generation, when, while the (+) mycelium remained normal, the activity of the (—) appeared to be checked, and, on January 5, the mycelium of $B(-)_{25}$ had developed scarcely sufficient growth to enable a transfer to be made, and for several days thereafter underwent but little increase. Eventually, however, this tube produced sporangia, and when tested a month later showed by forming zygospores in contact with a (+) strain that its sexual character had not disappeared.

The growth of the (—) series was renewed by taking a mycelial transfer from tube $B(-)_{23}$ from the upper part of the tube where a mycelium had evidently been developed from fallen sporangia, and for some time the mycelial transfers grew vigorously. While, however, the (+) tubes remained unaffected the growth in the (—) tubes gradually became again weaker until February 15, when the mycelium in tube B(—)$_{61}$ was almost at a standstill and did not develop sufficiently for a transfer until two days later. The (—) series was then continued on potato agar to which four per cent grape sugar had been added, but the increased nutriment was not a sufficient stimulus to carry the growth well beyond the sixty-fourth generation, when, on February 19, the series was discontinued. The (+) series had then reached the seventy-second generation, and to all appearances had been unchanged either in vegetative or sexual vigor. On the other hand it was ten days before tube B(—)$_{64}$ had developed sufficiently for the formation of sporangia and transfers, made from its spores, in contact with (+) inoculations produced but a feeble mycelium which was unable to take part in the production of zygospores. The same was true of a similar contrast made March 8, but when the experiment was repeated March 13, the growth was better, and a line of zygospores, though somewhat scanty, resulted. From an earlier sporangial culture made from the tube B(—)$_{64}$ zygospores could be obtained by contrast against (+) strains, and it is possible that some of the spores of the March 13th transfer were derived from a mycelium which had developed as a result of a resowing of the tube by the first stunted sporangia formed.

The experiment demonstrates that the continuous cultivation of the mycelium, in so far as conducted, has no apparent influence on the vegetative vigor nor on the sexual capacity of the (+) strain, while it decreases to a marked degree the vegetative vigor, and, as a consequence, the sexual capacity of the (—) strain. The sexual character is not, however, lost, but is merely unable to exert itself when the vital force in the mycelium is thus reduced. The series further offers another example of the general law that unfavorable conditions inhibit the formation of zygospores before that of sporangia.

Germination of Zygospores.

Mucor Mucedo is the only heterothallic species the zygospores of which the writer has succeeded in germinating. In April, 1903, a number of washed zygospores which had been taken from a tube culture of the previous February were sown in a van Tieghem cell. When they were for the last time inspected in the following June, no apparent change had taken place in them, but in October, when the cell was re-examined, it was found that zygospores had germinated and pushed their sporangiophores down into the water below, but no growth from any sporangial spores was observed. Transfers from this culture were made into three agar tubes, but in only one of them was there any mycelial growth, and in this single case it was accompanied by bacteria probably derived from the water in the bottom of the van Tieghem cell. By a single sporangial transfer a pure tube culture, however, was obtained from this infected growth, and tests to determine its thallic condition were made during October with five (+) and six (—) cultures, but with negative results. Subsequent tests were made in January of this same tube and of later members of a series of which this tube was the first. All were decidedly (—) in character except the most recent cultures in the series, which appeared to be neutral.

In so far as can be ascertained from the behavior of the cultures in a single experiment of this nature, one may infer that the zygospores from which the series was derived had, in germinating, produced spores, in the mycelium arising from which, the sexual character was dormant, but that the capacity for conjugation became evident after a certain lapse of time. The experiment is unsatisfactory in that the origin of the cultures cannot be traced with absolute certainty to an individual zygospore, although there is not much doubt that they were thus derived. A determination of the time at which the segregation of the sexual characters occurs in strains derived from zygospore germinations, and the nuclear condition associated with it, is a matter of considerable interest. The writer hopes soon to be able to present more satisfactory evidence bearing on the question, and until a greater body of facts is known it will not be prudent to attempt any general discussion of the subject.

Differentiation between the (+) *and* (—) *Strains.*

The (+) and (—) strains of this species offer no morphological differences by which as yet they can be distinguished. In the conjugative apparatus certain differences may exist in the size and time of appearance of the gametes, in the size of the mature suspensors, or in the length of the zygophoric branches, but none of these characters have been found to be correlated with the sexual differentiation of the strains. The shape and size of their spores and their gross appearance in pure cultures are essentially alike in both instances. Although no morphological difference has been determined, yet, as has been shown by the more rapid attenuation of the (—) strain when the two were grown at an unfavorable temperature and by the cessation of growth in the same strain in the mycelial series when the (+) was unaffected, there is a difference in vigor between the two opposite strains which, as is shown in the section on hybridization, is correlated with their sexual differentiation.

PHYCOMYCES NITENS.

The zygospores of this heterothallic species are extremely rare, having been reported, in so far as is known, by but two observers, although the sporangial condition is not uncommon in spontaneous cultures, especially on horse dung. After the secret of zygospore formation was discovered in *M. Mucedo*, the writer endeavored to apply the principle involved to obtaining zygospores of other forms then at his disposal, and contrasts were therefore made between the different cultures of a number of species then growing in the laboratory. By a fortunate chance, the first two cultures contrasted of Phycomyces contained the opposite strains which exist in this species, and consequently a black line of its huge thorny zygospores was produced at the region of contact of the two mycelia.

The zygospores are peculiar, not only on account of the dichotomously branched outgrowths, which arise from either suspensor, but also from the method in which the progametes are developed. Van Tieghem and Bainier, the two investigators who have done the most work on the group here considered and the only ones who have described the zygospores of Phycomyces, have figured stages, showing both suspensors in connection with the same mycelial hypha. Though such an account is incorrect, the condition seemed, *a priori*, not improbable at the time the observations were made, and the fact that the lower portions of the progametes are intricately convoluted and closely adherent renders a determination in every case of their basal connections a task of unusual difficulty, especially when one is dependent on stages obtained from spontaneous cultures. With the two sexual strains in hand, however, it has been comparatively easy to control the production of zygospores and to follow the main steps in the process of their development.

The stages figured in Plate III, Figures 45 to 50, were taken from van Tieghem cell cultures, from the regions of contact of the (+) and (—) strains. In these cultures, no doubt because of the small amount of nutriment afforded in the thin layer of agar used, zygospores have not been formed in abundance, but their development has been none the less easily observed.

When a (+) mycelial hypha is met by one from the (—) mycelium, stout much-lobed branches arise from both at their places of meeting, and become adherent with interlocking of their lobes (Figures 45 to 47), and from the development of two branches thus in contact the progametes assume their ultimate form. In *M. Mucedo* and in the majority of species in which the process is known, the progametes are in contact only at their extremities. In Phycomyces, however, they grow for a considerable distance more or less intimately adherent with interdigitation of their convoluted lobes, and at this stage form stout yellowish pillars, projecting from the substratum (Figure 48). By the more rapid elongation of the subterminal portion, a separation of the individual components finally occurs below their extremities (Figure 49), but, as is the case in all other forms in which the process has been carefully followed, the ends of the progametes are in contact from the very outset, and never become separate at any stage of their development. The production of the two gametes (Figure 50), and their union to form a zygote, which, by the absorption of nourishment through the suspensors, assumes the form of the mature zygospore, is essentially the same as has been already described in *M. Mucedo.* At about the time of the dissolution of the wall between the uniting cells, outgrowths arise from one or both of the suspensors near their junction with the gametes (Figure 50), and, by their elongation and repeated dichotomy, surround the mature zygospore with a loose envelope of forked spines. Septation in the suspensors or in the afferent hyphae may occur as the zygospore matures, and is also very generally found in arrested stages of development (Figure 49).

The adherence of the basal portions of the progametes is probably due to the fact that the mycelial hyphae are firmly imbedded in the substratum, and their separation by the growth of their gametes is therefore prevented. The presence of a curvature in the progamete of Spinellus when their points of origin are closely adjacent, renders it possible that such a curvature in Phycomyces is also a matter of no great significance and connected merely with the fixed position of the conjugative hyphae. The condition is frequently more simple than is shown in Figure 48, and cases have been observed in which the bases of the suspensors are simply crossed with little or no development of interlocking lobes.

Under the somewhat unfavorable conditions which exist in cell cultures, progametes are not produced at the places of contact between all of the sexually opposite hyphae of the two mycelia, and apparently the quantity of available nutriment has much influence in determining whether a single system of mycelial branches will take part in the formation of more than one zygospore.

The continued growth and branching of the apposed (+) and (−) mycelia would ultimately bring sexually opposite hyphae into contact.

Whether they are also zygotactic, with a mutual attraction influencing their direction of growth, as in *M. Mucedo*, is at present uncertain.

Van Tieghem ('73) was of the opinion that a difference, which is usually apparent in the time of origin of the outgrowths from opposite suspensors, indicates a sexual differentiation. It has been found, however, that this priority in the development of the spines is in some cases on the (+), and in others on the (—) side, and is probably, therefore, associated with a purely accidental inequality in the amount of nutriment received from the afferent hyphae.

Although no differences in structure have been observed, the character of growth of the (+) and (—) strains is distinctly unlike, and it is therefore possible to distinguish them when they are grown side by side. The mycelium in the (—) strain is slightly less vigorous, and the sporangiophores which arise from it are fewer and later in appearance (Plate IV, Figure 52). Moreover, while there is a great variation in the size of the sporangia and sporangiophores derived from the same strain, yet in the (—) the more delicate at first predominate, although in a later period of development no great difference may be discernible between the two strains. On certain nutriments the (—) strain is further distinguishable by an abundant production of sterile filaments from its mycelium, giving to the growth a characteristic white felted appearance.

The effect of external conditions on the activity of conjugation in this species has not been carefully investigated. Zygospores have been secured on all the substrata tested, which fact disproves the generally accepted opinion that oily material is necessary for their production. On nutrients with such low concentrations as are available in potato agar, zygospores are not numerous, but if the concentration is rendered higher by the addition of 4 per cent grape sugar, their formation is considerably increased. They have been also obtained on flour paste, milk agar, and potato agar acidulated with orange juice.

In the warm oven at 26°–28° C., zygospores are produced much less abundantly than at the room temperature.

Neither the apparent inequality in vegetative vigor between the (+) and (—) strains, nor their sexual capacity has been affected by cultivation through twenty sporangial generations.

Since the zygospores of this species were first synthetically obtained, transfers have several times been secured of Phycomyces which has appeared in dung cultures from different sources. Of these strains, two have shown a vigorous (+) character, two a weak (+) character, and one is apparently neutral.

ABSIDIA CAERULEA Bainier.

Mucor Saccardoi Oudemans.
Proabsidia Saccardoi (Oud.) Vuillemin.

This heterothallic species is decidedly common in occurrence, but has been reported in so far as is known by but two observers. Although Bainier's description is given in Saccardo's Sylloge, Vol. IX this fact seems to have escaped the attention of later writers, and one finds no mention of it, for example, in the recent paper of Vuillemin ('03^b), in which the genus is considered in some detail and subjected to further subdivision.

As has already been mentioned under the Citations in Part I, the sporangial condition of this species has been frequently found in this country. The first appearance of its zygospores in the laboratory, however, occurred on an old culture of rabbit dung which the writer had collected near Mt. Ktaadn, Me., in the summer of 1902. Less than half a dozen zygospores were found, but by making gross transfers from the region of their formation, their production was increased. Separation cultures, however, made in May, 1903, and contrasts between twelve pure sporangial transfers failed to give the two sexual strains.

In the following fall an attempt was again made to resolve the species into its (+) and (—) strains by obtaining germinations of the hyphae connected with the two suspensors, but the brittle zygosporic branches would not continue their growth when dissected out sufficiently to allow their connections to be followed. By making thirty-eight contrasts between pure sporangial transfers from cultures in which the formation of zygospores was very abundant, four zygosporic lines were finally obtained, and, by transfers from either side of these, the two strains were secured. Later tests have demonstrated that thirty-four of the thirty-eight inoculations contrasted were (—) in character.

The formation of zygospores in this species seems largely independent of external conditions, and zygospores have been obtained on all the substrata tested. In the warm oven at 26°–28° C., the rapidity of growth is increased and zygospores may be secured a day earlier than at the room temperature.

No marked differences have been observed in morphological characters nor in habit of growth between the (+) and (—) strains, although they have been carried in separate tube cultures to the thirteenth generation. It frequently happens that the circinate outgrowths arise from one suspensor before they do from the other, or those on one side may even entirely fail to develop. Although the matter has not been investigated, it is probable that this peculiarity, as in Phycomyces, has no connection with the sexual differences between the two strains.

Whether there is a mutual attraction between the hyphae from which, at their points of contact, progametes are developed, is at present uncertain. That such an assumption is not necessary, however, to account for the facts observed, is shown by the results of an experiment similar to that already described under Rhizopus (p. 268). In this instance bags of nutriment were freely suspended so that their opposite faces were 3 cm. apart and were inoculated respectively with (+) and (—) strains of the Absidia. Zygospores formed in abundance where the hyphae from the opposite strains met in the air space between the suspended masses of nutriment and thereby showed that, if any orienting of the conjugative hyphae occurs, the directive influence must lie outside the substratum and in the hyphae immediately affected.

Sufficient evidence is not at present available to determine the relative abundance in nature of the (+), (—), and neutral strains. Of three additional strains of this species derived from different sources, however, one is weakly (—) in character, and two are apparently neutral.

MUCOR N.

This heterothallic species, which is the type of a new genus, was found by Professor Thaxter together with its zygospores on a laboratory culture and kindly given the writer for experimentation.

External factors have more influence on zygospore formation in this species than in most of the other heterothallic forms under cultivation. At the room temperature under rather dry conditions, a zygosporic line is slow and feeble in development. If, however, the cultures are conducted in a nearly saturated atmosphere, zygospores are more abundant, while, in the warm oven at 26°–28° C., the line is decidedly thicker and earlier in appearance. Concentrated nutrients, in so far as they have been tried, fail to yield zygospores.

When contrasted, the (+) and (—) strains show a striking difference in habit which readily enables one to distinguish them. The growth of the (—) strain is lower and almost white, while that of the (+) strain is higher and dirty yellow in color. These differences between the (+) and (—) strains as well as their sexual character have remained unchanged through sixteen sporangial generations.

Mucor III.

The production of the zygospores of this undescribed Mucor, which apparently belongs to the *M. racemosus* group, has been continued under cultivation probably longer than those of any other heterothallic form, with the exception of the "Harvard strain" of Rhizopus.

The zygospores of this species were first found in a test-tube culture infected by bacteria, which had been made from an impure transfer from rat dung in an attempt to obtain an inconspicuous Mucor which was concealed below the growth of what appeared to be a larger form. Since it seemed not impossible that the production of zygospores might in some way be connected with a change in the character of the substratum induced by the accompanying growth of bacteria, tests were made by starting cultures from sporangial transfers to which bacteria were added. The results of these cultures were, however, negative, since no zygospores appeared. The original mixed culture was eventually freed from bacteria by successive mycelial transfers to solid media, and continued to produce zygospores whenever masses of mycelia were transferred from the zygosporic region.

Similar experiments as to external conditions were carried on for Mucor III, which have been already described for Rhizopus, but the zygospores were more uncertain in their occurrence. In general, nutrients with higher concentrations seem to support a greater abundance of zygospores than those with lower concentrations. Few substances have been tested, but flour paste, plain bread and bread soaked with dilute prune decoction, potato agar and potato agar plus 4 per cent grape sugar, have all been successfully used as substrata for the production of zygospores. On plain agar, horse dung agar, and potato agar acidulated with orange juice, however, it has not been possible to obtain zygospores.

When the cultures are grown in the warm oven at 26°–28° C., the formation of zygospores appears to be more abundant than when the same substratum is used at the temperature of the laboratory.

This species was finally resolved into its two sexual strains by making separation cultures of zygosporic material which had been preserved in a dried condition on bread. Five paired colonies were obtained separated by zygosporic lines, and in all cases there was a more or less marked difference apparent in habit of growth between members of each pair. Later tests have shown that, in every instance, the cultures with the lower development of sporangia have belonged to the strain which has been called (—). The separate (+) and (—) strains have now been carried to fourteen sporangial generations, and neither the vegetative difference between them nor their individual sexual character has shown any alteration.

Plate IV, Figure 58, is a photograph from an agar culture in which the two opposed strains were in the process of forming zygospores at their line of contact. The growth of the (+) strain on the left is much taller, looser, and lighter in general appearance than the other, while that of the (—) on the right is low and dark. Although, when the two strains are grown side by side, they are always distinguishable by their vegetative appearance, the difference is not equally marked under all conditions. A microscopic examination shows that this diversity in aspect between cultures of the two strains is correlated with a corresponding and striking difference in size of the vegetative reproductive bodies, the smaller spores being formed by the (—) strain. Plate I, Figures 23 and 24, are camera drawings representing the normal variations in size and shape of the spores taken from (+) and (—) potato agar tubes which had been kept under the same conditions of cultivation. It is here seen that the spores produced by the low growth of the (—) strain are smaller than those produced by the more luxuriant growth of the (+) strain. The difference in vegetative luxuriance between the (+) and (—) strains of Mucor III is greater than between those of any other form whose strains have yet been separated, and coupled with the difference in spore characters would almost certainly have been considered by systematists as of specific value.

MUCOR IV.

The zygospores of this heterothallic species were found in the fall of 1902 in an old tube culture of potato agar into which the previous summer at Cold Spring Harbor, L. I., the writer had made a transfer of one of the Cephalideae together with a weft of Mucor hyphae to furnish a host for its development.

Separation cultures on potato agar, in the attempt to resolve this species into its sexual strains, failed to produce zygospores in any of the dilutions. The (+) and (—) strains were finally separated, however, by making thirteen contrasts on horse dung agar, from pure sporangial transfers taken from a gross dung culture in which zygospores were abundant. Inoculations taken from either side of the single scanty line of zygospores that resulted formed the starting points for a sporangial series which has already been carried to the fourteenth generations without having suffered any apparent change in their sexual character. No essential differences have been observed between the (+) and (—) strains aside from that shown in their capacity for conjugation.

At the temperature of the laboratory, zygospores do not form on the artificial substrata usually employed as culture media. In the warm oven, however, at a temperature of 26°–28° C., zygospores can be obtained on potato agar the concentration of which has been increased by the addition of 4 per cent grape sugar, but on the same substratum at the room temperature zygospores are not produced. In so far as experiments have been tried, therefore, the optimum of temperature and of concentration is higher for Mucor IV than for the other heterothallic forms under cultivation.

MUCOR V.

Zygospores of this heterothallic species were first found in a small patch on a spontaneous dung culture in the laboratory, and by separation cultures and pure sporangial transfers it was eventually resolved, March 7, into its (+) and (—) strains. Mucor V seems to be sexually the most vigorous heterothallic form that the writer has under cultivation, and no substrata have been tried on which it will not form its zygospores in abundance when growths from the (+) and (—) strains are apposed.

In prepairing the culture photographed (Plate IV, Figure 53), inoculations of the two opposite strains were made at the places indicated by the (+) and (—) signs. The zygophoric hyphae seem strongly zygotactic, and, recurving downward where the (+) and (—) growths come in contact, form a furrowed line of crowded zygospores. The zygophores are of a more or less indefinite growth, and the zygosporic region is soon extended on either side of the original areas of contact (Figures 53 and 57) in curved lines of light brown zygospores which ultimately cover the whole surface of the culture as the two strains become completely intermingled. White lines are observable in the photographs at the region of contact between the opposite strains. They are always present shortly after the first zygospores have been formed, and consist of zygospores in arrested stages of development.

A detailed investigation of the process of conjugation in this species has not been attempted, but essentially the same condition prevails as has been already described in *Mucor Mucedo*. Plate III, Figures 40–44, are camera drawings taken at the intervals noted from a van Tieghem cell culture which had been inoculated with a mixture of spores from the (+) and (−) strains. The zygophores which gave rise to the progametes shown in Figure 40 a were observed, about an hour before the drawings were commenced, to be in the same relative position to each other as those figured in Figure 40 b. In both conjugations (a) and (b) the zygophoric hyphae met nearly terminally, while the free hypha at (c) grew into contact with its complementary zygophore from which three progametes had already been developed. In Mucor v, a continuation of the growth of the zygophores after the first contact is apparently the rule, and a consequent scalariform arrangement of the zygospores very frequently occurs (Figure 44 b).

The most noticeable vegetative difference between the (+) and (−) strains lies in the time of appearance of the sporangia. When the culture photographed (Plate IV, Figure 53) was two days old, sporangia had appeared in the growths from all the (+) inoculations, but even an examination under the low power of the microscope failed to show any from the (−) growths, and it was not until the day following that they were observed.

MUCOR VI.

The zygospores of this heterothallic species were first observed in February last in a separation culture of a Mucor from a laboratory dung culture, and the zygospores have been subsequently found on a later spontaneous culture.

Except in point of size, the individual zygospores resemble those of *M. Mucedo.* The zygophoric hyphae are commonly branched, longer, and less distinguishable from the sporangiophores. They are strongly zygotactic, and the zygophores which developed in the region of contact between the (+) strains in the culture photographed (Plate IV, Figure 54) showed from the centre of these neutral lines where the filaments were upright, an increasing curvature toward the (−) growth, separated from them by the lines of young zygospores.

The photograph fails to show the greater height of the (+) growth, which is often distinctly noticeable under certain conditions of cultivation, but the difference between the two strains is never so marked as in the case of Mucor III.

SUMMARY OF HETEROTHALLIC FORMS.

In the above nine species, which represent five distinct genera of the Mucorineae, it has been shown that the formation of zygospores in all cases results only through the interaction of two differing unisexual mycelia which have been termed (+) and (−).

In the majority of the forms, the (+) strain is characterized, in comparison with the (—), by a greater vegetative luxuriance. In Mucor III the difference is most distinct, being markedly shown in the size of the spores and in the height of the sporangial growth; in Mucor N the difference is in color as well as in the height of the sporangiophores; in Mucor V, the sporangia from the (—) strain are produced later; in *M. Mucedo*, the difference is discernible only through cultivation under unfavorable conditions; while in some others, as Rhizopus, no differences in vegetative characters have yet been observed.

In *Mucor Mucedo* the sexual strains have been rendered neutral in their action by cultivation under unfavorable conditions, and in Phycomyces, Absidia, and Rhizopus, strains have been found which from the first have shown themselves neutral in character.

External conditions have only a secondary influence on the formation of zygospores and affect the various species differently. Thus, for example, while a temperature of 26°–28° C. favors zygospore production in Mucor III and Mucor N, it entirely prevents their production in *Mucor Mucedo*, and, although a comparatively high concentration seems to be necessary for the formation of the zygospores of Mucor IV, it is detrimental to the production of those of Mucor N.

In all the forms the stimulus for the development of the progametes is the contact between sexually opposite hyphae. In *Mucor Mucedo* these arise from the mycelium as differentiated zygophores, and seldom or never bear sporangia; in Absidia and Rhizopus there is no evident differentiation into zygophoric hyphae, and apparently any of the undifferentiated aerial hyphae may take part in the formation of zygospores; and finally in Phycomyces the progametes arise from the contact of branches of the vegetative mycelium. In *M. Mucedo* and others of the genus Mucor, a mutual attraction between the zygophores is demonstrable, but in some forms, as Rhizopus, it has not been determined.

HOMOTHALLIC FORMS.

As has been stated in the Introduction, those species have been called homothallic the zygospores of which originate from a single mycelium in distinction to the heterothallic species, the zygospores of which are formed by a union of gametes which have originated necessarily from two different mycelia. It has been currently assumed that all species belong to the first class, and consequently the distinction above given has not been hitherto recognized. An enumeration, however, of those forms in which the thallic condition is known or strongly suspected will show that the homothallic group undoubtedly comprises a very small minority of the species in which zygospores have ever been found. These forms may be recognized by the fact that they can be induced to develop zygospores constantly on the same substrata when grown from a single spore, and, moreover, by the fact that through the zygophoric hyphae the opposed progametes may be traced to the same branches of the mycelium.

The members of the homothallic group may be conveniently divided into two subsections according to the presence or absence of a constant morphological difference in the gametes and zygophoric branches. The first division comprises Sporodinia, Spinellus, and Mucors I and II, and the second comprises Zygorhynchus and Dicranophora, both of which have been classed together as heterogamic on account of the morphological differentiation in their gametes. The homothallic forms will be considered in the order just given.

SPORODINIA GRANDIS.

Inasmuch as *Sporodinia grandis* is by far the most abundant and widely distributed homothallic form among the Mucorineae, and therefore the only one from which earlier workers could be sure of obtaining zygospores, it has been the subject of considerable investigation, as may be seen by reference to the literature cited under this species (p. 231). By Klebs ('98) the fungus could be made to produce zygospores on plain bread if the relative humidity in the surrounding air was sufficiently high. For Falck ('01), zygospores would form only when the concentration of the nutrient was sufficiently increased by the addition of certain solutions to the bread, and the relative humidity of the air seemed to be of slight importance.

Sporodinia is very common in this country on a great variety of Basidiomycetes of various groups. Dr. Farlow has been accustomed to use in his classes the zygospores of this species, and has sometimes obtained them from sowings on plain bread; and from all of several different collections the writer has been able to obtain an almost exclusive production of zygospores on the same substratum. A pure culture of this species was obtained in the summer of 1902 from material collected at Cold Spring Harbor, L. I., and has been kept running since in the laboratory. It has been noticed in general that in tube cultures sporangia have been exclusively present except in freshly made tubes where there was presumably a greater humidity in the confined air and in which zygospores were also formed.

Further to test the effect of humidity on the type of fructification, a series of cultures was made from the material above mentioned under conditions which were made to correspond to those described by Falck in connection with his experiments. On March 17, discs of bread were sterilized dry in stender dishes, soaked, five of them with sterilized water and five with equal amounts of dilute prune decoction, and, after having been subjected for eight minutes to steam at 100° C., were inoculated with drops of water containing spore material. The stenders were left in a culture drawer until March 20, when a vegetative growth was first visible. One of the prune juice cultures showed young zygospores in process of formation, but in none of the other stenders had the growth advanced further than a production of the mycelium. The cultures were disposed according to the varying degrees of atmospheric moisture indicated in Table XI.

TABLE XI.

	Plain Bread.	Prune Bread.
Air very moist. Cultures open in moist chamber with caps of wet filter paper.	Only zygs.	Zygs. with tuft of sporangia.
Air normal. Cultures closed on laboratory table.	Only zygs.	Only zygs.
Air dry. Cultures open on laboratory table.	Only sporangia on surface, zygs. on sides below.	Only sporangia on surface, zygs. on sides below.
Air very dry. Cultures open in a sealed vessel containing calcium chloride.	Only sporangia.	Only sporangia on surface; zygs. and azygs. on sides below.

In general, the series demonstrates that increased relative moisture is a condition favoring formation of zygospores, while decreased relative moisture is a condition favoring the formation of sporangia. With the exception of the tuft of sporangia in the prune bread culture in very moist air, none of the cultures in normal, nor those in very moist air, produced any sporangia, while in the cultures in dry, and in very dry, air sporangia predominated. It was observed that the stenders in the sealed vessel with calcium chloride suffered but little more desiccation than those exposed on the laboratory table.

From the fact that zygospores are readily produced on plain bread and from the observed effects of relative humidity on the type of fructification produced, the material investigated agrees with that used by Klebs, and the results obtained seem to give support to the suggestion that Falck worked with a form at least physiologically somewhat different, if not unusual.

SPINELLUS FUSIGER.

Next to Sporodinia, this species seems to have been more frequently found than any other homothallic form. It has not been cultivated by the writer, but he has been enabled to examine dried material from a number of different stations, as has been mentioned under this species in the Citations in Part I. The zygospores in all cases were found to be abundant between the gills of the host, and the connections of their suspensors could frequently be traced to the same branch of the spiny mycelium.

Its homothallic character is thus determined beyond question.

MUCORS I AND II.

This section will treat of a homothallic species or of a group of perhaps three forms which are very closely related. In October, 1901, the writer made a pure transfer from a Mucor growing apparently unaccompanied by zygospores on a culture of a decaying agaric and obtained the homothallic species which has been called in the present paper Mucor I. Mucor II was found by Professor Thaxter, January, 1898, at Daytona, Florida, growing spontaneously with its zygospores on a decaying Polyporus. A third form was obtained from a separation culture made in June, 1901, by Mr. E. E. Bogue from the "schleimflüss" on a recently cut birch stump. The mycelium was mixed on the soft agar with bacteria and yeasts, and produced on aerial hyphae only zygospores which were at first mistaken for sporangia. By making a number of mycelial transfers a pure culture was secured which produced sporangia as well as zygospores, and since this time no cultures have been obtained which would yield zygospores exclusively. In most cultures of this, as of the other two forms, zygospores and sporangia are generally both present in abundance.

It is possible that the form just mentioned is identical with one or the other of the first two forms. The three if not identical are sufficiently alike for the purposes of the present paper to be treated as one, and though more cultures have been made with Mucor I the facts here presented will apply to them all. Mucors I and II are alike as far as their method of conjugation is concerned. They apparently disagree slightly, however, in their action in cultures and in their relations to temperature; and in cell cultures Mucor I, moreover, produces chlamydospores which bud in a characteristic Oedocephalum-like manner not possessed by Mucor II, but it is uncertain whether the slight differences which may exist are sufficient to separate these forms as distinct species.

The process of conjugation is similar to that determined in some detail in *Mucor Mucedo.* That there is a mutual attraction between the zygophoric hyphae is probable, and progametes which here as in all other cases always result from hyphal contacts, are frequently produced between the terminal portions of hyphae of such considerable length as to suggest that contact was brought about by the mutual orientation of the hyphae affected. In a few instances also in cell cultures hyphae have been observed growing toward each other and producing progametes at their point of contact. Not enough observations have been made, however, to demonstrate that this meeting may not be accidental. Attempts to follow the zygophoric hyphae is an uncertain task, since in cell cultures sporangial formation predominates and the production of zygospores cannot be limited to a single line, as can conveniently be done in the heterothallic forms. Moreover the zygophoric hyphae are not always readily distinguishable from the sporangiophores, and it has thus several times happened, during the course of an observation, that all the hyphae whose positions had been mapped and followed by the aid of a camera lucida have later been found to produce only sporangia. It can, however, be stated from observation that the contact of zygophoric hyphae is the stimulus for the outgrowth from them of the progametes, and that the latter never arise independently.

That the thallus of this plant is in fact bisexual has been shown by its response to both the sexual strains of a heterothallic species in the process of "hybridization" subsequently described (cf. p. 311 and Plate IV, Figure 56). No experiments have as yet been made, however, which will enable one to determine where the segregation of the sexes occurs before the formation of gametes. It is possible that any zygophoric hyphae may produce progametes when brought into contact with any other, and the segregation might then occur at the moment when the gametes were cut off. That such a process may occur is suggested by the condition represented in Plate I, Figure 19, in which the zygophoric hyphae can be seen to have arisen in such a manner that there is an open protoplasmic connection between the two progametes, and in this respect the process is in marked contrast to that in Zygorhynchus, where the more delicate zygophore is always normally distinguished at the end of a filament by a septum below which the more vigorous zygophore arises as a branch. That the zygophoric hyphae may be primarily bisexual is suggested by the fact that, in one case observed, both of the zygophoric hyphae connecting with a zygospore had grown on to produce sporangia each of which we must assume contained bisexual spores. It is possible that the segregation may be partial in the zygophoric hyphae, giving to one a predominatingly (+) character and to the other, with which it forms progametes, a predominatingly (—) character. So far as they have been investigated, however, the mycelia developed from single spores are always equivalent as regards their sexual character. Separation cultures have been made both from transfers from single sporangia and from mixed transfers from a number of sporangia, and no difference could be observed in the zygosporic production of the different mycelial "colonies" developed, nor was there a line of greater zygosporic activity where the mycelia of two colonies came in contact. It is perhaps fruitless to speculate on these questions without more data than are at present available; but the writer hopes in the near future to be able to present additional experimental evidence bearing on the subject.

Homothallism appears to be a fixed condition in the species in which it has been found. During the six years Mucor II has been under cultivation in the laboratory, it has been continued to many generations of sporangial spores without suffering any abatement in its zygosporic activity, and the same is true, though to a less extent, of the other forms of the homothallic group. The thallic character, moreover, is not lost through the production or germination of zygospores, since although in water germinating zygospores produce only sporangia, in nutrients their germination gives rise to a mycelium from which are formed both zygospores and sporangia.

Since Mucor I was the first homothallic form that the writer had found and brought under cultivation, it has been the subject of a considerable number of cultural experiments, the object of which was to discover if possible the factors influencing the formation of zygospores in this species, as Klebs had done in Sporodinia, and to apply the results obtained in an attempt to secure zygospores in other forms. In these cultural investigations nearly two hundred pure gross cultures and somewhat over fifty cell cultures were made, but the production of zygospores was so little affected by the external conditions tested in the series which were conducted, that it seems undesirable to give more than a summary of the chiefly negative results obtained. In general, it may be said that zygospores were produced together with sporangia on all the substances tested, when the growth of the fungus was not obviously impeded.

Thus on such widely different substances as egg albumen, solution of peptone on sponge, solution of grape sugar on the same substance, and horse dung, zygospores were produced together with sporangia. Concentration of the available nutrient, in so far as tested, has practically no effect in determining the kind of fructification produced. On such dilute nutrient as could be procured from tap water with pure agar, or from sponge soaked with 0.5 per cent solution of grape sugar, with 0.5 per cent solution of peptone, or even with tap water, zygospores have been formed in small numbers along with a feeble development of sporangia. It sometimes has happened in the nutrients enumerated, that a few sporangia without zygospores were produced by the poorly developed mycelium. When spores are sown in such nutrient fluids as dilute prune decoction, an active fermentation takes place with an abundant liberation of bubbles of gas. Yeast-like forms develop, and although the mycelium generally remains submerged and septate, it may at times reach the surface and produce a scanty growth of sporangia sometimes accompanied by zygospores. Banana is the only substance upon which sporangia alone constantly form, and apparently this substratum is unfavorable to the growth of the fungus, for the sporangia are low in their development. In van Tieghem cells especially, when the substratum is disposed in a thin layer or tends to become dry, sporangia alone may be formed.

The relative humidity of the air seems within normal limits to be a comparatively unimportant factor in this case, and open cultures of favorable nutrients exposed to the air of the laboratory as well as those placed in a sealed vessel with calcium chloride have invariably formed zygospores below the luxuriant sporangial growth. Although zygospores seem to be somewhat more abundant when the cultures are exposed to a relatively moist air, desiccation, in the experiments tried, has not been able to check their formation to any considerable extent.

Temperatures up to at least 29° C. hinder the production of neither sporangia nor zygospores, and the difference between diffused light and absolute darkness is also without influence in this connection. Except in the separation culture from "schleimflüss," already referred to above, no cultures have given an exclusive production of zygospores. In this instance, the soft agar used and the probable fact that the mycelium developed from a single spore may have especially favored zygospore-formation.

From the foregoing experiments it is evident that, in cases where the vegetation is interfered with, the formation of zygospores is apparently inhibited before that of the sporangia. In this connection it is interesting to note that, while in one of the forms obtained from soil by Hansen ('02) a similar relation existed between zygospore formation and unfavorable conditions at least of temperature, in another form from the same source the sporangia under similar conditions were found to be inhibited before the zygospores. In conclusion it may be said, in so far as concerns the three homothallic Mucors above discussed, that, in general, external conditions not obviously unfavorable to the growth of the fungus have no influence in determining which form of reproduction will occur

ZYGORHYNCHUS MOELLERI.

This homothallic species, as already mentioned in the Citations in Part I (p. 227), has been found a number of times, the form with which the writer has experimented having been obtained at New Haven in a separation culture from soil by Professor Sturgis. It is similar in its mode of conjugation to *Z. heterogamus*, the other member of the genus which Vuillemin has separated from Mucor on account of the constant dissimilarity in the gametes. It may not be out of place in the present paper to give a brief account of the process of conjugation observed by the writer in *Z. Moelleri*, although in the main it will be but a repetition of the detailed description already given by Vuillemin in the other species.

In Mucors I and II, as has been seen, the zygophores generally arise from comparatively distant parts of the mycelium, and although their origin may be close together on the same mycelial filament, yet so far as has been observed zygospores are never formed between branches of a single aerial hypha. In Zygorhynchus, however, the conditions are reversed, and the two zygophoric branches almost invariably not only originate from a single aerial hypha, but have a definite position on the filament, and conditions in which zygospores have been observed to form between filaments connected with distant parts of the same mycelium are comparatively rare.

In the simpler case illustrated by the more common mode of conjugation, a terminal portion of an erect hypha is distinguished by a septum from the portion below. Immediately beneath this septum is produced a branch which, growing upward, recurves to meet the side of the slender zygophoric filament cut off by the septum already mentioned (Plate I, Figure 1). The two zygophores are from the beginning unlike in character as well as in origin. While the first, which contains but a small amount of protoplasm that becomes massed at the point of contact with the other, undergoes no further development, the second, which has arisen immediately below it, is from the outset richly supplied with dense protoplasm. Immediately after contact a progamete is developed as a perpendicular outgrowth from the slender erect zygophore, and in juxtaposition to this a progamete is formed by the terminal enlargement of the more vigorous zygophoric branch (Figures 2 and 3). In each of these progametes a transverse septum is formed, distinguishing the gametes which are unequal in size, the larger being formed on the side of the vegetatively more vigorous zygophore (Figures 4 and 5). This difference in size is always distinct, though in some cases (Figure 6), less marked than in others. The contents of the two gametes become united through the disappearance of the intervening wall, and the zygote here formed (Figure 7), by the gradual enlargement of the two cells thus united, assumes the shape of a mature zygospore (Figure 8). The supply of nutrient for this ripening process comes almost entirely by way of the more vigorous zygophoric branch, and, although the stretched wall of the larger gamete makes up the greater part of the outline of the zygospore, still the stretched wall of the smaller contributes to it. Although it may show a certain tendency in this direction, the condition here is thus not comparable to an oögamous fertilization where the male gamete furnishes protoplasm to, but forms itself no essential part of, the mature oöspore. There are neither cytological nor physiological data which will enable us to say with any degree of certainty which of these two unequal gametes is to be considered male in character, yet from the condition in oögamous Phycomycetes, it would be natural to assume that the male is represented by the smaller of the two uniting cells. The inequality in the size of the gametes is associated with a still greater inequality in their suspensors; while the suspensor subtending the larger is always swollen, a suspensor is seldom ever formed on the side of the smaller, since the septum which distinguishes the gamete just mentioned is usually directly continuous with the wall of the zygophoric filament. The mature zygospore thus appears to be borne terminally by the swollen suspensor and merely appressed against the slender zygophoric filament. It sometimes happens that a short stalk remains after the smaller gamete is distinguished (Figure 4), and although it fails to enlarge and in the mature condition is hardly noticeable, its origin warrants the use of the term suspensor.

In some old cultures, cells have not infrequently been found in different stages of development, ranging from a condition in which the wall is only slightly discolored and papillate, to a condition with dark distinctly denticulate thick walls resembling, except in size, a mature zygospore (Figure 9). They are apparently formed from the smaller gamete developed from the less vigorous zygophoric filament, and on this supposition may be called azygospores. Though it is possible that azygospores are formed from the more vigorous zygophoric branches, such a condition has not been observed. It has not been possible to follow in detail their production, and to determine the immediate stimulus to their formation. The only explanation known for the origin of progametes is contact between the opposing zygophoric hyphae, and although no cases have been observed in which a contact could be considered responsible for the origin of the cells in question, their position on the filament and their general appearance suggests that they have been cut off from single progametes.

Although the species considered has been under cultivation for about ten years, and continued in pure cultures through many generations of non-sexual spores, no substratum nor external condition has been found which would prevent the formation of zygospores, notwithstanding the fact that all nutrients were tried for its cultivation which were used in the tests with Mucor I and II (p. 293) and Rhizopus (p. 248). The production of sporangia has been always less than that of zygospores, and it is often difficult to obtain preparations showing the former in any abundance.

In fluid nutrients like dilute decoction of prune, there is no fermentation nor production of yeast forms, and fructification only occurs when the mycelium reaches the surface, where it gives rise to a scanty growth of sporangia and zygospores. Nutrients of high concentration have not been used, but on such dilute nutrients as can be obtained from tap water in pure agar, or even from tap water alone on a sponge, zygospores and sporangia are formed together, although their production is feeble.

In the dryest atmospheres obtained, namely, in a sealed vessel with calcium chloride, zygospores are produced in abundance, as is also the case when the surrounding air is rendered nearly saturated by layers of wet filter paper covering the culture. The maximum and minimum temperature limits have not been determined for the growth of the species, yet it is certain that both sporangia and zygospores are produced up to 29° C.

It may be seen from the foregoing cultural experiments that the formation of zygospores is practically independent of the external conditions to which the growth of the fungus is subjected.

Dicranophora sp.

This homothallic form, an undescribed species of the genus Dicranophora, has been given the writer by Professor Thaxter, who found it growing on Boleti at Kittery Point, Maine.

Few cultural experiments have been tried with the form, but such as have been made indicate that external conditions are more influential in determining the form of fructification than was the case in Mucor I and II and in Zygorhynchus. On the ordinary nutrient agar used in its cultivation, zygospores form abundantly on the substratum, and even in its interior, when soft agar is used. Upon tap water with pure agar growth is slow, and after several weeks the culture shows only a meagre production of sporangia without zygospores. Although when grown under favorable conditions of cultivation, sporangia and zygospores are formed simultaneously from the same mycelium, yet it is possible, by a decrease in the relative humidity of the surrounding air, to decrease the formation of zygospores and increase that of sporangia. Thus of three stender dish cultures on nutrient agar in which the mycelium had just become visible, the one left covered in a culture drawer produced abundant zygospores and comparatively few sporangia; while the other two, one of which was placed uncovered on a laboratory table, and the other in a sealed vessel with calcium chloride, where the surrounding air was comparatively dry, produced sporangiophores but no zygospores.

From the abundantly branched mycelium are developed numerous stubby, much-lobed branchlets the majority of which remain sterile. On account of the difficulty of distinguishing the sterile outgrowths from the parts destined to produce zygospores, and on account of the fact that the smaller zygophoric branch is usually beneath and concealed by the larger, it is a matter of some difficulty to obtain the early stages of conjugation. Enough has been seen, it is believed, to determine the main points of the process. The form is distinctly heterogamic, and a difference is from the very start apparent between the two zygophoric hyphae. They may both arise close together, with a proximal protoplasmic connection between them; and while one remains but little greater in diameter than the mycelial filament from which it arises, the other is distinctly swollen (Plate I, Figure 10). Stimulated by the contact, a bulge develops in the large zygophoric branch at its point of meeting with the other (Figures 10 and 11). When the gamete is distinguished from the slender branch, the bulge opposite is about equal in size, but in the case figured a septum has not yet formed distinguishing it as a gamete. Apparently a rapid increase in size takes place before this septum forms, and by the dissolution of the walls in contact an open communication is established between the two conjugating cells (Figure 12). At this stage of development the condition is essentially similar to that figured in Zygorhynchus (Figure 7). In the species under discussion, however, the smaller gamete forms a less proportionate part of the zygote than does the corresponding gamete in Zygorhynchus. The zygospore is developed almost entirely within the stretched and finally cutinized walls of the larger gamete, and a small beak which is generally present on the zygospore is the cutinized remnant of the wall of the smaller gamete. If the zygospore be crushed, a thick hyaline wall will be found to have formed within the boundaries of the original membrane of the zygote, and on its surface a protuberance of the same material of varying size may often be seen which apparently corresponds to the beak just mentioned (Figure 14).

SUMMARY OF HOMOTHALLIC FORMS.

The above five types of the homothallic group have been examined by the writer and, with the exception of Spinellus, all have been grown in pure cultures for several years. Although the figures and descriptions of many of the species reviewed under the Citations in Part I would indicate that these were also homothallic, yet in several instances, as in Phycomyces and Rhizopus, the account is shown by the results of the present paper to have been inaccurate as regards the thallic condition. The writer does not feel justified, therefore, in including in the homothallic group any forms, with the exception of *Zygorhynchus heterogamus*, which have not been examined with the distinct purpose of determining the thallic condition present.

If we compare the members of the homothallic group as thus limited, we shall find that they offer varying degrees of differentiation between the hyphae bearing zygospores and those bearing sporangia. In Spinellus, the zygospores are borne on a distinct spiny aerial mycelium.

In Sporodinia, although the aerial hyphae which bear sporangia and those which bear zygospores are somewhat alike in their method of branching, yet they are distinct, and each form of fructification is produced alone on individual filaments. In Dicranophora the zygospores are not aerial, but are formed between special short branches of the superficial or even slightly immersed mycelium. In Mucors I and II and Zygorhynchus, however, both zygospores and sporangia may be formed from the same aerial hyphae.

In so far as is known, external conditions have a marked influence on the kind of fructification produced only when the sporangia and zygospores are developed separately upon differentiated hyphae. Even in such forms as Sporodinia, in which external factors are most effective, under ordinary conditions zygospores and sporangia occur side by side, and it is only under extreme conditions that it is possible to obtain an exclusive production of either form.

The homothallic species thus exhibit a type of sexual reproduction which seems exceptional, and is distinctly different from that which is predominant in the Mucorineae as a whole; but although in several cases this process is not associated with any differentiation of the conjugative apparatus, such a differentiation is distinctly indicated in other instances, as has already been pointed out. This differentiation, the sexual nature of which can hardly be disputed, will be further discussed in connection with the phenomena of hybridization which afford further evidence in support of this view.

All the homothallic species studied have been kept under cultivation for a considerable time, — in the case of *Zygorhynchus Moelleri* for about ten years, — and no change in their zygosporic activity has been observed. No neutral strains of these forms, moreover, have been found, and homothallism therefore may be assumed to be a fixed condition in the forms in which it is known to occur.

It is a noticeable fact that no homothallic species have been found growing spontaneously on animal excrement, which is the most prolific source for forms of the Mucorineae. Sporodinia, Spinellus, and Dicranophora, in which alone a differentiation is apparent between the sporangiophores and zygophoric hyphae, have their natural habitat on the fructifications of certain fleshy fungi. Zygorhynchus has been found growing spontaneously on bread and on cultures from soil origin. The forms represented by Mucors I and II were obtained from Hymenomycetous fungi and from the schleimflüss of recently cut birch; and from Hansen's ('02) investigation it seems probable that soil is not an uncommon source of homothallic forms.

HYBRIDIZATION.

In the Introduction it has been already stated in some detail that there is a sexual response not only between the (+) and (—) strains of the same species but also between (+) and (—) strains of different species, and that no interaction ever occurs between strains with like signs. Although the process has not been observed to go further than a cutting off of the two gametes, and more frequently stops at the production of the two progametes or the formation of a gamete on but one side, the word "hybridization" has for convenience been applied to these imperfect attempts at conjugation. Just as, when the hyphae of two sexually opposite strains of the same species are allowed to come in contact, a dark line may be apparent by the accumulation of zygospores there formed (Plate IV), in a similar manner, when hyphae of sexually opposite strains of two different species are allowed to come in contact a white line may be apparent, which results from the accumulation of these attempts at hybridization above mentioned. Plate IV, Figure 57, is a photograph of a culture made to illustrate these lines of zygospores and hybrids. Where the (+) and (—) strains of Mucor v have come in contact, a very wide, dark line resulting from the zygospores there formed is apparent, while between the (+) growth of Mucor v and the (—) growth of a different heterothallic species, Mucor x, as well as between the (—) growth of Mucor v and the (+) growth of Mucor x, a white line resulting from the imperfect hybrids there formed is distinctly evident. The absence of such a white line between the growths with like signs results from the fact that no hybridization has occurred at their areas of contact.

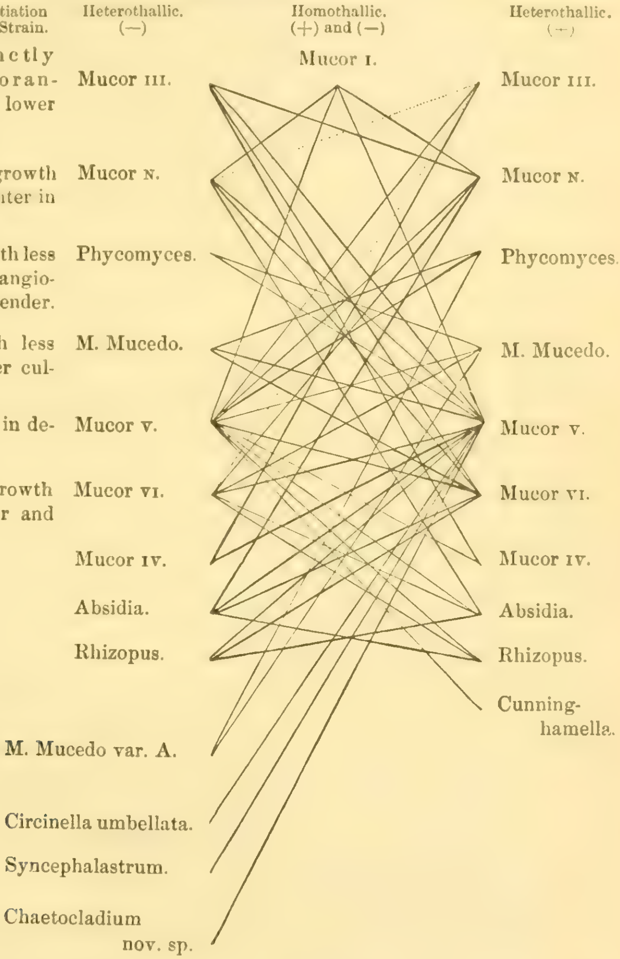
The first suggestion that an interaction is possible between sexual strains of different species arose during the investigation of the natural occurrence of sexual strains of *Mucor Mucedo*. A pure sporangial transfer from an unbranched Mucor growing on rabbit dung was inoculated into a stender dish together with inoculations of (+) and (—) strains of *M. Mucedo* so arranged that the mycelia in developing would come in contact with one another. In two days an unusually broad light line, apparently of young zygospores, resulted between the growth from this pure transfer and the (+) strain of *M. Mucedo*, and a similar line of less width occurred between the (+) and (—) strains of *M. Mucedo*. The day following the zygospores of *M. Mucedo* had become black in maturing, but the line between the unknown Mucor and the (+) strain of *M. Mucedo* remained white, and the conjugations showed no further development than the production of progametes which in some cases had distinguished their gametes on one side. Pure cultures were made of this form which showed such a peculiar reaction, and labelled *M. Mucedo* var. A. Subsequent contrasts on a large variety of substances containing available nutriment in more or less concentrations showed that this arrest in the development of zygospores between var. A and the (+) strain of *M. Mucedo* was constant, and not due, as was at first thought possible, merely to the inability of the progametes, for some reason unusually abundant, to obtain from the substratum sufficient nourishment for their further development. Var. A in tube cultures is much slower and less luxuriant in growth, and is distinguished at once from *M. Mucedo* by the light yellow coloration of sporangia, hyphae, and mycelium. In addition there are certain slight structural differences which made it seem improbable that the form could be other than a distinct species, and suggested that the imperfect conjugations observed with the (+) strain might be considered analogous to a process of hybridization.

The opposite strains of the nine heterothallic species under cultivation, which represent five different genera, were accordingly contrasted with one another, and it was found that the opposite strains of every species are capable of hybridizing with strains of other forms. In this way it has been possible to arrange the opposite strains of all the different species in two series according to their interaction with one another. Mucor v has been found most active in the production of hybrids, and has consequently been the form chosen as a standard with which the other species have been tested. All strains that have been found to hybridize with the strain of this species, which for reasons to be shown presently has been called (+), have been placed together in Table XII in the column on the left, while those that hybridize with the strain called (—) have been placed in the column on the right. Continuous lines connect those strains between which hybridization has been accomplished, and dotted lines connect those strains between which attempts at hybridization have been unsuccessful. It will be noticed that in no case has the position of a strain in either column been determined by positive hybridization in tests with less than two other species. By an examination of the table it will be seen that in every instance when a differentiation is apparent in the opposite strains, the strain showing the less vegetative luxuriance has been located by the hybridization tests in the column on the left. In view of their less luxuriant character the strains in this column have been called (—), while those in the

TABLE XII.

Thallic Condition of Forms Investigated as Determined by Hybridization.

Vegetative Differentiation Shown by the (—) Strain.	Heterothallic. (—)
Spores distinctly smaller. Sporangial growth lower and darker.	Mucor III.
Sporangial growth lower and lighter in color.	Mucor N.
Sporangial growth less dense. Sporangiophores more slender.	Phycomyces.
Mycelial growth less vigorous under cultivation.	M. Mucedo.
Sporangia later in development.	Mucor V.
Sporangial growth slightly denser and lower.	Mucor VI.
	Mucor IV.
	Absidia.
	Rhizopus.



other have been called (+). Before the possibility of correlating the opposite strains of different species was discovered, they were represented in cultures by arbitrary symbols, and though it seemed not impossible that the difference in strain luxuriance characteristic of certain species was in all cases similarly connected with their sexual differentiation, there was no satisfactory basis for such a belief. It is now, however, hardly to be doubted that in the (+) and (—) columns are represented the two opposite sexes.

In testing the hybridization between two different species the four possible contrasts have in all cases been made. Generally the strains of one species have been successively inoculated in two stender cultures between the opposite strains of the other, but with spreading forms like Rhizopus and Absidia, in order to avoid the possibility of conjugation between the strains of the same species, it has been necessary to oppose but two strains in a single dish, and consequently in hybridizing such forms four stender cultures have been made. Although the presence of hybrids at the line of contact between (+) and (—) strains of different species is often indicated by a more or less distinct white line, this is not always the case, and with such forms as Rhizopus and Absidia no line is apparent, although a microscopic examination may show that hybridization has occurred. In every instance, whether a line has been visible or not, the growths at all four lines of contact have been microscopically examined before the position of the strains has been listed in the table.

In tests made between certain species no hybrids were obtained, but had the external conditions been altered it is possible that hybridization would have occurred. By changing the nutriment, hybrids were obtained, for example, between Phycomyces and *Mucor Mucedo*, but if, as sometimes happens, it is impossible to satisfy on the same substratum the conditions which are necessary in each of the two species for the formation of zygospores, no hybridization is to be expected when their strains are contrasted.

The difference in the abundance of hybrids and in the distinctness of the lines resulting between the sexually opposite strains of two species contrasted frequently shows a difference in the sexual stimuli exerted by the (+) and (—) strains of the individual species. In the most marked instance of this nature observed, a characteristic white line was formed between the (+) strain of Mucor N and the (—) strain of Mucor III, while between the (—) strain of Mucor N and the (+) strain of Mucor III on the same substratum, no hybrids were found, even though a microscopic examination was made of the region of contact where they might have been expected.

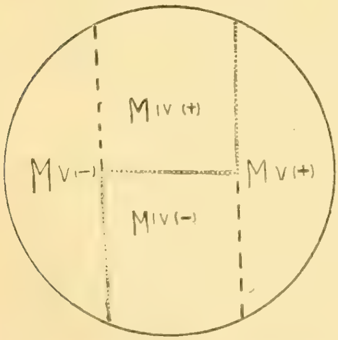
In the case of some species hybridization is more active with the (+) strain of Mucor V, while in other species the hybridization is more active with its (−) strain. A difference in the sexual activity of the (+) and (−) strains of Mucor v thus seems apparent. If, however, instead of assuming, as has been done in the previous sentence, that the strains of all the other species are mutually equal in the strength of the sexual stimulus which they exert, we consider the strains of Mucor v as equal in this respect, it will be found that with some species the (+) strains and with others the (−) take the more active part in hybridization. This inequality in the sexual activity of the opposite strains in contrast between two given species has been constant on the limited variety of the substrata tested, but what significance should be attributed to it is not at present entirely clear. As will be later pointed out in the discussion of hybridization with homothallic forms (p. 311), the inequality in the lines of hybrids may be correlated with the sexual differentiation of the (+) and (−) strains.

The difference which has just been under consideration is one between the individual strains of a single species, and can be seen only when hybridization is incited between two forms. Individual species, however, show varying intensities of sexual vigor, and while some can be induced to form zygospores only under special conditions, others will produce them under every condition to which they have as yet been subjected whenever their two strains are allowed to grow in contact. This condition is well illustrated by the fact that on certain substrata the production of the zygospores by (+) and (−) strains of Mucor IV does not occur, while distinct lines of hybrids are produced between strains of this species and of Mucor v respectively which are distinguished by unlike signs. Under somewhat different conditions of temperature and food supply contrasts of (+) and (−) strains of this form yield zygospores, but it is certainly remarkable that under any conditions the sexual response between them should be less intense than when they are contrasted against strains of a different species.

Mucor Mucedo var. A is the only form in which the process of hybridization has been followed on slide cultures. The formation of progametes with (+) strains of *M. Mucedo* appears to be essentially similar to the process where the contrast is made between the (+) and (−) strains of the latter species. In only a few instances have terminal cells been observed to have been cut off by septa in both progametes (Plate I, Figure 22). It frequently happens, however, that the gamete on one side only is distinguished (Figure 21), and a careful search in a number of slide cultures shows that, in the material examined, when only one gamete was formed, it invariably has been derived from the mycelium of var. A. Whether a similar relation exists in other hybrids between the strains characterized by the greater development has not been determined. Var A has been grown in pure cultures and continued to the twentieth generation without suffering any apparent change in its growth or in its sexual activity.

Rhizopus and Absidia were the first species after *Mucor Mucedo* and var. A between which hybrids were obtained. On flour paste, which is a suitable substratum for the production of zygospores of Rhizopus but less favorable for those of Absidia, hybrids were meagre in development, and could be found only between Absidia (+) and Rhizopus (—). On potato agar, however, which is more favorable for the zygospores of Absidia but less favorable for those of Rhizopus, hybridization was more active. The commonest condition in the process of this hybridization is represented by Figures 16 and 17, Plate I. The hyphae belonging to Absidia can readily be distinguished from those of Rhizopus by the blue coloration of the former, but the difference in nature of the two progametes is strikingly apparent in the more advanced stages, where, as is shown in Figure 18, the characteristic outgrowths have arisen from the Absidia suspensor. In the instance figured a gamete has been developed on the Absidia side and septa have formed in either suspensor. The production of gametes on both sides, however, is rare.

That the sexual character in Rhizopus is not confined to special zygophores is confirmed by the fact that when the (+) strain, for example, of this species is contrasted with the (—) strain of Mucor V, the stolons of the former are thickly beset with hybrids which have arisen in response to the contact between hyphae of the sexually opposite strains. The hyphae of the Mucor recurve from all sides and apply themselves to the stolons of Rhizopus, presenting in cultures an appearance of yellow festoons radiating from the line of contact.



Mucor IV does not produce its zygospores at the room temperature of the laboratory on the nutrients ordinarily used for the cultivation of other forms. The accompanying diagram represents the condition in a stender dish culture of potato gelatine. Mucor IV (+) and Mucor IV (—) are contrasted with each other in the centre of the dish, and on the sides are contrasted with Mucor v (+) and Mucor v (—). Broken lines represent areas of hybridization and dotted lines areas of contact between the different mycelia where no sexual response was apparent. It will be seen that lines of hybridization have occurred between the (+) and (—) strains of the two different species, but no zygospores have formed between those of Mucor IV. The strains of Mucor V seem to be characterized by an unusually marked activity in contrast to those of Mucor IV, which in this instance seem to play a comparatively passive part in the hybridization. In the warm oven and on a different substratum favorable to the interaction of the (+) and (—) strains of Mucor IV a similar arrangement of the same strains has yielded both zygospores and hybrids.

Mucor N will not produce zygospores when grown on such concentrated nutrients as are frequently used for the cultivation of other forms, and for this reason hybridization does not take place on these nutrients. With the (—) strain of Mucor V the line of hybrids is better developed than with the (+) strain of this species, and with the (—) strain of Mucor III a line of hybrids is produced, while with its (+) strain no hybrids occur under the conditions present in the single test made.

Phycomyces nitens is unique among the heterothallic forms under cultivation in that its zygophoric hyphae are not necessarily aerial. For this reason it was thought that hybridization would not occur between it and other species with a different habit, and the first tests with *Mucor Mucedo* on potato agar by their negative results seemed to confirm this idea. On the substratum employed the two species will form their zygospores, but a higher concentration of nutriment than is afforded by such a substratum is more favorable to the production of those of Phycomyces. A nutrient agar made up with condensed milk has been found advantageous for the growth of this latter species, and was therefore used in subsequent tests, which gave positive results.

The response between the opposite strains of the two species when contrasted is shown by a light line at their areas of contact, but the process of hybridization has never been observed to go so far as a cutting off of gametes. With the (+) strain of *M. Mucedo* the response is more marked than with the (—) strain. The hyphae shown in Plate III, Figure 51, were taken from the line between the (—) strain of Phycomyces and the (+) strain of *M. Mucedo*. A filament of the latter is seen to be closely embraced by an irregularly lobed branch of Phycomyces and presents the appearance of being enclosed by the haustoria of some parasite. The hyphae of *M. Mucedo*, thus attacked by those of Phycomyces, seem to be but slightly modified themselves. They are more or less branched, and, although generally sterile, cases have been observed in which sporangia have been produced from them. Sporangia, however, have never been found in direct connection with the convoluted hyphae of Phycomyces. The branches of *M. Mucedo* may be distinguished from those of Phycomyces by the fact that the walls of the former are covered with granulations, while those of the latter are smooth.

Mucor Mucedo does not as readily hybridize as some of the other forms; with Mucor VI, its sexual action is at best feeble, and often there is no response observable; and with Absidia and with Mucor III, although tests have been made, no hybrids have been obtained. The (+) strain of *M. Mucedo*, after having been rendered neutral towards its (—) strain by cultivation (p. 277), has been found no longer to form hybrids with var. A. This further emphasizes the conclusion that the stimulus for the production of progametes is the same in the formation both of hybrids and of zygospores.

As has been already mentioned, Mucor V is more active in forming hybrids than any of the other heterothallic forms known, and the strains of none of them have failed to respond when a contact with the proper strain of this species has been offered. For this reason Mucor V has been used in the few tests that have been made to determine the thallic condition of species the (+) and (—) strains of which have not been differentiated. They have been listed in Table XII (p. 305) under the sign opposite that of the strain to which they show a sexual response.

The zygospores of *Circinella umbellata* have been reported only by Bainier ('03), and in his figure he has represented a zygospore with its two suspensors in connection with the same hypha. Such a condition would indicate that the species is homothallic, but the evidence from hybridization points to a contrary conclusion.

Cultures of a new species of Chaetocladium, found by the writer in Venezuela, which can be readily grown saprophytically, and of Syncephalastrum, hybridize with Mucor V (+), and it is thereby rendered probable that these as well as some other species of the genus Mucor tested are individual (—) strains of heterothallic forms.

Cultures of *Cunninghamella echinulata*, a species originally described by Thaxter ('91) under Oedocephalum and recently redescribed as *C. Africana* by Matruchot ('03), who referred it to the Mucorineae largely on the ground that it acts as a host for Piptocephalis, have responded to the hybridization test with Mucor V (—), and have thus been shown to be (+) in character. In so far as may be determined, therefore, by the presence of a sexual response, the conclusion of Matruchot in regard to the true position of the fungus appears to be thus substantiated.*

The species which we have been just discussing are either known or supposed to be heterothallic. An extended investigation of the reciprocal action of different species with the homothallic forms will be reserved for a later research. It has been determined, however, that, as was to be expected in the light of the present research, one at least from this group responds sexually to the (+) *and* (—) strains of heterothallic species. A demonstration of this nature is shown in Plate IV, Figure 56, which has been already described somewhat in detail in the Introduction. It is a photograph of a culture in which a homothallic species, Mucor I, was sown between the (+) and (—) strains respectively of Mucor V. The growth of Mucor I appears dark in the photograph on account of the abundance of zygospores produced from its mycelium. At the areas of contact between the growth of the homothallic species and that of either strain of the heterothallic species, a white line is visible, indicating the presence of hybrids. That hybridization thus occurs with both (+) and (—) strains, which may be assumed to be unisexual, shows that the thallus of the homothallic form investigated is bisexual, inasmuch as it produces hyphae which form progametes as a result of the stimulus of contact with *both* the sexually opposite strains tested.

When this same homothallic form is contrasted between the (+) and (—) strains of Mucor N, the lines of hybridization are less well marked and unequally distinct, that between Mucor N (+) and the homothallic species being much more evident. A similar but slighter difference between the lines of hybridization with the species previously mentioned can be seen in the photograph. It is also here apparent that the line on the side of the (+) strain is better developed. A like difference in the lines of hybridization has been already discussed under the hybridization of heterothallic forms. The most natural conclusion from these facts seems to be that the (+) and (—) characters in the bisexual mycelium of the homothallic species are equally active, and such an assumption would force us to regard the (+) and (—) strains of Mucor V and Mucor N as unequal in sexual vigor with the (+) the more active. These are the only two species which have been contrasted with a homothallic form, and therefore it is at present imprudent to state more than that it seems strongly probable that the difference in question is correlated with the difference previously observed in the vegetative vigor of the strains tested. If such be the case it will further emphasize the conclusion that the two opposite sexes are represented in the series containing respectively the vegetatively more and less vigorous strains.

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* Since this paper was in press the (—) strain of this species has also been obtained, and contrasts of the two yielded abundant zygospores July 27.

By the facts presented in the foregoing section, it has been shown that a process of imperfect hybridization will occur between unlike strains of different heterothallic species in the same or even in different genera or between a homothallic form and both strains of a heterothallic species, and that, by taking advantage of this character, it has been possible to group together in two opposite series the strains of all the heterothallic species under cultivation. It has been seen, moreover, that when thus grouped the (—) or less luxuriant strains of all species known to show a vegetative difference between their opposite strains will be in one series, while the (+) or more luxuriant strains will be in the other. It has also been shown that when a contrast is made between the opposite strains of two different heterothallic species or between a homothallic form and both (+) and (—) strains of a heterothallic species the activity exhibited by these strains in the formation of hybrids at their lines of contact is generally unequal.

Hybridization may be assumed to indicate that the formation of progametes is determined by the stimulus of contact between the (+) and (—) hyphae whether of the same or of different species, and that the formation of zygospores is dependent on the union of gametes of the same species. From the facts observed it will be concluded that the formation of zygospores is a sexual process, that the mycelium of a homothallic species is bisexual, while the mycelium of a heterothallic species is unisexual, that differences in the hybridizing activity observed in opposite strains is correlated with the differences in vegetative activity which they exhibit, and further that in the (+) and (—) series of the heterothallic species are represented the two opposite sexes.

SUMMARY.

An outline of the general conclusions which have been reached as a result of the investigations embodied in the present paper has already been presented in the Introduction. A more extended discussion of the relations of the problems which have been suggested by the work of the past year the writer reserves until he has accumulated a greater body of facts on the subject. It now remains to give merely a brief recapitulation of the principal results considered in some detail in the foregoing pages.

(1) The production of zygospores in the Mucorineae is conditioned primarily by the inherent nature of the individual species and only secondarily by external factors.

(2) According to their method of zygospore formation, the Mucorineae may be divided into two main groups, which have been termed respectively homothallic and heterothallic.

(3) In the homothallic group, comprising the minority of the species, zygospores are developed from branches of the same thallus or mycelium, and can be obtained from the sowing of a single spore.

(4) In the heterothallic group, comprising probably a large majority of the species, zygospores are developed from branches which necessarily belong to thalli or mycelia diverse in character, and can never be obtained from the sowing of a single spore. Every heterothallic species is therefore an aggregate of two distinct strains, through the interaction of which zygospore production is brought about.

(5) These sexual strains in an individual species show in general a more or less marked differentiation in vegetative luxuriance, and the more and less luxuriant may be appropriately designated by the use of (+) and (—) signs respectively.

(6) In heterothallic species, strains have been found which from their failure to react with (+) and (—) strains of the same form have been called "neutral," and a similar neutrality may be induced by cultivation under adverse conditions.

(7) In all species of both groups in which the process of conjugation has been carefully followed the swollen portions (progametes) from which the gametes are cut off do not grow toward each other as currently believed, but arise from the stimulus of contact between more or less differentiated hyphae (zygophores), and are from the outset always normally adherent.

(8) In some species the zygophores have been demonstrated to be mutually attractive (zygotactic).

(9) In the heterogamic subdivision of the homothallic group a distinct and constant differentiation exists between the zygophoric hyphae and the gametes derived from them, but in the remaining homothallic forms and in all heterothallic forms no such differentiation is apparent.

(10) A process of imperfect hybridization will occur between unlike strains of different heterothallic species in the same or even in different genera, or between a homothallic form and both strains of a heterothallic species.

(11) By taking advantage of this character it has been possible to

group together in two opposite series the strains of all the heterothallic forms under cultivation.

(12) When thus grouped the (—) or less luxuriant strains will be in one series while the (+) or more luxuriant will be in the other.

(13) From the foregoing observations it may be concluded, —

(a) that the formation of zygospores is a sexual process;
(b) that the mycelium of a homothallic species is bisexual;
(c) while the mycelium of a heterothallic species is unisexual;
(d) and further, that in the (+) and (—) series of the heterothallic group are represented the two sexes.

A preliminary summary of the results of the present paper has been given by the writer ('04) in Science, June 3, 1904.

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The following additional reference to the American occurrence of *Spinellus fusiger* and *S. macrocarpus* came to the writer's notice too late for insertion in the body of this paper.

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EXPLANATION OF PLATES.

The figures in plates I, II, and III are camera drawings, made with the combinations of Zeiss and Leitz objectives noted and reduced about one third in reproduction. In Plate IV, Figures 52–55 are reproductions of Petri dish cultures, and were slightly reduced in the process of photographing; Figures 56–58 are reproductions of stender dish cultures, and were slightly magnified. The signs (+) and (−) are used to denote the opposite strains whenever these are distinguished.

PLATE I.

ZYGORHYNCHUS MOELLERI.

Figures 1–9. (Obj. 7. oc. 4.)

FIGURES 1–8. Stages in the development of the zygospore showing: —
- Figures 1–3, formation of progametes;
- Figures 4, 5, formation of gametes;
- Figure 6, union of gametes;
- Figure 7, zygote assuming the shape of a zygospore;
- Figure 8, mature zygospore.

FIGURE 9. "Azygospores."

DICRANOPHORA SP.

Figures 10–13. (Obj. C. oc. 4.) Figure 14. (Obj. 7. oc. 4.)

FIGURE 10. Zygophores in contact.

FIGURE 11. Formation of a progamete from the larger, and of a gamete from the smaller zygophore.

FIGURE 12. Zygote.

FIGURE 13. Mature zygospore.

FIGURE 14. Protuberance from the hyaline wall corresponding to the "beak" of the mature zygospore.

RHIZOPUS NIGRICANS.

Figures 15–18. (Obj. C. oc. 4.)

FIGURE 15. Three zygospores showing respectively a larger and a smaller gamete in connection with the same hypha.

FIGURES 16–18. Stages in hybridization between (+) hyphae of Rhizopus and (–) hyphae of Absidia.

MUCOR I.

FIGURE 19. Gametes showing origin of zygophoric hyphae from adjacent parts of the same mycelial branch. (Obj. C. oc. 2.)

MUCOR MUCEDO VAR. A.

FIGURES 20–22. Stages in hybridization between (+) hyphae of *M. Mucedo* and (–) hyphae of var. A. (Obj. C. oc. 4.)

MUCOR III.

FIGURE 23. Spores of the (–) strain. (Obj. J. oc. 4.)

FIGURE 24. Spores of the (+) strain showing greater dimensions. (Obj. J. oc. 4.)

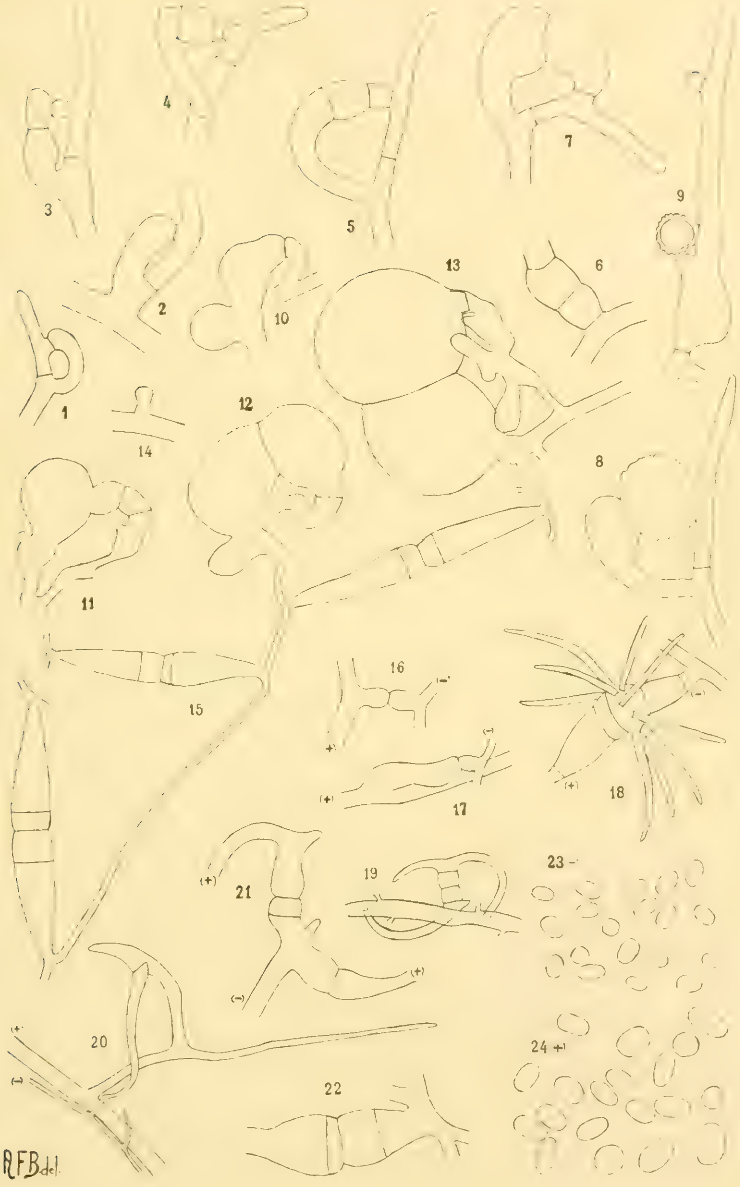

23 -
24 +
(+)
(-)
RFB del.

PLATE II.

MUCOR MUCEDO.

Figures 25-35. (Obj. A. oc. 4.)

FIGURES 25–32. Stages in conjugation between (+) and (–) zygophores from a zygosporic line. The drawings were made with the aid of a camera lucida and beginning with 10 A.M., the stages were taken at the intervals noted. The dots in Figure 30, (d) and (e), indicate the position of the ends of the zygophores at 2.15 P.M.

FIGURE 31. Condition two minutes before contact of zygophores at (d) showing slight bulging in the apposing walls.

FIGURE 33. Horizontal view of zygophoric hyphae from a zygosporic line.

FIGURE 34. Zygote showing centrifugal dissolution of intervening wall between the two gametes.

FIGURE 35. Mature zygospore.

CUNNINGHAMELLA ECHINULATA.

FIGURES 36–39. Stages of hybridization between (+) hyphae of *C. echinulata* and (–) hyphae of Mucor v. (Obj. 7. oc. 4.)

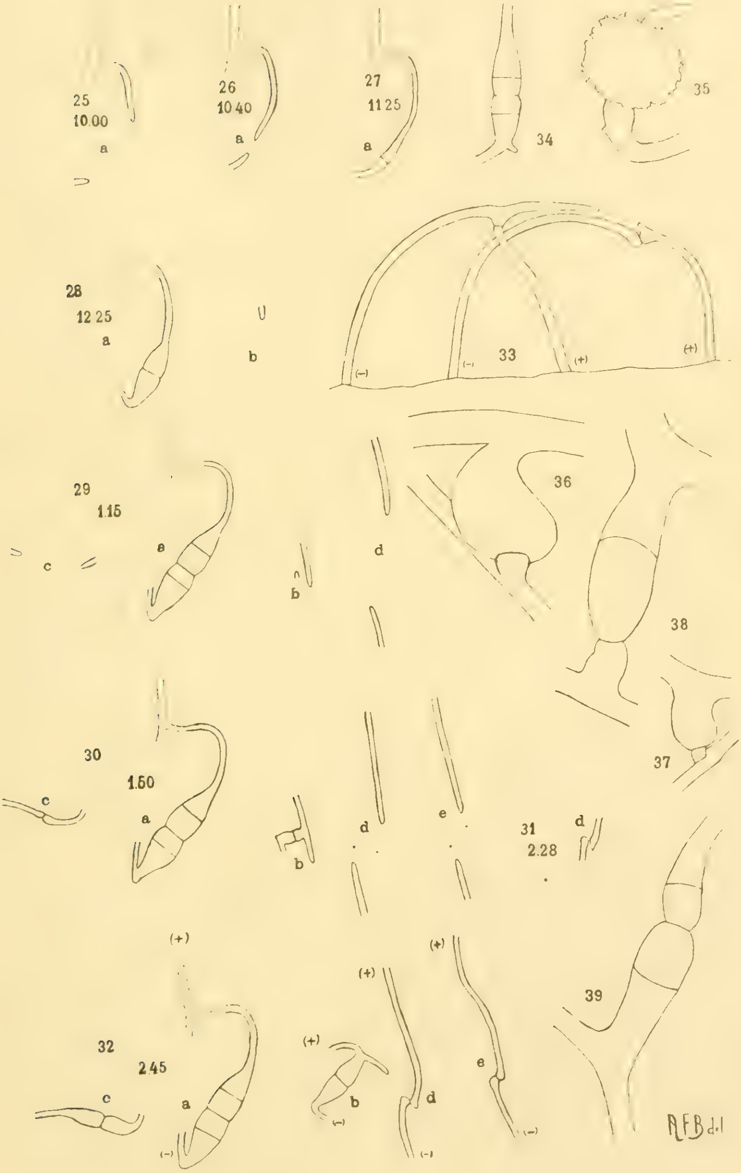

25
10.00
a
26
10 40
a
27
11 25
a
34
35
28
12 25
a
b
(−)
(−)
33
(+)
(+)
36
29
1.15
c
a
b
d
38
30
1.50
c
a
b
d
e
37
31
2.28
d
(+)
(+)
(+)
39
32
2.45
c
a
(−)
(+)
b
(−)
d
(−)
e
(−)
AFB del.

PLATE III.

MUCOR V.

Figures 40–44. (Obj. A. oc. 4.)

FIGURES 40–44. Stages in conjugation drawn at the intervals noted from a van Tieghem cell culture, into which a mixture of (+) and (–) spores had been inoculated.

PHYCOMYCES NITENS.

FIGURES 45–47. Branching at contact of (+), and (–) mycelial hyphae. (Obj. A. oc. 4.)

FIGURE 48. Progametes before their subterminal separation. (Obj. C. oc. 4.)

FIGURE 49. Progametes after subterminal separation. (Obj. A. oc. 4.)

FIGURE 50. Gametes and suspensors showing outgrowths arising from the (–) suspensor. (Obj. A. oc. 4.)

FIGURE 51. Stage in hybridization, between *P. nitens* and *M. Mucedo.* (Obj. 7. oc. 4.)

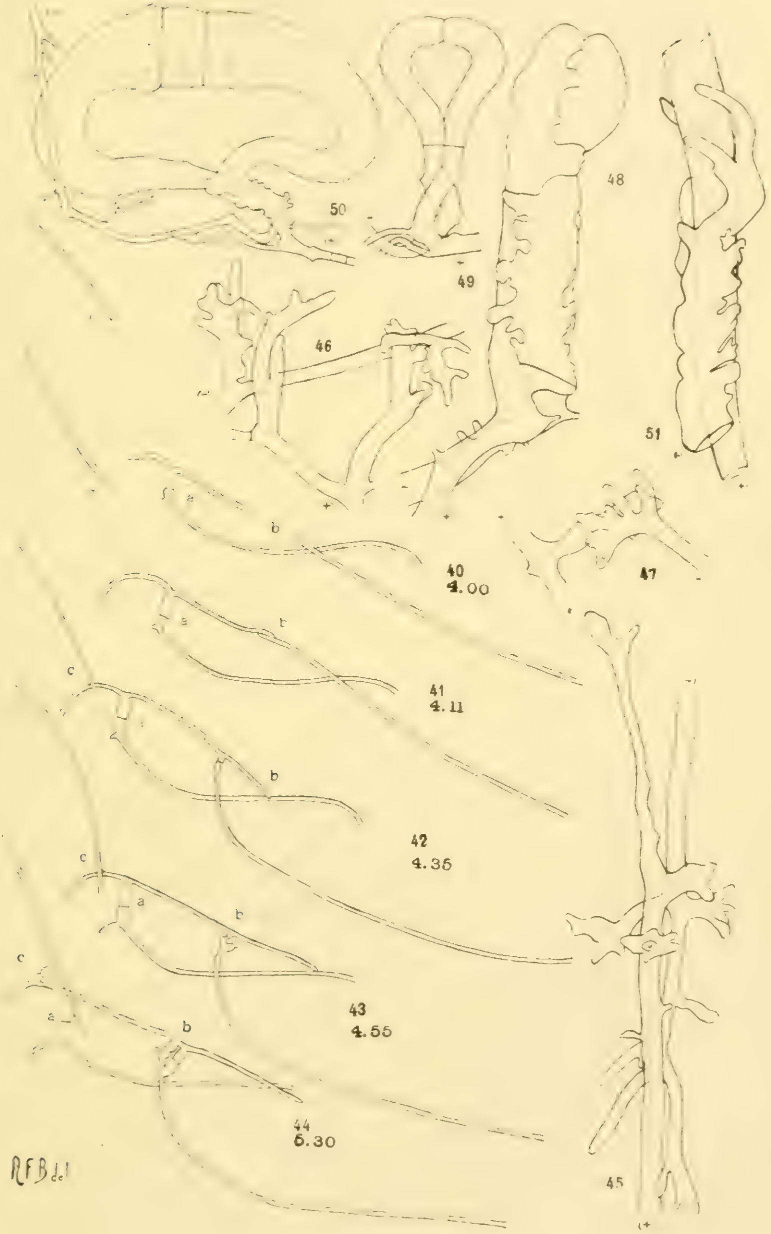

50
49
48
46
51
a
b
40
4.00
47
a
b
41
4.11
c
b
42
4.35
c
a
b
43
4.55
c
a
b
44
5.30
45
RFB del.

PLATE IV.

Figure 52. Phycomyces nitens showing zygosporic lines at regions of contact between (+) and (−) strains. The vegetative differences distinguishing the two strains are but slightly indicated in the photograph.

Figure 53. Mucor v. Zygosporic lines with light areas in their centre made up of zygospores in arrested stages of development.

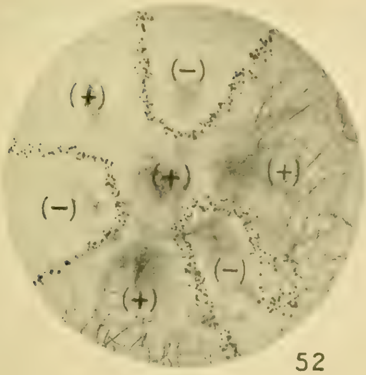
Figure 54. Mucor vi. Zygosporic lines.

Figure 55. *Mucor Mucedo.* Zygosporic lines.

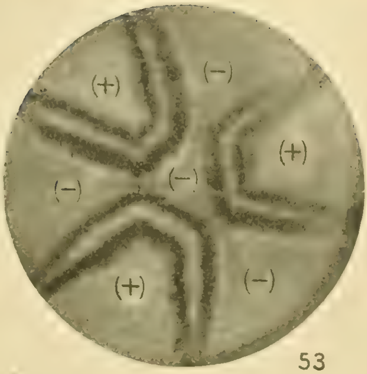
Figure 56. Mucor i and Mucor v (+) and (−). Lines of hybridization between a homothallic and a heterothallic species.

Figure 57. Mucor v and Mucor n. Zygosporic line between (+) and (−) strains of Mucor v and lines of hybridization between the opposite strains of Mucors v and n respectively (cf. diag. p. 212).

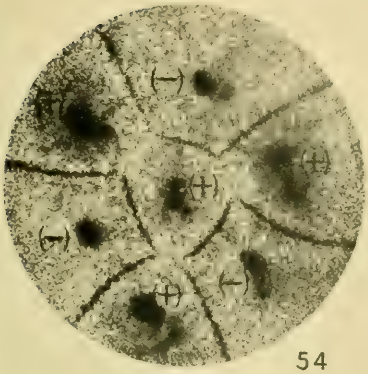
Figure 58. Mucor iii. A striking difference in habit is shown between the (+) and (−) strains which are forming zygospores at their regions of contact.



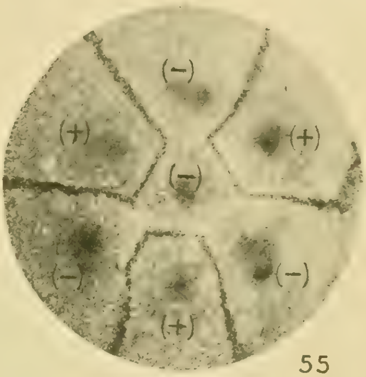
52



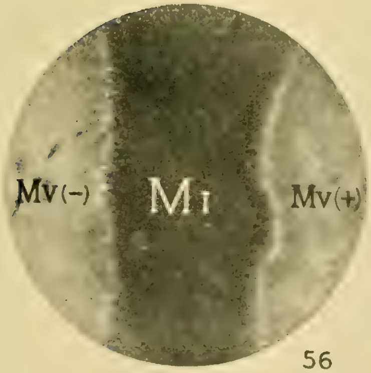
53



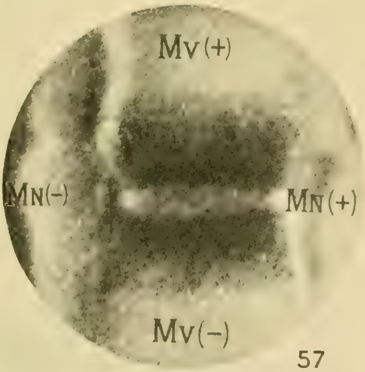
54



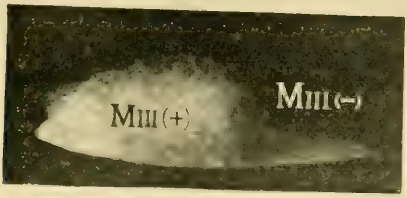
55



56



57



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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF
CASE SCHOOL OF APPLIED SCIENCE. — XLV.

ON THE COMPOSITION OF PETROLEUM.

By Charles F. Mabery.

I. *THE HYDROCARBONS IN OHIO TRENTON LIMESTONE PETROLEUM.* By Charles F. Mabery and O. R. Palm.
II. *THE HYDROCARBONS IN CANADIAN PETROLEUM WITH HIGH BOILING POINTS.*
III. *HYDROCARBONS IN SANTA BARBARA CRUDE OIL.*
IV. *SEPARATION OF SOLID PARAFFINE HYDROCARBONS FROM PETROLEUM WITHOUT EVAPORATION.* By Charles F. Mabery and Otto J. Sieplein.
V. *THE SOLID PARAFFINE HYDROCARBONS THAT COLLECT IN OIL WELLS IN PENNSYLVANIA.*
VI. *COMPOSITION OF COMMERCIAL PARAFFINE.*
VII. *COMPOSITION OF COMMERCIAL VASELINE, COSMOLINE, AND SIMILAR PRODUCTS.*

Aid in the Work described in these Papers was given by the Academy from the C. M. Warren Fund for Chemical Research.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — No. XLV.

ON THE COMPOSITION OF PETROLEUM.*

By Charles F. Mabery.

Received May 24, 1904.

I. THE HYDROCARBONS IN OHIO TRENTON LIMESTONE PETROLEUM WITH BOILING POINTS ABOVE 213°.

By Charles F. Mabery and O. R. Palm.

In a paper on the constituents of Pennsylvania, Ohio, and Canadian petroleum with boiling points between 150° and 220°, published several years ago,† it was shown that the distillate boiling at 212°–214° from Ohio Trenton limestone petroleum consisted of hendecane, $C_{12}H_{26}$, as a result of combustion, molecular weight determinations, and composition of its chlorine derivatives. Study of the portions with higher boiling points has since been continued for the purpose of ascertaining where the series C_nH_{2n+2} terminates and the series poorer in hydrogen appear.

The higher specific gravity of Trenton limestone petroleum and products prepared from it is well known, but the cause has not yet been explained. Since the series C_nH_{2n+2} has been found to continue in Pennsylvania petroleum in the solid forms as high as their molecular weights can be determined, and to be associated with series poorer in hydrogen above 260° (50 mm.), it was evidently interesting to ascertain the composition of the higher hydrocarbons in Ohio oil.

Results to appear later show that the series poorer in hydrogen form the chief portions of Canadian petroleum; and since the Trenton limestone oil stands in an intermediary position in its physical properties and in the properties of its distillates between Pennsylvania and Cana-

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* The work described in the following papers was carried on with aid granted by the Academy from the C. M. Warren Fund for chemical research.

† These Proceedings, **32**, 143 (1897).

dian crude oils, the series of hydrocarbons in Ohio oil should explain the difference in properties of commercial products obtained from it.

In resuming the study of Ohio Trenton limestone oil in 1901, a barrel of the crude oil was obtained at Welker, Ohio, which gave as its specific gravity at 20°, 0.8367. A combustion gave the following percentages of carbon, hydrogen, and sulphur: — carbon, 85.46; hydrogen, 13.91; sulphur, 0.48.

This crude oil is not so heavy as some specimens from Lima and Indiana, and the percentage of sulphur is not so high as specimens from some other fields, but it was believed to represent an average composition of Trenton limestone oil with reference to the principal series of hydrocarbons.

From 8 liters of the crude oil the portions distilling below 200°, equivalent to 1.8 liters, were removed under atmospheric pressure, and the residue was distilled *in vacuo* under a tension of 30 mm., at first within limits of 10°, which left a residue above 280° of 2000 grams. Subsequent distillation under the same tension brought the distillates together in larger quantities within certain well defined limits of temperature, indicating a greater proportion of individual hydrocarbons. After ten distillations, greater amounts collected within the following limits, and were found to represent definite hydrocarbons: 111°–113°; 129°–130°; 138°–140°; 152°–154°; 164°–168; 177°–179°; 187°–190°; 198°–202°; 213°–216°; 224°–227°; 237°–240°; 253°–255°; 263°–265°; 275°–278°. While it cannot be assumed that in a limited number of distillations the individual hydrocarbons can be separated completely, analysis and molecular weight determinations indicate that the separation is sufficiently complete to admit of identification, not only of members of the same series but of rather sudden changes from one series to the next poorer in hydrogen.

None of the fractions showed evidence of decomposition. The lower fractions were liquid, but the distillate 213°–217° was nearly solid at ordinary temperatures. The higher distillates were completely solid from paraffine, which formed a large part of these fractions. Since the solid hydrocarbons in Pennsylvania petroleum were identified as members of the series C_nH_{2n+2}, and the solid hydrocarbons in Ohio petroleum are without doubt composed of the same bodies, it was not thought worth while to examine these solids from Ohio oil. For their removal the oil was cooled to 0°, filtered, then cooled to —10°, and again filtered under pressure at the same temperature. In the higher distillates the solid hydrocarbons formed one-third or one-half of the total distillate.

The oils filtered from these fractions were very thick and viscous. Determinations of the specific gravity of the unpurified fractions at 20° gave values essentially higher than the corresponding fractions from Pennsylvania oil:

129°–130°	138°–140°	152°–154°	177°–179°	187°–190°	198°–202°
0.7998	0.8065	0.8187	0.8262	0.8351	0.8373

	224°–227°	237°–240°	253°–255°	275°–278°
(at 50°)	0.8586	0.8622	0.8849	0.8941

After removal of the solid hydrocarbon each fraction was thoroughly purified with sulphuric acid, agitating first with common concentrated acid and then with fuming acid, until the acid was nearly colorless. The heavier distillates were first dissolved in light gasoline, for they formed alone with acid an emulsion from which the oil did not readily separate. For complete removal of the acid the oil was finally washed with sodium hydrate, then dried first over calcium chloride and finally over sodium. If the oil remains unchanged after standing some time over sodium it is very well purified.

In the present state of our knowledge concerning the series of hydrocarbons poorer in hydrogen than the series C_nH_{2n+2}, definite names cannot be assigned to the members of these series. It is probable that the series C_nH_{2n} is composed of the methylene hydrocarbons, perhaps with complex side chains. The series still poorer in hydrogen may contain two or more methylene rings, but as yet there is no evidence as to what form these bodies may have.

HYDROCARBON $C_{12}H_{24}$, B. P. 211°–213°.

In a former paper* the fraction 212°–214° atmospheric pressure was shown by analysis and further identification to be dodecane, $C_{12}H_{26}$, and its specific gravity was found to be 0.7727 at 20°. This hydrocarbon was separated with others from a specimen of crude oil obtained from a well at Findlay, Ohio. Its specific gravity and composition corresponded to that of dodecane, which was separated from Pennsylvania petroleum.

The fraction 211°–213° now under examination gave as its specific gravity after thorough purification with sulphuric acid, as described above, 0.7970. A combustion gave percentages of carbon and hydrogen corresponding to the series C_nH_{2n}.

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* These Proceedings, **32**, 154.

I. 0.1462 gram of the oil gave 0.4590 gram CO_2 and 0.1860 gram H_2O.

II. 0.1281 gram of the oil gave 0.4020 gram CO_2 and 0.1635 gram H_2O.

III. 0.1563 gram of the oil gave 0.4906 gram CO_2 and 0.2000 gram H_2O.

	Calculated for C_nH_{2n}	Found.		
		I.	II.	III.
C	85.70	85.62	85.53	85.61
H	14.30	14.25	14.27	14.31

The molecular weight of this hydrocarbon determined at the freezing point of benzol supported the formula $C_{12}H_{24}$.

1.1097 gram of the oil and 29.394 grams benzol gave a depression of 1°.067.

Calculated for $C_{12}H_{24}$.	Found.
168	173

The same formula was verified by the index of refraction, 1.4350, from which the molecular refraction was calculated.

Calculated for $C_{12}H_{24}$.	Found.
54.24	55.00

It is therefore evident that this hydrocarbon has the composition of the series C_nH_{2n}. The only explanation that can be offered as to the difference between it and the product with the same boiling point separated before is that the different specimens of oil coming from different sources must have differed with respect to this hydrocarbon.

HYDROCARBON $C_{13}H_{26}$, B. P. 123°–125° (30 mm.).

After ten distillations under 30 mm., 27 grams collected at 129°–130° that distilled under atmospheric pressure at 223°–225°. After careful purification the specific gravity of this distillate was found to be 0.8055 at 20°. It was shown to belong to the series C_nH_{2n} by the following combustions:

I. 0.1536 gram of the oil gave 0.4824 gram CO_2 and 0.1902 gram H_2O.

II. 0.1336 gram of the oil gave 0.4195 gram CO_2 and 0.1710 gram H_2O

	Calculated for $C_{13}H_{26}$	Found.	
		I.	II.
C	85.70	85.65	85.63
H	14.30	13.85	14.32

The molecular weight of this hydrocarbon determined at the freezing point of benzol corresponded to the formula $C_{13}H_{26}$.

0.9896 gram of the oil and 25.6727 grams benzol gave a depression of 1°.040.

Calculated for $C_{13}H_{26}$.	Found.
182.0	181.6

A determination of the index of refraction gave 1.4400, which corresponded to the molecular refraction:

Calculated for $C_{13}H_{26}$.	Found.
59.84	59.55

HYDROCARBON $C_{14}H_{28}$, B. P. 138°–140° (30 mm.).

After the tenth distillation 45 grams collected at 138°–140° that gave as its specific gravity at 20°, 0.8129, after thorough purification with acid. A combustion gave values for the series C_nH_{2n}.

0.1407 gram of the oil gave 0.4435 gram CO_2 and 0.1787 gram H_2O.

	Calculated for $C_{14}H_{28}$.	Found.
C	85.70	85.96
H	14.30	14.18

The formula was verified by the molecular weight found at the freezing point of benzol:

0.9552 gram of the oil and 24.5822 grams benzol gave a depression of 0°.975.

Calculated for $C_{14}H_{28}$.	Found.
196	195.2

The index of refraction was found to be 1.4437 and the molecular refraction supported the same formula:

Calculated for $C_{14}H_{28}$.	Found.
64.44	64.10

HYDROCARBON $C_{15}H_{30}$, B. P. 152°–154° (30 mm.).

A fraction of 75 grams collected at first within these limits that gradually diminished to 20 grams after the tenth distillation. After purification with acid, it gave as its specific gravity at 20°, 0.8204. A combustion of this product gave values for carbon and hydrogen required for the series C_nH_{2n}.

0.1511 gram of the oil gave 0.4750 gram CO_2 and 0.1920 gram H_2O.

	Calculated for $C_{15}H_{30}$.	Found.
C	85.70	85.74
H	14.30	14.21

The molecular weight determined at the freezing point of benzol corresponded to the formula $C_{15}H_{30}$.

0.9370 gram of the oil and 24.2115 grams benzol gave a depression 0°.910.

Calculated for $C_{15}H_{30}$.	Found.
210	208.3

This formula was further confirmed by the index of refraction, 1.4480, which gave the following molecular refraction:

Calculated for $C_{15}H_{30}$	Found.
69.05	68.50

HYDROCARBON $C_{16}H_{32}$, B. P. 164°–168° (30 mm.).

18 grams of the oil collected within these limits after the tenth distillation that gave as its specific gravity at 20°, 0.8249. After filtration at –10°, and purification with acid, the specific gravity found was 0.8254. The proportions of carbon and hydrogen by combustion indicated the series C_nH_{2n}.

0.1536 gram of the oil gave 0.4848 gram CO_2 and 0.1945 gram H_2O.

	Calculated for $C_{16}H_{32}$.	Found.
C	85.70	86.08
H	14.30	14.16

The molecular weight at the freezing point of benzol corresponded to the formula $C_{16}H_{32}$.

1.1300 gram of the oil and 20.3767 grams benzol gave a depression of 1°.202.

Calculated for $C_{16}H_{32}$.	Found.
224	226

The index found, 1.4510, confirmed the same formula:

Calculated for $C_{16}H_{32}$.	Found.
73.65	73.08

HYDROCARBON $C_{17}H_{34}$, B. P. 177°–179° (30 mm.).

After the tenth distillation 35 grams collected within these limits with a specific gravity 0.8251 at 20°. After filtration at —10° and treatment with acid the specific gravity was raised to 0.8335. The percentages of carbon and hydrogen by combustion indicated the series C_nH_{2n}.

0.1501 gram of the oil gave 0.4720 gram CO_2 and 0.1881 gram H_2O.

	Calculated for $C_{17}H_{34}$.	Found.
C	85.70	85.76
H	14.30	14.02

The molecular weight at the freezing point of benzol determined the formula $C_{17}H_{34}$.

1.0880 gram of the oil and 24.9900 grams benzol gave a depression of 0°.896.

Calculated for $C_{17}H_{34}$.	Found.
238	238

The index of refraction, 1.4545, supported the same formula:

Calculated for $C_{17}H_{34}$.	Found.
78.25	77.50

A fraction collected at 188°–190° which should contain the next homologous hydrocarbon $C_{18}H_{36}$, but the oil was unfortunately lost by an accident during filtration from the solid.

HYDROCARBON $C_{19}H_{36}$, B. P. 198°–202° (30 mm.).

30 grams of this fraction were collected after the tenth distillation, which gave as its specific gravity at 20°, 0.8364. On cooling to —10° and filtration, 16 grams of the filtered oil was obtained with specific gravity 0.8471 at 20°. Determinations of carbon and hydrogen gave percentages which indicated a change from the series C_nH_{2n} to the series C_nH_{2n-2}.

0.1528 gram of the oil gave 0.4830 gram CO_2 and 0.1845 gram H_2O.

	Calculated for $C_{19}H_{36}$.	Found.
C	86.36	86.24
H	13.64	13.52

A determination of the molecular weight at the freezing point of benzol gave the formula $C_{19}H_{36}$.

0.9063 gram of the oil and 24.1120 grams benzol gave a depression of 0°.700.

Calculated for $C_{19}H_{36}$.	Found.
264	263

This formula was further verified by its index of refraction, which was found to be 1.4614, corresponding to the following molecular refraction:

Calculated for $C_{19}H_{36}$.	Found.
85.35	85.57

HYDROCARBON $C_{21}H_{40}$, B. P. 213°–217° (30 mm.).

The hydrocarbon $C_{20}H_{38}$ apparently did not appear, since the distillates were small, with no appearance of accumulating below 213°, at which point a larger amount was collected that proved to be the hydrocarbon $C_{21}H_{40}$. After the tenth distillation 25 grams collected that gave as its specific gravity at 20°, 0.8417. After filtration at −10° the specific gravity was raised to 0.8546 at 20°. The percentages of carbon and hydrogen corresponded to the series C_nH_{2n-2}

0.1362 gram of the oil gave 0.4330 gram CO_2 and 0.1675 gram H_2O.

	Calculated for $C_{21}H_{40}$.	Found.
C	86.30	86.70
H	13.70	13.66

A determination of the molecular weight at the freezing point of benzol gave a value required for the formula $C_{21}H_{40}$.

1.1020 gram of the oil and 25.1400 grams benzol gave a depression of 0°.734.

Calculated for $C_{21}H_{40}$.	Found.
292	292.6

The index of refraction found, 1.465, also confirmed the same formula:

Calculated for $C_{21}H_{40}$.	Found.
94.56	94.45

HYDROCARBON $C_{22}H_{42}$, B. P. 224°–227° (30 mm.).

22 grams collected after the tenth distillation at 224°–227°, whose specific gravity was found to be 0.8480 at 20°. After filtration at —10° and treatment with acid in gasoline solution, the specific gravity was raised to 0.8614. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n-2}.

I. 0.1403 gram of the oil gave 0.4438 gram CO_2 and 0.1670 gram H_2O.
II. 0.1571 gram of the oil gave 0.4970 gram CO_2 and 0.1901 gram H_2O.

	Calculated for $C_{22}H_{42}$.	Found. I.	Found. II.
C	86.25	86.27	86 27
H	13.75	13.31	13.53

The molecular weight determined at the freezing point of benzol corresponded to the formula $C_{22}H_{42}$.

0.8385 gram of the oil and 21.6023 grams benzol gave a depression of 0°.602.

Calculated for $C_{22}H_{42}$.	Found.
306	309

This formula was still further confirmed by the molecular weight determined at the boiling point of benzol, which gave 310.

The index of refraction, 1.4690, supported the same formula, as shown by the corresponding molecular refraction :

Calculated for $C_{22}H_{42}$.	Found.
99.16	99.60

HYDROCARBON $C_{24}H_{46}$, B. P. 237°–240° (30 mm.).

The hydrocarbon $C_{23}H_{44}$ did not appear in the small amount of distillates between 227° and 235°. Within the limits 237°–240°, 30 mm., 40 grams collected which was solid with paraffine at ordinary temperatures. After filtration at —10° the thick viscous oil gave as its specific gravity 0.8639. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n-2}.

0.1515 gram of the oil gave 0.4804 gram CO_2 and 0.1855 gram H_2O.

	Calculated for $C_{24}H_{46}$.	Found.
C	86.23	86.48
H	13.77	13.70

The molecular weight determined at the freezing point gave the value required for $C_{24}H_{46}$.

1.0900 gram of the oil and 28.0340 grams benzol gave a depression of 0°.570.

Calculated for $C_{24}H_{46}$.	Found.
334	334

With hydrocarbons of such high molecular weights, the limits of accuracy in determinations by the freezing point method with the aid of solvents commonly used are reached on account of the slight depression of the freezing point.

A determination of the index of refraction gave 1.4715, which corresponds to the molecular refraction of $C_{24}H_{46}$.

Calculated for $C_{24}H_{46}$.	Found.
108.4	108

Several higher fractions were collected at points where the oil showed a tendency to accumulate, and the results of analysis indicated a series still poorer in hydrogen, C_nH_{2n-4}. 25 grams of the oil collected at 253°–255°, from which the solid was separated by filtration at −10°, and the heavy viscous oil was purified by sulphuric acid in gasoline solution. Specific gravity of the purified oil at 20° was found to be 0 8842. A combustion gave values for carbon and hydrogen required for the series C_nH_{2n-4}.

0.1424 gram of the oil gave 0.4511 gram CO_2 and 0.1718 gram H_2O.

	Calculated for $C_{23}H_{42}$.	Found.
C	86.79	86.42
H	13.21	13.49

Several attempts to determine the molecular weight of this hydrocarbon by the freezing point method failed on account of the slight depression, and the amount of oil was not sufficient for a determination by the boiling point method. The index of refraction was found to be 1.4797, which corresponds to the following molecular weight:

Calculated for $C_{23}H_{42}$	Found.
101.7	162.1

A fraction that collected in considerable quantity at 263°–265°, 30 mm., after treatment with sulphuric acid in gasoline solution and filtration at −10°, gave as its specific gravity at 20°, 0.8864. A combustion gave the following percentages of carbon and hydrogen:

0.1470 gram of the oil gave 0.4670 gram CO_2 and 0.1752 gram H_2O.

	Calculated for $C_{24}H_{44}$.	Found.
C	86.75	86.68
H	13.25	13.32

The molecular weight of this hydrocarbon was not determined for the same reasons as the preceding. A determination of the index of refraction gave 1.4802, corresponding to the following molecular refraction:

Calculated for $C_{24}H_{44}$.	Found.
106.5	106.3

A fraction collected in quantity of 40 grams at 275°–278°, 30 mm., that was filtered at −10°, and the thick viscous oil purified with acid. It gave as its specific gravity at 20°, 0.8912. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n-4}:

0.1547 gram of the oil gave 0.4914 gram CO_2 and 0.1814 gram H_2O.

	Calculated for $C_{25}H_{46}$.	Found.
C	86.71	86.66
H	13.29	13.11

The molecular weight of this hydrocarbon was not determined. The index of refraction was found to be 1.4810, and the molecular refraction as follows:

Calculated for $C_{25}H_{46}$.	Found.
110.9	110.5

While the precise formulas of the last three hydrocarbons are not determined with as complete data as those preceding, it is evident that the determinations establish the series C_nH_{2n-4}.

Ohio petroleum is therefore composed chiefly of the series C_nH_{2n+2}, C_nH_{2n}, C_nH_{2n-2}, and the series C_nH_{2n-4}. It resembles Pennsylvania petroleum in the large proportion of solid paraffine hydrocarbons. But the greater portion of the portions boiling above 216° consists of the series C_nH_{2n}, which together with the series poorer in hydrogen explains the higher specific gravity of the crude oil and distillates prepared from it. It is probable that series still poorer in hydrogen are contained in the less volatile portions that cannot be distilled without decomposition.

The following table gives a summary of the hydrocarbons described in this paper with their boiling points and specific gravity at 20° :

Hydrocarbons separated from Ohio Trenton Limestone Petroleum.

Series.	Symbol.	Boiling Point.		Specific Gravity at 20°.
C_nH_{2n}	$C_{12}H_{24}$	211°–213°	at. pressure	0.7970
"	$C_{13}H_{26}$	223°–225°	" "	0.8055
"	$C_{14}H_{28}$	138°–140°	30 mm.	0.8129
"	$C_{15}H_{30}$	152°–154°	" "	0.8204
"	$C_{16}H_{32}$	164°–168°	" "	0.8254
"	$C_{17}H_{34}$	177°–179°	" "	0.8335
C_nH_{2n-2}	$C_{19}H_{36}$	198°–202°	" "	0.8364
"	$C_{21}H_{40}$	213°–217°	" "	0.8417
"	$C_{22}H_{42}$	224°–227°	" "	0.8614
"	$C_{24}H_{46}$	237°–240°	" "	0.8639
C_nH_{2n-4}	$C_{23}H_{42}$	253°–255°	" "	0.8842
"	$C_{24}H_{44}$	263°–265°	" "	0.8864
"	$C_{25}H_{46}$	275°–278°	" "	0.8912

II. THE HYDROCARBONS IN CANADIAN PETROLEUM WITH HIGH BOILING POINTS.

By Charles F. Mabery.

In a former paper* the constituents of Canadian Corniferous limestone petroleum distilling below 216° were described and were shown by analysis and determination of molecular weights to belong to the series C_nH_{2n+2}, to the hydrocarbon $C_{11}H_{22}$ boiling at 196°. The hydrocarbon boiling at 216° was found to be $C_{12}H_{24}$. Since several series have appeared in Pennsylvania and Ohio Trenton limestone crude oil, it seemed of interest to ascertain the series that form the main constituents of Canadian petroleum in the portions with high boiling points. The distillates separated as a part of the former work were available for the study of the higher constituents.† Examination of the fraction collected at 216° was repeated, and further results were obtained in verification of the former work, which assigned the formula $C_{12}H_{24}$ to this hydrocarbon.

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* These Proceedings, **32**, 156 (1897).

† A part of this work was performed by Mr. E. T. Numsen, who selected it as the subject of a thesis for the degree of Bachelor of Science.

A determination of molecular established the formula $C_{12}H_{24}$.

0.4528 gram of the oil and 23.375 grams benzol gave a depression of 0.555.

Calculated for $C_{12}H_{24}$	Found.
168	171

The chlorine product was formed also, by the action of chlorine on the hydrocarbon standing over water. On fractioning the product it collected for the most part at 160°, 15 mm., and this distillate gave as its specific gravity at 20°, 0.9145.

The percentage of chlorine corresponded to the formula $C_{12}H_{23}Cl$.

0.1910 gram of the oil gave 0.1338 gram AgCl.

Calculated for $C_{12}H_{24}$.	Found.
17.52	17.32

The molecular weight of the chlorine product at the freezing point gave the required value for the same formula.

0.361 gram of the oil and 20.79 grams benzol gave a depression of 0°.425.

Calculated for $C_{12}H_{23}Cl$.	Found.
202.5	200

Distillation of the higher fractions was repeated several times, to collect so far as possible the hydrocarbons within narrow limits.

HYDROCARBON $C_{13}H_{26}$, B. P. 228°–230°.

The next fraction examined was the one which collected at 228°–230° after the 20th distillation. Without purification it gave as its specific gravity at 20°, 0.8087. After thorough agitation with concentrated and fuming sulphuric acid its specific gravity was reduced to 0.7979, which agrees closely with the specific gravity of the hydrocarbon $C_{13}H_{26}$ separated from Ohio petroleum.

A combustion gave values for carbon and hydrogen required for the series C_nH_{2n}.

0.1614 gram of the oil gave 0.5068 gram CO_2 and 0.2060 gram H_2O.

	Calculated for $C_{13}H_{26}$.	Found.
C	85.70	85.64
H	14.30	14.28

A determination of the molecular weight of this hydrocarbon gave values required for $C_{13}H_{26}$.

0.4631 gram of the oil and 17.79 grams benzol gave a depression of 0°.689.

Calculated for $C_{13}H_{26}$.	Found.
182	185

This formula was further verified by its index of refraction; the index found, 1.444, corresponded to the molecular refraction:

Calculated for $C_{13}H_{26}$.	Found.
59.84	60.59

The formula was still further confirmed by its chloride, which was formed by passing chlorine through the purified oil over water in diffuse sunlight until sufficient was absorbed to form the greatest yield of monochloride. After several distillations *in vacuo* a considerable portion collected at 165°, under a tension of 15 mm. A Carius determination gave the required percentage of chlorine for the monochloride.

0.1548 gram of the oil gave 0.1014 gram AgCl.

	Calculated for $C_{13}H_{25}Cl$.	Found.
Cl	16.40	16.20

The molecular weight of the chloride was determined by the freezing point method.

0.3069 gram of the oil and 19.14 grams benzol gave a depression of 0°.355.

Calculated for $C_{13}H_{25}Cl$.	Found.
216.5	220.4

The formula of the chloride was also shown by its index of refraction; the index found, 1.465, corresponded to the molecular refraction:

Calculated for $C_{13}H_{25}Cl$.	Found.
64.78	65.02

A determination of the specific gravity of the chloride gave 0.9221.

HYDROCARBON $C_{14}H_{28}$, B. P. 141°–143° (50 mm.).

After the twentieth distillation, a fraction collected in considerable quantity within the limits 141°–142°, 50 mm., that gave without puriﬁcation as its specific gravity at 20°, 0.8206. After purification with concentrated and fuming sulphuric acid, its specific gravity was reduced to 0.8099. This hydrocarbon was shown to be of the series C_nH_{2n} by determinations of carbon and hydrogen.

I. 0.1657 gram of the oil gave 0.5185 gram CO_2 and 0.2135 gram H_2O.
II. 0.1656 gram of the oil gave 0.5201 gram CO_2 and 0.2079 gram H_2O.

	Calculated for $C_{14}H_{28}$.	Found.	
C	85.70	85.34	85.65
H	14.30	14.35	14.04

The formula was shown by its molecular weight.

0.3121 gram of the oil and 18.46 grams benzol gave a depression of 0°.418.

Calculated for $C_{14}H_{28}$.	Found.
196	198

This hydrocabon gave 1.449 as its index of refraction, which corresponded to the molecular refraction of the same formula:

Calculated for $C_{14}H_{28}$.	Found.
64.44	65.02

This hydrocarbon distilled under atmospheric pressure with some decomposition at 244°–248°. Further confirmation of the formula was obtained by the composition of the chloride, which was formed in the same manner as described above. The product of chlorination was submitted to fractional distillation, which caused a considerable portion to collect at 180°, 15 mm. Specific gravity of the chloride at 20°, 0.9288. A determination of chlorine gave the percentage required for the monochloride.

0.1690 gram of the substance gave 0.1039 gram AgCl.

	Calculated for $C_{14}H_{27}Cl$.	Found.
Cl	15.40	15.21

The molecular weight of the chloride was found at the freezing point of benzol.

0.7890 gram of the substance and 23.80 grams benzol gave a depression 0°.718.

Calculated for $C_{14}H_{27}Cl$.	Found.
230.5	226.4

The index of refraction, 1.471, corresponded to the same molecular weight.

Calculated for $C_{14}H_{27}Cl$.	Found.
69.39	69.35

Another fraction of the chlorine product collected in considerable quantity at 197°–200°, 15 mm., which, after purification, distilled close at 200°, and gave as its specific gravity at 20°, 1.0066. It gave a percentage of chlorine indicating a dichloride.

0.1669 gram of the substance gave 0.1680 gram AgCl.

	Calculated for $C_{14}H_{26}Cl_2$.	Found.
Cl	26.80	24.90

The low percentage of chlorine is doubtless due to incomplete chlorination. The index of refraction found, 1.489, corresponds to the molecular refraction of the dichloride.

Calculated for $C_{14}H_{26}Cl_2$.	Found.
74.33	76.12

HYDROCARBON $C_{15}H_{30}$, B. P. 159°–160° (60 mm.).

Since a considerable portion of the distillates collected constant at 159°–160°, 50 mm., after the twentieth distillation, this fraction was selected for the identification of the next hydrocarbon, which proved to be $C_{15}H_{30}$. It gave as its specific gravity at 20°, before purification, 0.8311, and after thorough agitation with concentrated and fuming sulphuric acid, 0.8192. Determinations of carbon and hydrogen indicate the series C_nH_{2n}.

0.1586 gram of the oil gave 0.4961 gram CO_2 and 0.2007 gram H_2O.

	Calculated for $C_{15}H_{30}$.	Found.
C	85.70	85.30
H	14.30	14.16

This formula was indicated by the molecular weight determined at the freezing point of benzol.

0.3624 gram of the oil and 19.276 grams benzol gave a depression of 0°.443.

Calculated for $C_{15}H_{30}$.	Found.
210	208

The same formula was confirmed by the molecular refraction, calculated from the index found, 1.452.

Calculated for $C_{15}H_{30}$.	Found.
69.04	69.15

A portion of this hydrocarbon was chlorinated, and the product distilled until it came together in large amount at 190°, 15 mm. Its specific gravity at 20° was found to be 0.9358. Determinations of chlorine established its composition.

I. 0.1579 gram of the substance gave 0.0951 gram AgCl.
II. 0.2032 gram of the substance gave 0.1158 gram AgCl.

	Calculated for $C_{15}H_{29}Cl$.	Found.	
		I.	II.
Cl	14.45	14.55	14.09

For further identification the molecular weight of the chloride was ascertained.

0.4520 gram of the substance and 17.44 grams benzol gave a depression of 0°.529.

Calculated for $C_{15}H_{29}Cl$.	Found.
244.5	240

The index of refraction found, 1.455, corresponded to a molecular refraction of the same formula:

Calculated for $C_{15}H_{29}Cl$.	Found.
73.99	74.83

It would have been interesting to include in this examination the hydrocarbons with still higher boiling points to ascertain where the series C_nH_{2n-2} and C_nH_{2n-4} begin, for comparison with the corresponding distillates from other petroleum. But the work described above exhausted the supply of distillates prepared from the large distillation of crude oil, and the great amount of labor involved in working up to the well distilled fractions from the crude oil prevented a repetition on account of the attention demanded by more important lines of work.

The hydrocarbons described above, representing considerable proportion of Canadian petroleum, correspond in boiling points and in composition to those separated from Ohio Trenton limestone petroleum. An explanation of the higher specific gravity of Canadian crude oil and commercial products obtained from it over Ohio, Pennsylvania, West

Virginia, Kentucky, and similar crude oils, is given by the larger proportions of the series of hydrocarbons C_nH_{2n} and series poorer in hydrogen in the Canadian oil. This series begins in the fraction 196° in the latter crude oil, much lower than in any of the other petroleums mentioned, and no doubt forms a much larger proportion of the crude oil. Doubtless the same holds true of the series still poorer in hydrogen. As shown in a former paper, the series C_nH_{2n} does not appear in Pennsylvania petroleum until the hydrocarbon $C_{17}H_{34}$ is reached; but in Canadian petroleum the first hydrocarbon of this series is $C_{11}H_{22}$. In Ohio petroleum the series begins with the hydrocarbon $C_{12}H_{24}$.

Doubtless the changes in series in the crude oil are more gradual than appears from our results. One distillation as conducted in a refinery affords only a partial separation. But in long continued fractional separations the tendency toward collection in heaps indicates an accumulation of the hydrocarbons in the vicinity of their boiling points. The smaller quantities which collect between are composed of mixtures of higher and lower boiling hydrocarbons, and doubtless the larger heap is not entirely free from higher and lower boiling constituents. Perhaps this causes a balancing of molecular weights, but it is evident that the contaminating higher and lower bodies are not sufficient in amounts to interfere with determinations that establish the formula of the principal hydrocarbon.

The following table gives the hydrocarbons separated from Canadian petroleum, and the chlorides prepared from them, with their boiling points and specific gravity:

	Symbol.	Boiling Point.		Specific Gravity at 20°.
Hydrocarbon	$C_{12}H_{24}$	216°		
Chloride	$C_{12}H_{23}Cl$	160°	15 mm.	0.9145
Hydrocarbon	$C_{13}H_{26}$	228°–230°		0.8087
Chloride	$C_{13}H_{35}Cl$	165°	" "	0 9221
Hydrocarbon	$C_{14}H_{28}$	141°–143°	50 "	0.8096
Chloride	$C_{14}H_{27}Cl$	180°	15 "	0.9288
Hydrocarbon	$C_{15}H_{30}$	159°–169°	50 "	0.8192
Chloride	$C_{15}H_{29}Cl$	190°	15 "	0.9358

III. HYDROCARBONS IN SANTA BARBARA CRUDE OIL.

Composition of California Petroleum (*continued*).

In a paper published two years ago,[*] allusion was made to a specimen of peculiar heavy petroleum that we had received from Mr. J. B. Bardwell, Summerland, Santa Barbara County, Cal., representing the heavy oil that is taken from wells sunk below the Pacific Ocean. An examination of this petroleum seemed desirable, especially since it is the heaviest specimen, with one exception, that has been examined in this laboratory. It represents the last member of a series beginning with the light yellow Berea Grit oil from Southern Ohio, which was found by O. C. Dunn and me * to be the lightest petroleum that had come to our knowledge.

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[*] These Proceedings, **36**, 255.

This work was undertaken by Mr. C. V. Zoul.† The oil as it was received had the consistency of heavy tar; it would scarcely flow at ordinary temperatures.

Its specific gravity at 20° was found to be 0.9845. It contained 0.84 per cent sulphur and 1.25 per cent nitrogen. A combustion gave 86.32 per cent carbon, and 11.70 per cent hydrogen, showing that the crude oil is composed, at least to a large extent, of hydrocarbons poorer in hydrogen than petroleum from most other sources. In attempting to ascertain the composition of this petroleum, 2000 grams were fractioned under a tension of 60 mm., collecting within 5°. The first distillate came over at 200° atmospheric pressure, and only small amounts were collected below 175°, 60 mm. The distillation was carried to 365°, without attempting to collect closer than in 5° fractions, since the principal object was to ascertain the form of the hydrocarbons, although, as will appear, the fractions gave fairly close values for their molecular weights.

The heavy character of the constituents of this petroleum is shown by the specific gravity of several distillates after the fifth distillation.

150°-155°	305°-310°	325°-330°	240°-245°
0.8649	0.9641	0.9739	0.9778

The residue at 365° from 2000 grams was 200 grams, which showed that the greater portion of the oil distilled between 150° and 360°, or within a range of about 200°. Only 85 grams came over below 150°, 60 mm.; at higher temperatures the fractions collected in larger amounts at certain points, and these distillates were selected to ascertain the series of hydrocarbons:

	155°-160°	175°-180°	195°-200°	210°-215°
Weights,	25	70	50	75
	223°-230°	250°-255°	310°-315°	340°-345°
Weights,	75	50	80	45

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* Am. Chem. Journ., **18**, 1 (1896).

† Mr. Zoul selected this work as the subject of a thesis for the degree of Bachelor of Science.

HYDROCARBON $C_{13}H_{24}$, B. P. 150°–155° (60 mm.).

Before treatment with acids, the distillate 150°–155° gave as its specific gravity at 20°, 0.8649 and 0.8621 after purification with concentrated and fuming sulphuric acid and washing with caustic soda.

A combustion gave percentages of carbon and hydrogen corresponding to the formula $C_{13}H_{24}$.

0.1357 gram of the oil gave 0.4312 gram CO_2 and 0.1653 gram H_2O.

	Calculated for $C_{13}H_{26}$.	Found.
C	86.67	86.68
H	13.33	13.62

The molecular weight of this hydrocarbon at the freezing point of benzol indicated the same formula.

0.3641 gram of the oil and 18.17 gram benzol gave a depression of 0°.541.

Calculated for $C_{13}H_{24}$.	Found.
180	181.5

A determination of the index of refraction gave 1.4681, from which the molecular refraction was calculated:

Calculated for $C_{13}H_{24}$.	Found.
57.74	58.05

HYDROCARBON $C_{16}H_{30}$, B. P. 175°–180° (60 mm.).

The refraction 175°–180° was rather thick and viscous, with a specific gravity, unpurified, at 20°, 0.8832. After purification its specific gravity was reduced to 0.8808 with comparatively little change. Percentages of carbon and hydrogen by combustion corresponded to the formula $C_{16}H_{30}$.

0.1533 gram of the oil gave 0.4879 gram CO_2 and 0 1797 gram H_2O.

	Calculated for $C_{16}H_{30}$.	Found.
C	86.48	86.82
H	13.52	13.11

A determination of molecular weight at the freezing point of benzol gave the same formula.

0.7134 gram oil and 16.56 gram benzol gave a depression of 0°.974.

Calculated for $C_{16}H_{30}$.	Found.
222	216.7

The index of refraction was found to be 1.470, which gave the molecular refraction for the formula $C_{16}H_{30}$.

Calculated for $C_{16}H_{30}$.	Found.
71.6	70.3

Hydrocarbon $C_{17}H_{30}$, B. P. 190°–195° (60 mm.).

Very little difference in specific gravity of this distillate was found before and after purification. The unpurified distillate gave 0.8969, the purified, 08919. The proportions of carbon and hydrogen by combustion indicated a change in series from C_nH_{2n-2} in the preceding hydrocarbons to C_nH_{2n-4}.

0.1444 gram of the oil gave 0.4628 gram CO_2 and 0.1664 H_2O.

	Calculated for $C_{17}H_{30}$.	Found.
C	87.18	87.40
H	12.82	12.90

Determination of the molecular weight at the freezing point of benzol gave a value for the formula $C_{17}H_{30}$.

0.8220 gram of the oil and 18.20 grams benzol gave a depression of 0°.921.

Calculated for $C_{17}H_{30}$.	Found.
234	240

The index of refraction was found to be 1.4778, corresponding to the following molecular refraction:

Calculated for $C_{17}H_{30}$.	Found.
74.03	74.27

Hydrocarbon $C_{18}H_{32}$, B. P. 210°–215° (60 mm.)

The viscous distillate 210°–215° had the specific gravity without purification 0.9085, and was not materially changed after thorough treatment with concentrated and fuming sulphuric acid in gasoline solution 0.8996. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n-4}.

0.1494 gram of the oil gave 0.4787 gram CO_2 and 0.1713 gram H_2O.

	Calculated for $C_{18}H_{32}$.	Found.
C	87.10	87.38
H	12.90	12.83

An 18-carbon compound was shown by a determination of its molecular weight at the freezing point of benzol.

0.8281 gram of the oil and 18.94 grams benzol gave a depression of 0°.864.

Calculated for $C_{18}H_{32}$.	Found.
248	248

This formula was also shown by the index of refraction, which was found to be 1.484 at 20° and the molecular refraction :

Calculated for $C_{18}H_{32}$.	Found.
78.64	78.86

Hydrocarbon $C_{24}H_{44}$, P. B. 250°–255°, 60 mm.

The viscous distillate 250°–255° gave as its specific gravity at 20° 0.9299. A combustion gave values for carbon and hydrogen for the series C_nH_{2n-4}.

0.1481 gram of the oil gave 0.4745 gram CO_2 and 0.1628 gram H_2O.

	Calculated for $C_{24}H_{44}$.	Found.
C	87.42	87.40
H	12.58	12.29

Determinations of the molecular weight of this hydrocarbon at the freezing point of naphthaline gave results corresponding to the formula $C_{24}H_{44}$.

I. 0.5682 gram of the oil and 11.66 grams naphthaline gave a depression of 1°.022.

II. 0.4777 gram of the oil and 12.21 grams naphthaline gave a depression of 0°.819.

Calculated for $C_{24}H_{44}$.	Found.	
	I.	II.
332	329	330

HYDROCARBON $C_{27}H_{46}$, B. P. 310°–315° (60 mm).

This distillate was very thick and viscous. For purification it was dissolved in light gasoline, and after treatment the gasoline was distilled off with the aid of a current of CO_2. The original distillate gave the specific gravity 0.9518, which was not changed by the treatment with acid; it then gave 0.9451.

Determinations of carbon and hydrogen gave values required for the series C_nH_{2n-8}.

I. 0.1479 gram of the oil gave 0.4757 gram CO_2 and 0.1588 gram H_2O.
II. 0.1486 gram of the oil gave 0.4769 gram CO_2 and 0.1613 gram H_2O.

	Calculated for $C_{27}H_{46}$.	Found. I.	Found. II.
C	87.59	87.72	87.52
H	12.41	12.00	12.13

A determination of its molecular weight at the freezing point of benzol gave a value required for a 27-carbon compound.

0.3985 gram of the oil and 17.488 grams benzol gave a depression of 0°.300.

Calculated for $C_{27}H_{46}$.	Found.
370	372

The same formula was supported by the index of refraction, 1.5146, from which the molecular refraction was calculated.

Calculated for $C_{27}H_{46}$.	Found.
116	118

HYDROCARBON $C_{29}H_{50}$, B. P. 340°–345° (60 mm.).

This distillate was purified as before in light gasoline. After the gasoline was removed there remained an extremely viscous oil, light yellow in color, that would scarcely flow at ordinary temperatures. Specific gravity at 20°, 0.9778. The original fraction had scarcely any odor, consequently contained no phenol derivative.

A combustion of the purified oil gave the following values for carbon and hydrogen:

0.1489 gram of the oil gave 0.4781 gram CO_2 and 0.1634 gram H_2O.

	Calculated for $C_{29}H_{50}$.	Found.
C	87.44	87.76
H	12.56	12.28

The higher distillates were not examined, since the results described above are sufficient to establish the composition of the main body of the crude oil. The difficulties in obtaining hydrocarbons at these high temperatures at all well fractioned, and analytical values to decide their composition, are very great. It is evident that the series of hydrocarbons which compose Santa Barbara petroleum explain the peculiar nature of this oil, which is unlike any other petroleum that has been examined in this laboratory. The most volatile distillates are composed of the series C_nH_{2n-2}, and the proportions of hydrogen gradually fall off through the series C_nH_{2n-4} and C_nH_{2n-8}. This change in series is accompanied with a corresponding increase in specific gravity. These viscous hydrocarbons evidently approach the constituents of the asphaltic oils and natural tars. It is easy to see how petroleum is converted by slow evaporation in nature into the great beds of tars and asphalts, and it affords the most probable explanation of their formation.

The hydrocarbons described in this paper, with their boiling points and specific gravity, are brought together in the following table:

HYDROCARBONS SEPARATED FROM SANTA BARBARA (CAL.) PETROLEUM.

	Symbol.	Boiling Point.		Specific Gravity at 20°.
Hydrocarbon	$C_{13}H_{24}$	150°–155°,	60 mm.	0.8621
"	$C_{16}H_{30}$	175°–180°,	"	0.8808
"	$C_{17}H_{30}$	190°–195°,	"	0.8919
"	$C_{18}H_{32}$	210°–215°,	"	0.8996
"	$C_{24}H_{44}$	250°–255°,	"	0.9299
"	$C_{27}H_{46}$	310°–315°,	"	0.9451
"	$C_{29}H_{50}$	340°–345°,	"	0.9778

IV. SEPARATION OF SOLID PARAFFINE HYDROCARBONS FROM PETROLEUM WITHOUT EVAPORATION.

BY CHARLES F. MABERY AND OTTO J. SIEPLEIN.

By the method of distillation, which is the only means for the separation of the petroleum hydrocarbons, it is not possible to determine whether the solid paraffine hydrocarbons are contained in the crude oil or whether they may not be formed by decomposition.

As a direct proof that the solid hydrocarbons are actually contained in petroleum we have separated them without distillation. Three kilos of an average specimen of Pennsylvania crude petroleum was allowed to stand in a shallow pan exposed to a current of air in a strong flue draft for thirty days. The change was shown by the increase in specific gravity:

Specific gravity at 15° before exposure. Dec. 9.	Dec. 12.	Jan. 8.
0.000	0.840	0.862

When taken from the flue, January 8, the residue weighed one kilo, which showed a loss of 66.67 per cent. The original oil distilled as follows:

50°–150°.	150°–200°.	200°–250°.	250°–300°.	+300°.
21 per cent.	11 per cent.	11 per cent.	13 per cent.	42 per cent.

The residue above 300° is, therefore, about the same in weight as the residue left by evaporation. That the loss by evaporation had practically ceased at the end of one month was shown by allowing the residue to stand one year in the same place, exposed to draft as before. But there was no change in weight nor in specific gravity.

A combustion of the crude oil gave the following percentages of carbon and hydrogen:

0.1968 gram of the oil gave 0.4915 gram CO_2 and 0.2512 gram H_2O.

C	85.51
H	14.18

Combustion of the residue after 30 days' evaporation:

0.1559 gram of the oil gave 0.4925 gram CO_2 and 0.1559 gram H_2O.

C	86.16
H	13.69

Evaporation of the portions with lower boiling points, which form the greater part of the series C_nH_{2n+2}, leaves a larger proportion of series poorer in hydrogen, C_nH_{2n} and C_nH_{2n-2} corresponding to the composition shown by analyses.

On cooling with ice the residue from evaporation became quite solid.

On distillation, 28 per cent came over below 30°, 6 per cent at 300°–360°, with a residue above 360° of 66 per cent.

The amount of solid hydrocarbons in the residue was determined by the method suggested by Zalioziecki. 50 grams of the residue with fusel oil, mostly isoamyl alcohol, was allowed to stand at 0° for 24 hours; 250 grams alcohol was then added and the precipitate collected on a filter, and washed with a mixture of fusel oil and alcohol. The precipitate on the filter was then placed in an extraction apparatus, and the solid extracted with benzol. After evaporation of the benzol, the residue was heated to 140° for one hour to remove the last traces of fusel oil. 19.8 grams of a greenish black solid remained, melting at 32°, equivalent to 39.6 per cent of the heavy residue or to 14 per cent of the original oil before evaporation.

The solid was again dissolved in fusel oil and precipitated by alcohol, giving a smaller quantity of solid melting at 45°. This treatment repeated gave a light brown solid melting at 57°. The latter was then dissolved in ether and precipitated by alcohol, which gave a lighter colored solid; another repetition gave a white solid melting at 61°. Its specific gravity was 0.7966 at 70°, corresponding to that of paraffine, and a combustion gave percentages of carbon and hydrogen required for paraffine hydrocarbons.

0.1511 gram of the solid gave 0.4730 gram CO_2 and 0.1984 gram H_2O.

C	85.37
H	14.69

The specific gravity, melting point, and composition of this substance indicate that it was composed of paraffine hydrocarbons, and demonstrate the presence of the solid paraffine hydrocarbons in Pennsylvania petroleum.

Paraffine collected from heavy distillates in the refinery is therefore obtained as it exists from the original crude oil, and it is not formed during distillation. Indeed, as explained in another paper, it cannot be formed; for any decomposition of petroleum hydrocarbons by heat removes hydrogen with a conversion of the hydrocarbon into a lower series; for example, of the paraffine hydrocarbons of the series C_nH_{2n+2} into the series C_nH_{2n}, C_nH_{2n-2}, etc., and perhaps also forming the methylenes and condensed methylenes. Such changes do not involve an absorption of oxygen into the hydrocarbon molecule, but a removal of hydrogen by oxygen as water. That oxygen compounds of the nature of phenols are found in petroleum from various fields has been fully demonstrated independently by Richardson and by Mabery (data not published), and it is quite probable that most petroleum contain these bodies to a greater or less extent; but they have been formed by natural processes.

In ordinary refinery distillation, there is probably little decomposition below 300° C., but at this point cracking begins and increases rapidly to the end. There are doubtless reactions at higher temperatures between nitrogen, sulphur, and oxygen compounds. Then the breaking down in series begins with loss of hydrogen, through the asphaltic hydrocarbons, until finally coke is reached, which forms a considerable proportion of the products of refining.

V. THE SOLID PARAFFINE HYDROCARBONS THAT COLLECT IN CERTAIN OIL WELLS IN PENNSYLVANIA.

In attempting to determine the series of hydrocarbons, and members of the series that form the solid constituents of Pennsylvania petroleum, it seemed of interest to examine the semi-solid product that collects in considerable quantites in some of the Pennsylvania wells, especially at Coreopolis, Pa. The Petrolatum Company collects large quantities of this pasty substance from their wells and from it manufacture various preparations such as vaseline and cosmoline. The crude product is obtained in the form of a light-yellow pasty mass, from which it is impossible to separate a solid by pressure in filter paper or by filtration. It evidently consists of an emulsion of the high boiling oils with the solid hydrocarbons which no doubt are in part dissolved in the oily constituents.

Having obtained several gallons of this substance through the kindness of the Coreopolis Petrolatum Company in October, 1898, I undertook an examination of it with the principal object of proving the presence of solid hydrocarbons, and identifying their composition as to the series and molecular formulas, for comparison with the solid hydrocarbons which I have separated from Pennsylvania crude oil and from commercial paraffine. These semi-solid emulsions have no doubt been formed by evaporation of the lighter constituents of petroleum, and the solid hydrocarbons are therefore what were contained in the original petroleum. It also afforded an opportunity to confirm the presence of the solid constituents of paraffine in crude petroleum, a question concerning which a doubt is often expressed by refiners.

The specimen received was in the form of a thick paste, with a specific gravity at 60° of 0.8345. In beginning the distillation, 9380 grams were distilled in a porcelain still under a tension of 50 mm. and the distillates collected in the following quantities:

	-195	195°-200°	200°-245	245°-265°	265°-285
Grams	905	865	895	850	830
	285°-315°	315°-330	330°-342		Residue.
Grams	200	790	885		3000

Distillation of these fractions was continued within 10°, 5°, and 2°, until after the sixth distillation larger quantities collected at the following temperatures :

242°–244° 268°–270° 280°–282° 308°–310° 328°–330°

340°–342° 352°–354 ° 368°–370° 382°–384°

Since the distillates below 268° showed by their thin pasty condition that they contained only small proportion of solid hydrocarbon, no attempts were made to separate their constituents. The fraction 272°–274° gave as its specific gravity at 20° 0.8116. A combustion gave the following percentages of carbon and hydrogen :

0.1600 gram of the substance gave 0.5058 gram CO_2 and 0.1963 gram H_2O.

Calculated for $C_{25}H_{48}$.	Found
86.21	86.22
13.79	13.73

This fraction, therefore, consisted mainly of a hydrocarbon of the series C_nH_{2n-2}, and the same will appear in the higher fractions to be described.

On cooling the distillates with salt and ice, and attempting to separate the solid by filtration under pressure, scarcely any of the solid remained on the filter, so complete was the emulsion. But on trying the solubility in ether and alcohol, such a wide difference was observed between the crystalline and amorphous solid, it afforded an easy means of separation.

The distillate 272°–274°, 30 mm., was therefore dissolved in a suitable mixture of ether and alcohol cooled with salt and ice and filtered cold. The solid collected was pressed and the crystallization repeated, which gave a perfectly white crystalline hydrocarbon, melting point 50°–51°. It gave as its specific gravity at 60°, 0.7900. Determinations of carbon and hydrogen gave percentages required for the series C_nH_{2n+2}.

0.1516 gram of the substance gave 0.4728 gram CO_2 and 0.1973 gram H_2O.

	Calculated for Tetracosane, $C_{24}H_{50}$.	Found.
C	85.21	85.06
H	14.79	14.56

The quantity of the solid hydrocarbon obtained was small, evidently not sufficient to influence the proportions of carbon and hydrogen in the unfiltered distillate. While the amount obtained was not enough for a determination of its molecular weight, its melting point and specific gravity, the same as those of the hydrocarbon tetracosane separated from paraffine distillate 272°–274°, indicate that it has the same composition.

Tetracosane obtained by Krafft,* by reduction of the ketone derived from barium stearate and barium heptylate, melted at 51°. But the specific gravity of Krafft's hydrocarbon was somewhat lower than that of the hydrocarbon from petroleum.

The quantities of the distillates between 274° and 316° were too small to permit of a separation of crystalline hydrocarbons in sufficient amounts for identification of individual constituents.

The pasty solid collected at 316°–318°, 50 mm., yielded a solid hydrocarbon by crystallization from alcohol and ether that melted at 66°. The crude distillate gave as its specific gravity at 20°, 0.8212. A combustion gave values required for the series C_nH_{2n-2}.

0.1564 gram of the substance gave 0.4935 gram CO_2 and 0.1930 gram H_2O.

	Calculated for Hentricontane, $C_{28}H_{54}$.	Found.
C	86.15	86.05
H	13.85	13.80

The purified solid gave as its specific gravity at 70°, 0.7997.

A combustion gave numbers for carbon and hydrogen required for the series C_nH_{2n+2}.

0.1499 gram of the substance gave 0.4698 gram CO_2 and 0.1969 gram H_2O.

	Calculated for $C_{31}H_{64}$.	Found.
C	85.31	85.45
H	14.69	14.69

The formula of this hydrocarbon was further verified by its molecular weight.

0.3648 gram of this substance and 12.12 grams naphthaline gave a depression of 0.471.

Calculated for $C_{31}H_{64}$.	Found.
436	441

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* Ber. Deutsch. chem. Gesellsch., **15**, 1718 (1882).

From the ketone palmiton, $C_{31}H_{62}O$, Krafft * prepared Hentricontane, $C_{31}H_{64}$, M. P. 68°.1, which differs but slightly from that of the hydrocarbon separated from petroleum, 66°.

Since the solid hydrocarbon formed such a small proportion of the original distillate that it did not affect materially the composition of the latter, this solid evidently could not be compared with the hydrocarbon separated from the corresponding distillate from paraffine nor that from crude Pennsylvania petroleum, for the predominating constituents should materially affect the boiling points of all the minor solid constituents. This is shown to be true by the high molecular weights and high melting points of the hydrocarbons separated from Coreopolis distillates so far as they could be determined. The distillate collected at 328°–330°, 50 mm., after filtration from the solid, gave as its specific gravity at 70°, 0.8235. After crystallization from ether and alcohol, the white crystalline solid gave as its specific gravity at 70°, 0.7982. The proportions of carbon and hydrogen given by combustion supported the formula of the series C_nH_{2n+2}. The original distillate gave as its specific gravity 0.8217. A combustion gave the following percentages of carbon and hydrogen.

0.1586 gram of the substance gave 0.4995 gram CO_2 and 0.1925 gram H_2O.

	Calculated for $C_{29}H_{56}$.	Found.
C	86.14	85.86
H	13.86	13.57

The solid purified from ether and alcohol gave proportions of carbon and hydrogen corresponding to the series C_nH_{2n+2}.

0.1549 gram of the solid gave 0.4842 gram CO_2 and 0.2075 gram H_2O.

	Calculated for $C_{32}H_{66}$.	Found.
C	85.33	85.25
H	14.67	14.98

The melting point of the crystalline hydrocarbon was found to be 67°–68°. Its molecular weight at the boiling point of benzol and also at the freezing point of naphthaline indicated the next higher homologue, $C_{32}H_{66}$, dotricontane, but the small quantity of substance available for this determination was not sufficient to yield satisfactory results.

The specific gravity of the unpurified distillate 342°–344°, 50 mm., was 0.8288; after separation of the solid by freezing and purification

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* Ber. Deutsch. chem. Gesellsch., **15**, 1714.

from ether and alcohol its specific gravity was 0.8005 at 75°. Melting point of the purified hydrocarbon, 68°–70°. The unpurified distillate gave percentages of carbon and hydrogen corresponding to the series C_nH_{2n-2}.

0.1517 gram of the substance gave 0.4784 gram CO_2 and 0.1850 gram H_2O.

	Calculated for $C_{30}H_{58}$.	Found.
C	86.12	86.06
H	13.88	13.64

The purified solid gave proportions of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1544 gram of the solid gave 0.4816 gram CO_2 and 0.2060 gram H_2O.

	Calculated for $C_{32}H_{66}$.	Found.
C	85.33	85.06
H	14.67	14.92

The molecular weight as determined by the boiling point method confirmed the formula $C_{32}H_{66}$.

0.5127 gram of the substance and 12.23 grams benzol gave a depression of 0°.654.

Calculated for $C_{32}H_{66}$.	Found.
438	442

As mentioned above, the solid hydrocarbons form a small proportion of the original oil, and the small amounts that collect at 328°–330° and 340°–342° have doubtless nearly the same composition, $C_{32}H_{66}$.

The distillate 366°–368°, 50 mm., before treatment, had the specific gravity 0.8312, and the solid hydrocarbon separated by freezing, filtration, and crystallization from ether and alcohol, the specific gravity at 80°, 0.8009. The purified solid melted at 71°–72°. The proportions of carbon and hydrogen in the unpurified distillate corresponded to the formula C_nH_{2n-2}.

0.1485 gram of the substance gave 0.4982 gram CO_2 and 0.1851 gram H_2O.

	Calculated for $C_{31}H_{60}$.	Found.
C	86.11	85.96
H	13.89	13.94

The solid separated from the fraction 366°–368°, after purification, gave on analysis 85.41 per cent carbon and 14.74 per cent hydrogen, corresponding to the series C_nH_{2n+2}, was present in only small quantity, and probably differed but slightly from the hydrocarbon contained in the fraction 342°–344°.

The close agreement in composition of the untreated distillate, before removing the solid hydrocarbon, to the series C_nH_{2n-2}, is due, as explained above, to the very small proportion of the solid hydrocarbon, C_nH_{2n+2}. The oils separated by filtration are evidently composed for the greater part of the series C_nH_{2n-2}, although the low percentage of hydrogen may indicate the presence in small amounts of a series even poorer in hydrogen.

The untreated distillate 380°–384°, 50 mm., gave as its specific gravity 0.8336, and the solid hydrocarbon obtained by freezing, filtration, and crystallization from ether and alcohol, 0.8052 at 80°. Melting point, 76°. Determinations of carbon and hydrogen indicated the series C_nH_{2n-2}.

0.1560 gram of the substance gave 0.4898 gram CO_2 and 0.1943 gram H_2O.

	Calculated for $C_{34}H_{66}$.	Found.
C	86.08	95.82
H	13.92	13.93

A combustion of the purified solid gave percentages of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1501 gram of the solid gave 0.4695 gram CO_2 and 0.1974 gram H_2O.

	Calculated for $B_{35}H_{72}$.	Found.
C	85.37	85.33
H	14.63	14.71

The molecular weight of the solid as determined by the boiling point method corresponded to the same formula.

1.0012 gram of the substance and 24.72 grams benzol gave a rise of 0°.212.

Calculated for $C_{35}H_{72}$.	Found.
492	490

As the molecular weights of the hydrocarbons of these series increase the determinations become more and more difficult and uncertain. The values obtained support, together with other data, the formulas given. With the high boiling liquids there is less trouble than with solids.

As formerly explained, the freezing point of benzol cannot be relied on for solids with molecular weights much above $C_{20}H_{42}$. At its boiling point benzol can be depended on for a somewhat higher range, but becomes uncertain with hydrocarbons in the vicinity of $C_{30}H_{62}$. Other solvents are equally uncertain, on account of a lack of solubility or uncertain influence of the dissolved substance on the freezing point. After long trial with naphthaline, which promised reliable results on account of its free solvent power and large depression, we found that the substances that had melting points not far removed from that of naphthaline gave variations in depression too great for reliable calculations of molecular weights.

SOLID HYDROCARBONS IN PENNSYLVANIA OIL WELLS.

	Symbol.	Melting point.	Specific gravity.
Tetracosane	$C_{24}H_{50}$	50°–51°	0.7900 at 60°
Hentricontane	$C_{31}H_{64}$	66°	0 7997 at 70°
Dotricontane	$C_{32}H_{66}$	67°–68°	8.8005 at 75°
Tetratricontane	$C_{34}H_{66}$	71°–72°	0.8009 at 80°
Pentatricontane	$C_{35}H_{72}$	76°	0.8052 at 80°

Hydrocarbons of the series C_nH_{2n+2} have now been identified in Pennsylvania petroleum in continuous series, with but few members wanting from butane, C_4H_{10}, B. P. −10°, to pentatricontane, $C_{35}H_{72}$, B. P. 380°–384°, 50 mm.

VI. COMPOSITION OF COMMERCIAL PARAFFINE.*

It has been questionable, even with refiners of petroleum of long experience, as to whether paraffine is actually present in the crude oil or whether it is a result of change during the process of refining. It therefore seemed of interest to ascertain whether the hydrocarbons of which commercial paraffine is composed are identical with the solid constituents of the crude oils that yield paraffine.

The method of distillation was the same as that employed in distilling the crude oil, except that it was found necessary to pack the neck of the flask with a thick coating of asbestos. With exclusion of air it was found that paraffine could be distilled indefinitely with no decomposition. But introduction of air, either by leaks or by accident, caused serious decomposition, as shown by the dark color and disagreeable odor of the distillates.

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* A part of the work described in this paper was performed by Mr. H. R. Payne, and formed the subject of a thesis for the degree of Bachelor of Science.

1500 grams of the best commercial paraffine was distilled *in vacuo* under 40 mm., and the distillates were collected at first within 10°, then 5°, and finally within 2°, altogether seven times. The lowest distillate collected below 250°, and the final residue above 350° was only 30 grams. All the distillates were colorless, except the residue, which was slightly brown, and nearly odorless. The distillates collected in larger quantities at temperatures corresponding to those of hydrocarbons separated from the crude oil as follows:

256°–258°	90 grams	316°–318°	35 grams
272°–274°	45 "	332°–334°	20 "
282°–284°	70 "	346°–348°	40 "
292°–294°	30 "		

TRICOSANE, $C_{23}H_{48}$, B. P. 256°–258° (40 mm.).

The distillate 256°–258° melted without purification at 48°, and the melting point was not raised by crystallization from alcohol and ether. Tricosane obtained by Krafft * melted at 47°.7. Its specific gravity was ascertained above its melting point.

60°	0.7836	70°	0.7814	80°	0.7807

The specific gravity of tricosane from petroleum differs materially from the specific gravity of this hydrocarbon which Krafft obtained from the ketone, lauron.

47°.7	0.7785	80°.8	0.7570	88°.8	0.7456

A combustion of this hydrocarbon gave percentages of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1510 gram of the substance gave 0.4710 gram CO_2 and 0.1989 gram H_2O.

	Calculated for $C_{23}H_{48}$.	Found.
C	85.20	85.06
H	14.80	14.73

A determination of its molecular weight at the boiling point of benzol gave a value required for tricosane.

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* Ber. d. chem. Gesellsch., **15**, 1713 (1882).

0.8907 gram of the solid and 24.24 grams benzol gave a rise of 0°.295.

Calculated for $C_{23}H_{48}$.	Found.
324	319

TETRACOSANE, $C_{24}H_{50}$, B. P. 272°–274° (40 mm.).

This distillate melted without purification at 50°–51°, which was not changed by crystallization from alcohol and ether. Krafft's tetracosane obtained from the ketone derived from barium stearate and barium heptylate melted at 51°.1. The quantity obtained of tetracosane was not sufficient to determine its specific gravity.

This hydrocarbon was shown to belong to the series C_nH_{2n+2} by the following combustions:

I. 0.1424 gram of the substance gave 0.4451 gram CO_2 and 0.1923 gram H_2O.
II. 0.1460 gram of the substance gave 0.4551 gram CO_2 and 0.1955 gram H_2O.
III. 0.1443 gram of the substance gave 0.4502 gram CO_2 and 0.1924 gram H_2O.

	Calculated for $C_{24}H_{50}$.	Found. I.	II.	III.
C	85.21	85.25	85.00	85.08
H	14.79	14.96	14.97	14.91

The formula was verified by a determination of its molecular weight.

3.0316 grams of the oil and 22.42 grams benzol gave a rise of 1°.034.

Calculated for $C_{24}H_{50}$.	Found.
338	335.

PENTACOSANE, $C_{25}H_{52}$, B. P. 282°–284° (40 mm.).

The melting point of the unpurified distillate 282°–284° was 53°–54°. After crystallization from ether and alcohol, it melted at 54°.

Its specific gravity was determined above its melting point:

60°	0.7911	80°	0.7870
70°	0.7881	90°	0.7854

A combustion gave percentages of carbon and hydrogen required for the formula C_nH_{2n+2}.

0.1467 gram of the substance gave 0.4578 gram CO_2 and 0.1950 gram H_2O.

	Calculated for $C_{25}H_{52}$.	Found.
C	85.23	85.09
H	14.77	14.88

The formula was identified by a determination of its molecular weight.

1.4324 grams of the substance and 8.5282 grams benzol gave a rise of 1°.265.

Calculated for $C_{25}H_{52}$.	Found.
352	354

HEXACOSANE, $C_{26}H_{54}$, B. P. 294°–296° (40 mm.)

After the seventh distillation, 25 grams collected at 294°–296°, that gave as its melting point 55°–56°, and this was not changed by crystallization from ether and alcohol. It gave as its specific gravity above its melting point:

60°	0.7938	80°	0.7893
70°	0.7918	90°	0.7879

The percentages of carbon and hydrogen obtained by combustion corresponded to the series C_nH_{2n+2}.

0.1526 gram of the solid gave 0.4757 gram CO_2 and 0.2027 gram H_2O.

	Calculated for $C_{26}H_{54}$.	Found.
C	85.23	85.02
H	14.77	14.86

The molecular weight found at the boiling point of benzol verified the formula.

1.6742 grams of the substance and 13.151 grams benzol gave a rise of 0°.93.

Calculated for $C_{26}H_{54}$.	Found.
366	365

OCTOCOSANE, $C_{28}H_{58}$, B. P. 316°–318° (40 mm.).

At 316°–318°, 35 grams collected after the seventh distillation that melted at 60°, which was not changed by crystallization from ether and alcohol. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1586 gram of the oil gave 0.4954 gram CO_2 and 0.2122 gram H_2O.

	Calculated for $C_{28}H_{58}$.	Found.
C	85.28	85.17
H	14.72	14.96

A determination of its molecular weight corresponded to the formula $C_{28}H_{58}$.

I. 1.3337 gram of the solid and 23.73 grams benzol gave a rise of 0°.362.

II. 1.7964 gram of the solid and 21.96 grams benzol gave a rise of 0°.53.

Calculated for $C_{28}H_{58}$.	Found. I.	Found. II.
394	397	396

NONOCOSANE, $C_{29}H_{60}$, B. P. 346°–348° (40 mm.).

At 346°–348° 40 grams collected; melting point, 62°–63°. It gave on combustion percentages of carbon and hydrogen for the series C_nH_{2n+2}.

0.1520 gram of the solid gave 0.4752 gram CO_2 and 0.2062 gram H_2O.

	Calculated for $C_{29}H_{60}$.	Found.
C	85.28	85.27
H	14.72	14.87

Its molecular weight was found to be as follows:

I. 1.6181 gram of the substance and 23.57 grams benzol gave a rise of 0°.41.

II. 1.6236 gram of the substance and 23.99 grams benzol gave a rise of 0°.402.

Calculated for $C_{29}H_{60}$.	Found. I.	Found. II.
406	405	408

That the solid distilling above 350° has the composition of the series C_nH_{2n+2} is shown by the following combustion:

0.1469 gram of the solid gave 0.4581 gram CO_2 and 0.1947 gram H_2O.

	Calculated for $C_{30}H_{62}$.	Found.
C	84.31	85.04
H	14.69	14.82

Since only two per cent of the original paraffine taken for distillation remained as a residue at 350°, the solid hydrocarbons which compose the main body of paraffine produced in the refinery are evidently members of the series C_nH_{2n+2}. They do not contain oxygen, as frequently stated, — at least the hydrocarbons that can be distilled. It is not reasonable to assume that they have been formed by cracking from other constituents, since such decompositions give rise to hydrocarbons of lower molecular weights. It is well known that paraffine may be decomposed completely by heat when exposed to air into lower hydrocarbons. In mass we have found that it may be distilled indefinitely by exclusion of air, but it cannot be distilled in air without decomposition.

Thorpe and Young * subjected shale paraffine to distillation under pressure in a bent tube so arranged that the paraffine could be repeatedly distilled. The distillation was continued until the solid was converted into bodies that remained liquid at ordinary temperatures. This liquid proved to consist of two series of hydrocarbons; of the series C_nH_{2n+2}, the hydrocarbons C_5H_{12} to $C_{11}H_{24}$ were identified, and of the series C_nH_{2n} the ethylene hydrocarbons, C_5H_{10} to $C_{11}H_{22}$.

It is our intention to examine the products of the distillation of paraffine in air to observe the changes in cracking. It is an interesting question as to whether any relation exists between the paraffine hydrocarbons and the methylenes with reference to the formation of these bodies in their organic origin, and whether the series are interconvertible by natural agencies.

HYDROCARBONS IN COMMERCIAL PARAFFINE.

	Symbol.	Melting Points.	Specific Gravity.
Tricosane	$C_{23}H_{48}$	48°	0.7886 at 60°
Tetracosane	$C_{24}H_{50}$	50°–51°	
Pentacosane	$C_{25}H_{52}$	53°–54°	0.7941 at 60°
Hexacosane	$C_{26}H_{54}$	55°–56°	0.7968 at 60°
Octocosane	$C_{28}H_{58}$	60°	
Nonocosane	$C_{29}H_{60}$	62°–63°	

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* Ber. d. deutsch. Gesellsch., **5**, 556 (1872).

VII. COMPOSITION OF COMMERCIAL VASELINE, COSMOLINE, AND SIMILAR PRODUCTS.

After ascertaining the composition of the constituents of Pennsylvania petroleum with high boiling points, and the composition of the semi-solid product used at Coreopolis for the manufacture of the medicinal preparations, it seemed of interest to investigate the composition of the commercial products for comparison with specimens obtained directly from the wells. On account of the very high boiling points, it is only possible to apply distillation *in vacuo*. Under 15 mm., 100 grams of vaseline (Ceroleum) distilled in the following proportions.

	–250°	250–275°	275–300	300–325	325–350	350–360°	Residue.
Grams	14	22	14	18	7	20	6

The distillates were light yellow in color, and when melted showed a slight fluorescence. Below 250° the distillate was a heavy viscous liquid, the others all semi-solid, and the residue was colored brown. To ascertain whether solids could be separated from these distillates, the fraction 250°–275° was dissolved in ether and sufficient alcohol added to precipitate the solid hydrocarbon. The solid was again dissolved in ether and precipitated by alcohol. After filtration it showed no greasy feel and melted at 70°. After similar treatment, from the fraction 350°–360°, a solid was obtained which melted at 77°–78°. These solids resembled in all respects the solid paraffine hydrocarbons. It is therefore evident that vaseline is composed of heavy oils such as have been identified as forming the portions of Pennsylvania petroleum with high boiling points, and Coreopolis heavy oil, hydrocarbons of the series C_nH_{2n}, C_nH_{2n-2}, and C_nH_{2n-4}, together with solid paraffine hydrocarbons. The quantity of solid compounds present is sufficient to saturate the oil and in slight excess to form an emulsion of the desired consistency.

What is known as "scale" paraffine in the refineries is composed of the solid hydrocarbons, with enough of the heavy oils to form a greasy solid. But the solid constituents are hydrocarbons of the same series, C_nH_{2n+2}, which cannot be changed into other forms without decomposition. These hydrocarbons are only sparingly soluble in the liquid hydrocarbons of lower boiling points of the same series; but they dissolve more freely in the series with high boiling points, C_nH_{2n+2}, C_nH_{2n}, C_nH_{2n-2}, and C_nH_{2n-4}.

Since the work described in these papers was begun, ten years ago, I have received efficient aid from the following assistants in distillation:

Messrs. C. Lindmueller, D. H. Walker, C. R. Cummins, and H. A. Triebing; in distillation and analysis, Mr. C. S. Richards; analysis and molecular weight determinations, valuable aid from Mr. O. J. Sieplein.

It is my intention to make later a summarization in a separate paper of the results published from the beginning in this series of papers on petroleum.

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A QUANTITATIVE DETERMINATION OF THE ANOMALOUS DISPERSION OF SODIUM VAPOR IN THE VISIBLE AND ULTRA-VIOLET REGIONS.

By R. W. Wood.

With Three Plates.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

A QUANTITATIVE DETERMINATION OF THE ANOMALOUS DISPERSION OF SODIUM VAPOR IN THE VISIBLE AND ULTRA-VIOLET REGIONS.

By R. W. Wood.

Presented by C. R. Cross. Received May 31, 1904.

The dispersion of a medium with but a single band of metallic absorption is represented by the formula

$$n^2 = 1 + \frac{m\,\lambda^2}{\lambda^2 - \lambda_m{}^2},$$

in which λ_m is the wave-length at the centre of the absorption band, and m a constant. This formula, which is developed from electro-magnetic considerations, is practically identical with the original Sellmeier formula. Up to the present time it has never been tested experimentally, for the reason that no medium of suitable constitution is known which admits of accurate determinations of the variation of the refractive index with the wave-length. We possess such a medium in the vapor of metallic sodium, but the experimental difficulties which stand in our way are so great that only qualitative observations have been made up to the present time.

The formula given above, being the simplest dispersion formula which we possess, ought by all means to be tested experimentally, and it was with this end in view that the work reported in the present paper was undertaken.

After a large amount of fruitless work, methods have at length been developed which have made possible accurate determinations of the absolute refractive index of the metallic vapor at various densities, for all wave-lengths comprised between the extreme red and the remote ultra-violet. For wave-lengths immediately adjoining the D lines the refractive index has been found to have a value as high as 1.38, as great as that of some liquids, while the dispersion is so great even at the position of the D_3 line of helium, that, could we form a prism of the vapor giving the same deviation as a 60° glass prism, we could by its aid separate a close Zeeman doublet by an amount as great as the distance between the red and blue of the spectrum formed by the glass prism. The results obtained have been applied to the formula, and most excellent agreement found.

The anomalous dispersion of the vapor was first observed by Kundt, and has since been studied by Becquerel, Julius, Ebert, and others. These investigators have for the most part limited their observations to the dispersion produced by flames of prismatic form containing the vapor of sodium. Under these conditions the anomalous dispersion is only apparent in the immediate vicinity of the D lines, extending to a distance up and down the spectrum not much greater than the distance which separates the lines. Some four years ago it was shown by the writer that by employing the vapor of the metal in glass tubes, it was possible to obtain far greater deviations, and to extend the observations on the dispersion from the extreme red to the violet. These results were made possible by the very remarkable physical properties of the vapor, which will be discussed more in detail presently.

The prisms formed in heated glass tubes are in reality non-homogeneous cylinders of the vapor, the density being greatest along the heated floor of the tube, and least along the top. The definition given by non-homogeneous masses of vapor is surprisingly good, and relative values of the refractive indices can be determined without difficulty; but as no means could be found of determining the angle of the equivalent prism, it was impossible to do more than guess at the absolute values. An attempt made several years ago to obtain data by interference methods gave no results, owing to the fact that the fringes were immediately obliterated by the convection currents of heated hydrogen. By adopting the expedient of heating the metal in a high vacuum it has been found possible to obtain deviations as high as 1500 fringe-widths with the D_3 light of helium, without in any way altering the distinctness of the fringes. By combining the data obtained with the interferometer with the relative values obtained by prismatic deviation, it has been possible to obtain absolute values for a very wide range of wave-lengths. We will begin with a brief discussion of the properties of the medium.

Physical and Optical Properties of the Vapor of Metallic Sodium.

One of the chief difficulties which has stood in the way of measuring the optical constants of sodium vapor is the impossibility of confining the vapor in vessels bounded by flat surfaces of glass. As soon as the temperature is raised to such a point that vapor of any considerable density forms, the glass is attacked and rendered opaque by the reduction of silica. The vapor, however, has a most remarkable viscosity, which I am at the present time investigating, which makes it possible to form a mass of great density separated from the glass plates which close the ends of the tube by a high vacuum. If we place a number of pieces of clean sodium in a tube of hard Jena glass, the ends of which are closed with small pieces of thin plate glass, and exhaust the tube on a mercurial pump, on heating the under side strongly with small Bunsen flames, the sodium vapor shows very little inclination to distil to the cold parts of the tube. It condenses, to be sure, on the upper side of the tube, but is given off so much more rapidly from the surface of the molten metal than it can diffuse to the upper portion, that the density gradient is very steep. Observations on the deviation produced by the non-homogeneous cylinder show that the equivalent prism has a form similar to that shown in Figure 1, the density gradient being steeper near the bottom of the tube. To secure good definition it is therefore necessary to place in front of the tube an opaque screen perforated with a wide horizontal slit. The tube thus prepared, used in the manner to be presently described, shows the strong anomalous dispersion in the vicinity of the D lines with great distinctness. If the flames are made to play upon the upper surface of the tube, after some of the vapor has condensed there, the spectrum straightens out, and the dispersion presently occurs in the opposite direction, showing that the base of the prism is always against the heated portion of the tube regardless of gravity.

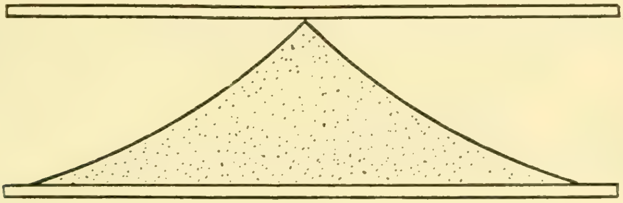

FIGURE 1.

In a tube 25 cms. long, 10 cms. of which are heated red hot on all sides by means of an iron wire carrying a heavy current, a mass of sodium vapor may form of sufficient density to give a displacement of several thousand helium (D_3) fringes; notwithstanding this, practically no trace of the vapor can be detected beyond the heated portion. This condition can be maintained for an hour or more, owing to the slowness of the diffusion. If mercury is heated under similar conditions it is impossible to obtain shifts of more than a fringe or two. Potassium distils almost in a flash to the colder parts of the tube. The other alkali metals have not as yet been investigated. The vapor of sodium appears to possess a cohesion almost as great as that of a liquid. Even in a vacuum tube it appears to have a free surface, while in an atmosphere of hydrogen it can be dipped up on a wire, a vigorous shaking of which is insufficient to dislodge it or cause its dissipation. A number of very interesting experiments with the vapor have already been made, and others are in progress at the present time. They will be reported in a subsequent paper; as for the present we are only concerned with the fact that a homogeneous cylinder of very dense vapor terminated with sharply defined ends can be formed in an exhausted tube.

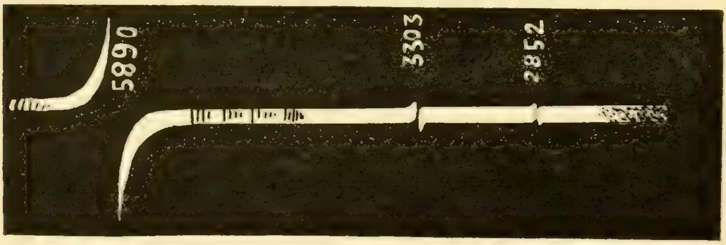

FIGURE 2.

Anomalous dispersion occurs at the absorption lines which belong to the principal series, being very strong at the D lines, feeble at the first pair of ultra-violet lines ($\lambda = 3303$), and almost imperceptible at the second pair ($\lambda = 2852$), as is shown in Figure 2. If the vapor is of considerable density, the D lines run together into a single absorption band, and the dispersion can be represented throughout the complete range of wave-lengths as resulting from electrons of a single period, for the ultra-violet bands only affect the refractive index in their immediate vicinity. These bands make their appearance in succession as the density of the vapor increases, the second not appearing until the band at the D lines has become three or four times as broad as the distance between the lines. The complicated channelled absorption spectrum appears to exert no influence on the refractivity of the vapor.

I feel certain, however, that the channelled absorption spectrum affects the refractivity of the medium in a small degree, and intend to test the matter in the near future by a modification of the method of striae. The channelled bands affect the magnetic rotatory dispersion in a powerful degree, and this subject is under investigation at the present time. The magnetic dispersion is many times greater than the ordinary dispersion, as has been found by examining the vacuum dispersion tubes in a powerful magnetic field with polarized light, and the whole subject will be fully taken up in a subsequent paper.

The absolute values of the refractive index for different densities have been determined with the interferometer for monochromatic light of different wave-lengths. Relative values (dispersion) have been obtained in the same way, as well as by the method of crossed prisms.

Determination of the Dispersion with the Interferometer.

The absolute determinations being the more difficult, it was decided to begin with an investigation of the dispersion, in order to get as much experience as possible with the behavior of the vapor under various conditions. This was a wise decision, as was subsequently found, for a number of very elusive sources of error were found, which were eliminated in the final work.

The general plan of the apparatus is show in Plate I. The interferometer was the form of Michelson instrument designed by Brace. The base of the instrument consisted of a heavy tube of brass 12 cms. in diameter and a little over a meter long. The arrangement of the mirrors is shown in the diagram, all but one being of speculum metal. Light from an arc lamp was focussed on the slit of the monochromatic illuminator. The beam issuing from the slit of this instrument comprised a range of wave-lengths about equal to the range between the D lines. Near the slit was mounted a helium tube, which furnished a light of constant wave-length, and the observations consisted in counting the fringe shifts for the two kinds of light, when a given amount of sodium vapor was introduced into the path of one of the interfering beams. A small mirror was mounted in front of the interferometer, which deflected a portion of the emergent light into a telescope, the remainder of the beam entering a second telescope. By a little adjustment it was possible to get the fringes formed by helium light distinct in one telescope, and those formed by the light from the monochromatic illuminator distinct in the other. The helium tube standing a little to one side of the slit, very little overlapping of the two systems occurred.

The sodium was contained in a seamless steel tube, covered with asbestos paper and wrapped with a helix of iron wire, through which a heavy current was passed. The ends of the tube were closed with plates of glass, which were of such a thickness that they served in place of the compensating plate, which is usually placed parallel to the transparent mirror. The tube was exhausted by means of a mercurial pump, with which it was kept in connection throughout the experiments in order to remove the occluded hydrogen which was given off as soon as the sodium was heated. This occluded gas gave a good deal of trouble, and the behavior of the fringes seemed to indicate that a small amount of the gas became entangled in the sodium vapor, no amount of pumping appearing to remove it entirely. Its effect on the fringe shifts will be discussed presently.

The D_3 line of helium was so strong in the tube used that no color screens were necessary, the fringes being as distinct as those obtained with sodium light. Owing to its proximity to the absorption band, very large fringe displacements were obtained with a small amount of comparatively rare sodium vapor.

In the electrically heated tubes the sodium vapor showed no tendency to form the non-homogeneous equivalent of a prism, and the fringes consequently remained straight, even after very great displacements. Two observers were necessary, one counting the helium fringes as they passed the cross-hair of the telescope, and the other noting the transit of the fringes formed by the light from the monochromatic illuminator. If the light used was on the same side of the D lines as the helium line, the two sets of fringes were shifted in the same direction; if on the other side, the shifts were in opposite directions. In addition to comparing light from the monochromatic illuminator with the helium light, a very careful comparison was made with the green mercury line, since this light was to be used in the absolute determinations.

At first very discordant results were obtained, but the source of error was soon discovered to lie in the heating, and consequent expansion of, the base bar of the interferometer by the radiation from the sodium tube. The instrument was then covered with layers of cotton and asbestos board, and consistent series were at once obtained. There was still a source of trouble which for a long time remained undiscovered. On heating the tube it was found that the relative shifts changed as the temperature rose, becoming constant after a shift of about a dozen helium fringes. Repeated heating of the sodium to a high temperature combined with pumping lessened the discrepancies, but the trouble was never completely eliminated. It is now believed that a small amount of hydrogen is held captive by the sodium vapor, condensing with it as the tube cools.* When the tube is again heated this occluded hydrogen is evolved with the first traces of sodium vapor, and modifies its refractivity. If we are working with light on the same side of the D lines as the helium line, for which the refractive index of the vapor is less than unity, the evolution of this gas will oppose the displacement of the fringes, holding both sets back by practically the same amount. Suppose we are working with a wave-length for which the relative shift with respect to helium light is 1 : 10. If we form a sufficient quantity of sodium vapor to shift the helium fringes through 30 fringe-widths, and the other set 3 widths, and an amount of hydrogen sufficient to give a shift of one fringe-width has been set free, it is obvious that our observed ratio will be 2 : 29, or in round numbers, 1 : 15. The errors observed were always at the beginning of the series when the tube was heated, and at the end when it was cooled, and were always of such a nature as to be explained perfectly by the hypothesis which I have advanced. On this account we always disregarded the relative values found when the sodium vapor was only present in very small quantities.

Many weeks of work were necessary before all sources of error were eliminated, and the best methods of working discovered. As a sample of one of the troubles met and overcome, the following will serve. When working with light from the monochromatic illuminator of a wave-length very near that of the D lines the fringes disappeared before shifts of more than one or two fringe-widths were observed. It had been our habit to set the mirrors of the instrument in such positions that the centre of the fringe system was seen in the telescopes. The disappearance was finally found to be due to the dispersion of the vapor, and by turning the screw of the instrument in such a way as to increase the length of the air path, they appeared again. The most advantageous way to work was therefore to begin the observations with the maximum air path consistent with visibility of the fringes; as the sodium vapor formed they increased in distinctness at first, and then gradually faded away. In this way it was possible to get a shift twice as great as before; if it was desirable to work with a denser vapor to eliminate the error due to evolved hydrogen, we have only to increase the length of the air path still further, and work further out in the system. Under these conditions the fringes are invisible at the beginning and end of the experiment, only coming into view when a considerable quantity of sodium vapor is present. The same thing is true to a less extent with helium light, the fringes disappearing after a shift of about two hundred fringe-widths has taken place. By increasing the length of the air path, either by turning the screw of the instrument, or introducing one or more thick plates of plane-parallel glass, they again come into view, and the counting can be continued. The latter method was preferred, as the glass plate could be dropped into place in a moment without interrupting the count. This trick was only necessary in making the absolute determinations with helium light, as in the comparison work the vapor never attained such a density as to cause the helium fringes to disappear. When working close to the helium line with the monochromatic illuminator it was necessary to work with very narrow slits, otherwise the enormous dispersion of the vapor obliterated the fringes almost immediately, in spite of all precautions. It was impossible to approach closer to the absorption band than the helium line, owing to this trouble, and ratios of 1 : 2 are about as large as we feel much confidence in, though on one or two occasions we set the monochromatic illuminator on the D_3 line, and obtained equal shifts in the two systems. It was impossible to observe a shift of more than one or two fringes in this case, before the system disappeared. The most accurate determinations are doubtless those where the ratio is between 1 : 3 and 1 : 15.

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* It is interesting to compare this with the aberrant solution of iodine vapor in the vapor of CS_2. Wood, Phil. Mag., 1896: On the Absorption Spectrum of Solutions of Iodine and Bromine above the Critical Temperature.

In the blue-green region the ratio is in the neighborhood of 1 : 40, and the values are not as accurate, as an accidental displacement of half a fringe-width, which may easily occur as the result of a slight change in the temperature of the instrument or the air, exercises a much greater effect on the results. On this account the determinations made by the prismatic method are to be considered more reliable in this region of the spectrum.

Determination of the Dispersion in the Immediate Vicinity of the D Lines.

The extraordinarily high value of the dispersion close to the absorption band makes it impossible to sift out from the spectrum light sufficiently homogeneous to yield interference fringes, when even a very small amount of sodium vapor is introduced into the path of the light. It was found difficult to get satisfactory values with the light from the monochromatic illuminator much closer to D_8 than the distance separating D_2 from D_3, the fringes becoming blurred as soon as the smallest trace of sodium vapor was formed. A different method was accordingly adopted, which yielded excellent results.

It is clear that if we employ light of two different wave-lengths in illuminating the interferometer, the introduction of the metallic vapor will shift the two fringe systems by different amounts, the fringes appearing and disappearing as they get into and out of step in succession. If both systems are shifted in the same direction, which is the case when both wave-lengths are on the same side of the absorption band, we can measure the dispersion by counting the number of fringes which pass the cross-hair of the telescope between two successive disappearances. If, for example, the number is found to be fifteen, we know that one set of waves is retarded or accelerated fifteen wave-lengths, under the same conditions which cause a shift of sixteen fringes for the other waves. All uncertainty as to the movement of the fringes during a period of minimum visibility was avoided in the method employed, as will appear presently.

To determine the dispersion close to the D lines we require highly homogeneous light of two different wave-lengths, which would yield a close double line at the point at which we wish to measure the dispersion.

This was accomplished by breaking up the radiation of a helium tube into a Zeeman triplet and cutting out the middle component by means of a Nicol prism placed with its plane of vibration perpendicular to the lines of magnetic force. A large Rhumkorff magnet with an ammeter in circuit was employed, the key for making and breaking the circuit being placed close to the observing telescope of the interferometer. The distance between the components of the double line obtained when a current of 30 amperes was flowing through the magnet was found to be $\frac{1}{23}$ of the distance between the sodium lines. This was determined very easily by counting the number of turns of the interferometer screw necessary to cause the fringes to pass through a given number of phases of maximum visibility, the method being the one commonly employed in the laboratory in the determination of the relative wave-lengths of the sodium lines. The magnetic field was now thrown off, and the temperature of the heating coil gradually raised. As the sodium vapor formed the fringes were counted as they drifted across the field. The current was thrown into the magnet at frequent intervals for a fraction of a second, and the fringes were seen to pass through successive phases of visibility and invisibility. The momentary disappearance caused by the magnetic field did not interfere with the counting, for the fringes moved slowly and with the regularity of clockwork. The exact moment at which the fringes disappeared could not of course be accurately determined, since they remained invisible during a period occupied by a shifting of several fringe-widths, but by observing six or seven successive disappearances, values were obtained which did not differ by more than two or three per cent. The average of a number of readings showed that the sodium vapor shifted the two sets of fringes in the ratio of 10.5 to 11, that is, fringes formed by waves of length 587487 are shifted 10.5 fringe-widths, while those formed by waves of length 587513 are shifted 11 fringe-widths. These numbers do not look very formidable until we translate them into prismatic deviations. A prism having the same dispersion giving a total deviation of only 11 degrees would seperate the two components of the Zeeman triplet half a degree, or if we could construct a sodium vapor prism giving the same deviation as a 60 degree glass prism, two lines in the spectrum twenty-three times as close together as the D lines would appear separated by a distance greater than the distance between the red and bluish-green of the spectrum formed by the glass prism.

The same method was applied to a determination of the dispersion between the yellow mercury lines, for the purpose of obtaining a check on the curve obtained by comparing the shifts of the helium fringes with fringes formed by the light from the monochromatic illuminator. A prism was placed in front of the observing telescope to separate the yellow fringes from the more intense green ones. On setting them in motion by forming sodium vapor in the path of the light they were found to disappear periodically in the same manner. It was impossible to get as accurate data as were obtained with the Zeeman lines, since there were moments of complete invisibility, but by counting steadily it was possible to tide over the few seconds when the field was a blank and get a fair estimate of the relative shifts. A number of observations were made, the mean of which gave 5 : 6 as the ratio of the shifts of wave-lengths 5770 and 5790. These values fitted almost exactly on the dispersion curve obtained with the monochromatic illuminator and the helium tube. Obviously a more accurate method would have been to provide a device by which the light of one of the yellow lines could have been cut off as desired. An attempt was made to do this with the monochromatic illuminator, but the loss of light was too great.

Some very curious results were obtained by illuminating the interferometer with sodium light. The fringes disappeared at the moment when the thermo-element indicated a temperature of 180. It was at first thought that the disappearance was due to absorption of the light by the vapor in one of the two paths, but on covering up the other path it was found that the field of the instrument was still brightly illuminated. On turning the screw of the instrument the fringes appeared again, and a little further experimenting showed that the effect of the vapor was to alter the visibility curve, its periodicity becoming less as the density of the vapor increased. No satisfactory explanation of the altered appearance of the fringe system could be found, and an attempt was made to get rid of one of the D lines in the illuminating light, thus simplifying the conditions. Prisms and gratings were tried, but the loss of light was so great that even with an oxyhydrogen-sodium flame the fringes could be made out only with the greatest difficulty. The desired result was finally obtained by mounting a quartz crystal between two crossed nicols. A number of crystals were examined with polarized light perpendicular to the axis, and one selected which showed dark bands in the spectrum, separated by a distance about double the distance between the D lines. The nicols were arranged so that the emergent light vibrated in such a plane as to be most copiously reflected by the interferometer mirror, i.e. the first nicol was mounted with its short diagonal horizontal, while the one next to the instrument was placed with its long diagonal horizontal. Between the nicols was mounted the quartz crystal with its axis making an angle of 45 degrees with the vertical. The light from an oxyhydrogen-sodium flame, after passage through the polarizing system, was brought to a focus on a cardboard screen by means of a lens, forming a series of bright bands separated by dark intervals. A narrow slit in the screen allowed light corresponding to D_1 or D_2 to enter the interferometer, according as it was set on one edge or the other of one of the bright bands. This arrangement worked admirably and yielded an abundance of light. No trace of the periodic invisibility was found in the fringe system, such as is always observed when ordinary sodium light is used. As soon as the smallest trace of sodium vapor was formed in the tube the fringes were found to have become quite invisible in certain parts of the system. The visibility curve changed rapidly as the density of the vapor increased, the period becoming smaller. The general behavior of the fringes did not differ materially from that observed when both sodium lines were used, proving that the presence of the two wave-lengths was not essential. The appearance of fringes under the conditions of the experiment depends upon the fact that the emission line is broader than the absorption line. The absorption is restricted to the centre of the line, the edge light only getting through the sodium tube and forming fringes. This edge light is most powerfully dispersed by the vapor, and the two fringe systems which it forms are shifted in opposite directions. It was hoped that a method might be worked out for determining the dispersion of the vapor in the immediate vicinity of the D lines by studying the behavior of sodium light when dispersed in the interferometer, and a great deal of time was lost in the endeavor to unravel the very complicated changes produced in the appearance of the fringe system. If we were dealing with strictly homogeneous radiations, lying to the right and left respectively of the absorption band, the two fringe systems would move in opposite directions at the same rate, and we should have simply rapid alternations of visibility and invisibility. By the simultaneous use of a helium tube we could determine the number of disappearances of the sodium fringes corresponding to a shift of one or more helium fringes, which would give us at once a measure of the shift of the system formed by the sodium light. This would give us a measure of the refractive index closer to the absorption bands than we could get by any other method. An attempt was made to utilize the Zeeman phenomenon in this case, the sodium flame being placed in a magnetic field and the interferometer illuminated with one or the other of the D lines by means of the polarizing system. There was no difficulty in finding positions of the interferometer mirror for which the fringes were visible with the field on, and invisible when it was off, but nothing resembling a rapid appearance and disappearance of the fringes was observed when the sodium vapor was formed. This is probably due to a number of causes. In the first place the light from the flame is by no means strictly homogeneous, for a pale flame containing only a little sodium and yielding narrow lines does not give light enough. In the second place, as is well known, the D lines do not break up into polarized triplets in the magnetic field, but into a quadruplet and sextuplet. At the time of trying the experiment this circumstance was not thought of, and the failure may have been due in part to the fact that only D_2 was utilized, which is obviously less suited to the purpose, for the reason that it gives us a double line on each side of the absorption band when the central components are cut out by the nicol. I am of the opinion, however, that the finite breadth of the normal line is chiefly responsible for the failure. Some experiments were made with vacuum tubes containing sodium vapor, which yield highly homogeneous light, but they were found troublesome to manage. It is quite possible that, by the use of these tubes, results could be obtained in the case of D_1, and utilizing the light emitted in the direction of the lines of force (Zeeman doublet) we might get data for both lines. This experiment would have been tried, were it not for the fact that almost as satisfactory results can be obtained by the method of prismatic deviation, with a much less expenditure of time.

As a discussion of the modifications of the appearance of the fringes formed by sodium light, by the introduction of the vapor of sodium into the path of one of the interfering beams of light, has no especial bearing upon the determination of the dispersion of the vapor, it will be postponed for the present. A fuller discussion will be given in a subsequent paper on "The achromatization of fringes formed by approximately homogeneous light by highly dispersing media." *

PRISMATIC DETERMINATION OF THE DISPERSION IN THE IMMEDIATE VICINITY OF THE D LINES.

As I have already shown, prisms of excellent defining power can be formed by heating metallic sodium in highly exhausted tubes of hard glass. In my earlier experiments, in which the metal was heated in an atmosphere of hydrogen, the definition was not good enough to obtain a smooth curve close to and between the D lines by the method of crossed prisms. The photographs illustrating my first paper, in which the dispersion between the lines was shown, were taken by employing a prismatic flame of hydrogen burned at a platinum jet, and fed with the vapor of sodium. By using vacuum tubes heated by very small flames far better results can be obtained, and perfectly smooth sharp curves obtained. Two different methods were used in the determination of the deviation produced by the prism for wave-lengths close to the D lines.

A reticulated mesh was ruled on a small plate of glass with one of the Rowland dividing engines. This plate was mounted in the eye-piece of the spectroscope, on the slit of which the image of an illuminated horizontal slit was thrown, after dispersion by the sodium tube. The spectroscope was provided with a plane grating ruled with twenty thousand lines to the inch, which gave a very brilliant 2d order spectrum. There were four squares of the reticulated mesh between the D lines, and nine squares between D_2 and D_3.

As the temperature of the sodium tube was raised the relative deviations in the different squares of the mesh were noted. The light immediately adjacent to the D lines was deviated entirely out of the field of the telescope before any measurable deviation at the helium line occurred. At a temperature at which the spectrum at the helium line was deviated

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* Phil. Mag., September, 1904.

the width of one square, the wave-lengths between the D lines, and for a distance of two squares to each side of them, were lost by absorption. By means of step by step measurements, taken at different temperatures, it was possible to get a very good idea of the dispersion curve within this very narrow range of the spectrum. The appearance of the spectrum on the reticulated background at two different temperatures is shown in Figure 3, the dispersion at D_2 being somewhat greater than at D_1. Observations were taken of the dispersion within a quarter of a square of D_2, that is, within a distance of the absorption line equal to $\frac{1}{16}$ of the distance between the D lines. The deviation at this point was calculated to be about eighty times the deviation at the helium line. If we number the wave-lengths at the lines of the mesh between D_3 and D_2 1 to 9, the observations can be recorded as follows. The light very close to D_2 ($\frac{1}{4}$ of a square distant) was seen to be distinctly pulled down through a distance of seven squares with a vapor density sufficient to deviate the spectrum at the first line (= 5888.3) through a distance of one square. The temperature was then raised a trifle. The light very close to D_2 now disappeared, partly as a result of absorption and partly through being deviated out of the field, when the spectrum at the first line was deviated through three squares, that at the second line one square. The temperature was now raised until the spectrum at the ninth line was deviated through one square. The deviation at the second was now five, that at the third three squares, and that at the fifth, two squares.

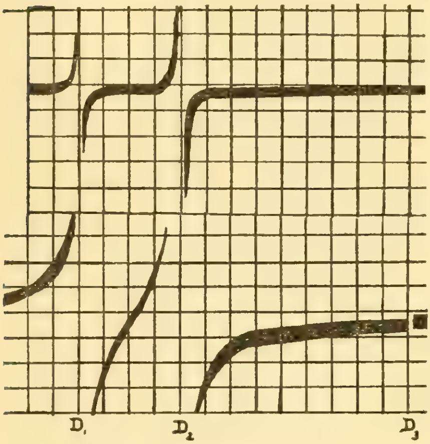

Figure 3.

These results for the different wave-lengths between D_3 and D_2 are given in the following table:

	Deviation.		Deviation.
D_3 5875	1	5888.3	12
5881.6	2	5889.6	84
5885	3	D_2 5890	. .
5886.6	5		

These values are of course only approximations, the errors becoming greater as we approach D_2.

The same experiment was repeated, using a filar micrometer in the eye-piece in place of the reticulated mesh, and values obtained which agreed fairly well with those given above. This method, however, did not admit of such a near approach to D_2 as the other.

From these results it is possible to calculate the values of the refractive index within this range for the very dense vapor used in the absolute determinations. These will be called observed values, and they will be compared with the values calculated from the dispersion formula for the corresponding wave-lengths.

I have already spoken of the enormous dispersion at the helium line, a dispersion great enough to separate a Zeeman doublet by an amount equal to nearly the width of the entire visible spectrum produced by a glass prism of sixty degrees, provided we were able to construct a prism of sodium vapor giving the same mean deviation, and get the light in question through it. Great as is this dispersion, it is as nothing when compared with that within say an Ångstrom unit of one of the D lines, as an inspection of the above table indicates.

The dispersion within the range between D_2 and D_3 is shown graphically in Curve 2, Plate II.

If this curve were completed up to wave-length 5889.6, at which the largest recorded deviation occurs, the point in question would be at the thirty-eighth square below the unity line. To include this point it would have been necessary to take the ordinates of the curve on about $\frac{1}{4}$ of the scale employed.

Visual Observations of the Dispersion in the Visible Spectrum by the Method of Crossed Prisms.

As a check on the results obtained with the interferometer, a series of measurements of the dispersion was made by the method of crossed prisms. The method was identical with that described in my previous paper, except that the metal was heated in a highly exhausted tube of hard Jena glass instead of in a tube filled with hydrogen. These tubes can be very easily prepared, and give no trouble at all. They are, however, apt to crack if reheated, after having been allowed to cool. It is not necessary to have the sodium in separate small lumps, as the action is not the formation of a number of prisms of sodium vapor, but a cylinder of vapor, very dense along the heated floor of the tube, and decreasing to very nearly zero along the top. If it is desired to prepare a tube for purposes of demonstration, a tube of thin steel is preferable, as it can be used over and over again. In this case it is necessary to lay a thick pad of wet asbestos paper along the top of the tube, since the more uniform heating which results from the better conductivity of the steel is detrimental to the formation of the non-homogeneous cylinder of vapor, which only takes place when the temperature gradient between the top and bottom of the tube is very steep. The plate glass ends are fastened on with sealing-wax, a small glass tube being sealed in in the manner described in my previous paper, through which the air is exhausted. See Figure 4.

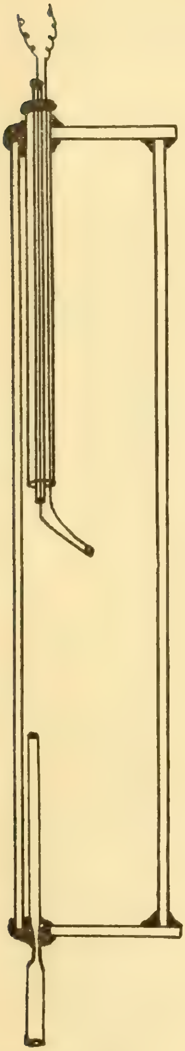

FIGURE 4.

Glass tubes were used in making the measurements, as there was no occasion for cooling the tube and reheating it and the temperature gradient takes care of itself in this case. The sodium was first melted in a small iron crucible and poured into an improvised sodium press; a piece of apparatus so easily made and so useful that a brief description of it may not be out of place. A steel cylinder is bored with a hole two or three cms. in diameter, to within a few millimeters of its end. A 3 mm. hole is then drilled through the end at the centre of the larger hole. A solid steel piston is turned so as to fit easily into the bore of the larger cylinder. The molten sodium is poured in, allowed to cool, and the piston inserted. The whole is then put into the jaws of a large vice, and the sodium wire squeezed out directly into the dispersion tube. In this way the formation of oxide is prevented and the sodium is freed from the hydrocarbon in which it is kept. The glass plate is immediately cemented on and the tube exhausted on a mercurial pump. An ordinary hand pump would doubtless answer every purpose, as the vacuum does not need to be below a few millimeters. I see no reason why a well made steel tube should not be used year after year in class demonstration, though I have not tried keeping one so long. With the tube once prepared the apparatus for showing the anomalous dispersion of the vapor, which is one of the most beautiful optical experiments that I know of, can be set up in five minutes. The former method which I described was rather troublesome, as it involved the use of a hydrogen generator and drying tubes, and it was often difficult to avoid the formation of smoke in the tube. The vacuum tubes make the experiment as easy to perform as the simplest of the laboratory demonstrations. A combustion furnace is not necessary, two or three Bunsen burners turned down low answering every purpose. The amount of dispersion can be regulated by turning the flames up or down. A full description of the disposition of the apparatus will be found in my first paper.* In brief, it consists of a horizontal slit on which the crater of an arc lamp is focussed. If a Nernst lamp is available it may advantageously be substituted for the illuminated slit, the glower being mounted in a horizontal position. The light from the slit is made parallel with a lens, and after traversing the sodium tube is focussed on the slit of a small spectroscope by means of a second lens. It is almost as satisfactory to use a single lens passing the convergent beam through the dispersion tube.

If it is desired to show the anomalous *spectrum*, a low power eye-piece is substituted for the spectroscope. The appearance of this spectrum is shown by a colored plate illustrating the earlier paper. In the present work it was desirable to have a normal spectrum, and the spectroscope was replaced with a collimator and plane grating. The light from the grating entered a camera made of a long wooden box, provided with an achromatic telescope lens of 2 meters focus. The spectrum, which was received on a plate of ground glass, was about 25 cms. long and 1 mm. wide. Its position was accurately recorded on the ground glass by means of a narrow pencil mark ruled along its centre, the ground surface having been sponged over with oil to increase its transparency. The burners of the furnace were now lighted and turned down very low, so that the tips of the flames fell a little short of the tube. As soon as the temperature of the tube became constant, the curved branches of the dispersed spectrum were outlined on the glass with the pencil. The burners were then turned up a little and the deviations of the oppositely curved branches increased, their new positions being recorded in the same manner. A number of records were made in this way, and the tube was then allowed to cool off. The spectrum straightened, and was found to occupy the same position on the plate as before, showing that no displacement of the apparatus had occurred. A cadmium spark and sodium flame were now placed before the slit of the grating spectroscope, and the position of the bright lines recorded on the plate. The deviations which different regions of the spectrum had experienced were then carefully measured. The object of making a number of records was to show whether the relative deviations depended on the temperature of the tube and density of the vapor. No indications of any such irregularity were found. The values found in this way agreed admirably with the results obtained with the interferometer. It moreover furnished data regarding the dispersion in the blue and violet region, which the interferometer failed to do with any accuracy for reasons which I have given. Photographs of the dispersed spectrum are reproduced in Figure 8 (Plate III).

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* Wood, The Anomalous Dispersion of Sodium Vapor. Proc. Roy. Soc: also Phil. Mag., 1901.

NUMERICAL RESULTS.

The results given by the three methods are given in the second column of the table in the section devoted to the refraction and dispersion of sodium vapor of great density, which follows. For the sake of uniformity they have all been reduced to the same scale. A shift of 100 helium (D_3) fringes being taken as the standard. A wave-length, for which the relative shift, with respect to helium light, of the interferometer fringes was found to be 1 : 4, is entered in the table as 25.

The shift ratio for the green line of mercury with respect to helium was found to be 1 : 25. The wave-length of this light is therefore represented in the table by the numeral 4, which means that a prism of sodium vapor which gives a deviation 4 for green hg. light, gives a deviation 100 for helium light. The results obtained by the methods of crossed prisms (both visual and photographic) were reduced to the same scale, being compared with the value found for the green mercury line. This was necessary, since if a vapor of sufficient density is used to give a measurable deviation in the blue and violet region, it absorbs almost entirely that portion of the spectrum occupied by the helium line. The values obtained by means of the Zeeman doublet and the two yellow lines of mercury, by the method of coincidences are not given in the table. As their values were purely relative, i. e. not compared with helium or any other standard wave-length, it was necessary to assign to one of the wave-lengths a shift taken from the curve determined by the other methods, and then calculate the shift of the other wave-length. For example, in the case of the yellow mercury lines, of wave-lengths 577 and 579, the ratio as obtained by the coincidence method was 5 : 6. The shift of 577 on the curve was found to be 14; i. e. 14 of these fringes to 100 of helium. The shift of wave-length 579 with respect to helium is given by the proportion $5 : 6 = 14 : x$, from which we get $x = 168.$, a value which fell exactly on the curve obtained with the interferometer.

The same thing was done in the case of the values obtained by breaking the helium line up into a Zeeman doublet, 11 fringes for $\lambda = 587487$, 11.5 fringes for $\lambda = 587513$. The values for the range of wave-lengths comprised between the helium line and D_2 were determined by the method of crossed prisms, the measurements being made with the reticulated mesh in the focus of the piece. For light of wave-length midway between D_3 and D_2 the deviation is twice as great as for D_3, consequently we enter this in the table as 200.

The Ultra-violet Dispersion.

A qualitative study of the dispersion in the ultra-violet was first made by the method of crossed prisms.

A tube of hard Jena glass, 40 cms. long and 1.5 cm. internal diameter, was fitted with end plates of quartz, loaded with sodium, and exhausted. The arrangement of the apparatus is shown in Figure 5.

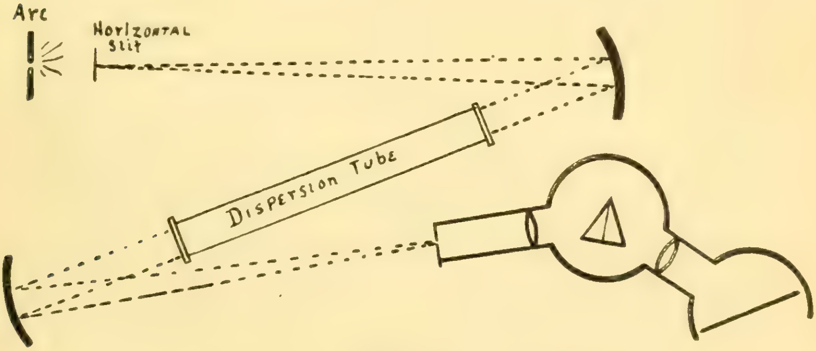

Figure 5.

As the method depends on forming a sharp image of a brightly illuminated horizontal slit upon the vertical slit of the spectrograph, quartz lenses could not be used, owing to their chromatic aberration. Concave silvered mirrors are free from this defect, but have abnormally low reflecting power in the ultra-violet region. Very satisfactory photographs were obtained with them, however, in the preliminary work, though they were replaced with magnalium reflectors later on. The light from the crater of an arc-lamp was focussed on the horizontal slit, and collimated with one of the concave mirrors. It then passed through the sodium tube, after which it was brought to a focus by a second mirror on the slit of a quartz spectrograph. The tube was heated by a row of small Bunsen burners, and as soon as a steady state was reached, which could be told by observing the curved branches of the spectrum with an eye-piece, the photographic plate was inserted and exposed. Owing to the low reflecting power of the silver the ultra-violet region was somewhat under-exposed. The curvature of the spectrum in the immediate vicinity of the first two ultra-violet lines of the principal series was very pronounced, however. The plates obtained with the small quartz spectrograph are too small to reproduce well, especially as the density is not very great in the most interesting region, and it has seemed better on this account to make drawings from them. The general appearance of the photographed spectrum is shown in Figure 2, which has been enlarged about two diameters. It is apparent that the effect of these lines on the refractivity of the medium is negligible except in their immediate vicinity.

Plates obtained in the manner described were useless for purposes of measurement, as no record was made of the position of the spectrum on the plate before its deviation by the sodium prism.

As the displacement is very slight in the remote ultra-violet, the two records would overlap in this region, even if the spectrum was very narrow. To overcome this difficulty the following method was devised. A slide was fitted to the plate holder, perforated with a number of vertical slits, each about 2 mms. wide, the widths of the opaque bars and slits being equal. This slide was left in the plate holder during the exposures, and screened off the spectrum at regular intervals. In order to obtain greater deviations a large tube of Jena glass 80 cms. long was used. It was heated in a combustion furnace, the burners of which were turned down low, the best results being obtained when the tops of the flames only partially enveloped the tube.

As it was desired to extend the observations as far down into the ultra-violet as possible the spark was substituted for the arc, and to avoid loss of light by reflection one of the mirrors was dispensed with.

The spark was furnished by a large induction coil operated with an alternating current of 110 volts, a Leyden jar of large size having been joined in parallel with the terminals. Cadmium electrodes were used, the spark gap being placed close to the horizontal slit, which was mounted at the centre of curvature of a concave reflector of magnalium of one meter radius. The convergent beam from the reflector passed through the sodium dispersion tube and came to a focus on the slit of the spectrograph. The bottom of the tube having been raised to a full red heat in the combustion furnace, the photographic plate was exposed with the slotted slide in position. The anomalously dispersed spectrum was thus recorded in small sections corresponding to the slits in the slide. The tube was then allowed to cool off, the slide drawn out a distance equal to the width of one of the slits, and a second exposure made which recorded the position of the undeviated spectrum. A drawing of the plate holder with its perforated slide, and the deviated and undeviated spectra obtained with it are shown in Figures 6 and 7.

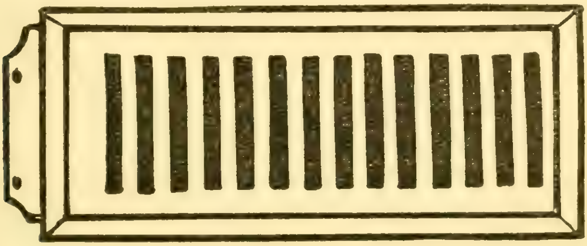

Figure 6.

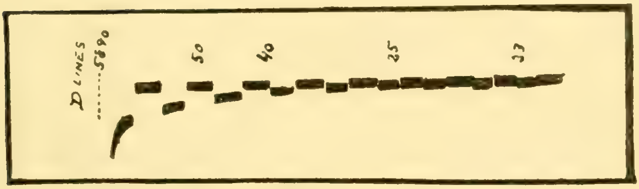

Figure 7.

Absolute Determinations of the Refractive Index.

The deviations of the light rays by the non-homogeneous cylinders of sodium vapor formed in exhausted tubes give us no measure of the absolute refractive index, for the reason that we have no way of determining the angle of the equivalent prism. Various methods were tried, designed to give to a mass of vapor of uniform density a prismatic form. None of them was very satisfactory, and until the discovery was made that a fairly uniform cylinder of vapor could be formed in an exhausted tube, the absolute value of the index for any given wave-length could only be guessed at.

The results obtained on the dispersion with the interferometer showed that results correct to within 5 per cent could in all probability be obtained by this method.

The density of sodium vapor at different temperatures has been investigated by Jewett * for a range comprised betweed 368 and 420. It was accordingly determined to take the temperature of the vapor as a measure of its density.

A thermo-couple of iron and constantan wire was made by holding the ends of the wires together, both being connected to a storage battery giving an E.M.F. of about twenty volts. The other wire of the battery was then touched momentarily to the tips of the two wires, the small arc which formed on separation fusing them together very neatly. I do not remember to have seen this method of making thermo-couples described, and have mentioned it as it was found very satisfactory. The fused bead which united the wires was then hammered out into a thin disc, and the junction mounted in the axis of the sodium tube, the wires being insulated with thin glass tubes which passed through a larger tube, as shown in Figure 4.

The couple was calibrated in baths of molten lead, zinc, tin, and aluminium before and after the experiment. The determinations at the lower temperatures and densities were made with the light from the helium tube; those at high temperatures, with the green light from a mercury tube. This was necessary, since a mass of very dense sodium vapor not only absorbs helium light strongly, but owing to its high dispersion makes the fringes invisible, the different wave-lengths in the D_3 line (which is of course of finite width) being shifted by different amounts. If the increase in the density with increasing temperature followed the same law as in the experiments of Jewett, the optically determined density curve should agree with that found by gravimetric method employed in his work. For example, he found that the density at 412° was six times as great as at 385°, and we should consequently look for a fringe shift at the higher temperature six times as great as at the lower. This, however, was by no means the case, the fringe shift being only about twice as great. This is on the whole exactly what we should expect under the conditions of the experiment, for we must remember that our

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* Phil. Mag., 4, 1902, page 551.

column of sodium vapor terminates in a vacuum, if the expression is allowable. If the sodium were heated in a tube provided with end plates which were not acted upon by the vapor, so that the whole could be brought to a constant temperature, it is probable that agreement with the results obtained by Jewett would be obtained. As it is, we have a cold condenser at each end of the column, and only obtain vapor of any considerable density through the circumstance that the rate of diffusion is comparatively slow.

I hope in the near future to determine the actual density under the conditions of the experiment.

The density at a given temperature was found to depend to a large extent upon the previous treatment of the tube. If the tube was heated for the first time to a temperature of say 350°, a shift of say ten helium fringes would be observed. If the tube was then raised to a red heat and allowed to cool, on reheating it to the same temperature (350°), a shift of perhaps fifteen fringes would be found. It appeared as if the density at a given temperature was much greater if a sodium "dew" had been formed over the entire inner surface of the tube, by the condensation of the dense vapor given off at the high temperature; in other words, the density at a given temperature was proportional to the surface of molten sodium which was exposed.

The determinations were made in glass tubes for the reason that in steel tubes the transition from the heated to the cool portion of the tube was not sufficiently sudden to insure uniform density in the cylinder of vapor. The Jena glass tubes could, however, be raised to a full red heat by means of the coil of iron wire, without showing any trace of sodium vapor in the portions protruding from the helix.

In order to obtain consistent results it was found necessary to heat the tube very gradually, in order to allow the thermo-element time to take the temperature of the vapor. One observer watched the scale of the galvanometer, the other counted the fringes as the tube was heated or cooled. The helium fringes disappeared after a shift of something over a hundred fringe-widths had occurred, but by dropping a plate of plane parallel glass into the air path of the interferometer they could be made distinct again and the count continued. In this way it has been possible to record shifts as high as four hundred fringe-widths. Beyond this point it is impossible to go with the the helium light, the components of helium light becoming too much spread out by the dispersion of the vapor to be brought together again by going out any farther in the fringe system. To extend the observations to higher temperatures and greater vapor density the green radiation from a mercury tube was used, with which fringe shifts as high as one hundred fringe-widths were observed, corresponding to a shift of twenty-five hundred helium fringes. In this way it has been possible to obtain measures of the optical density of the vapor under the specified conditions from a temperature of 280°, at which point the fringes commenced to move, to above 650°, at which temperature the dense vapor was luminous, the entire section of the tube appearing filled with a dull red glow. Plotting the fringe shifts as abscissae, and the temperatures as ordinates, we obtain curves of form similar to the density curve obtained by Jewett, though by no means coincident with it, for reasons which I have already given.

The observations were made both when heating and cooling. At first it was found that the shift obtained on cooling was greater than the one observed during the heating; in other words, the fringe system did not return to the starting point. This was found to be due to hydrogen gas liberated from the sodium, which from the circumstance that it shifts the fringes in the opposite direction from the shift produced by the sodium vapor, retards the motion of the system during its liberation, and by remaining unabsorbed in the tube prevents the fringes from returning to the starting point. This trouble was eliminated by heating the tube to a high temperature, and setting the mercurial pump in operation.

Temperature.	Fringe Shift.		Temperature.	Fringe Shift.
	Helium $\lambda = 5875$	Mercury $\lambda = 546$		Mercury $\lambda = 546$
274	1		560	14
282	2		584	19
314	3		595	23
334	4		608	32
348	5		628	38
360	6		644	50
376	8		650	55
389	9			
396	11			
408	14			
438	25	1		
452	35			
474	50	2		
496	77	3		
508	98	4		
518		6		
540		10		

The observations extended over several weeks, many tubes both of steel, porcelain, and glass being used. The length heated was varied and the different series of observations compared. Most of this work was only useful in determining and eliminating the sources of error. The final series, which was considered the best, was made with a tube of Jena glass, the length of the heated portion being 8 cms. The retardation, or rather the acceleration, since we are working with light on the blue side of the absorption band, is due to a column 16 cms. long, since it is twice traversed by the interfering beam. From the data given in this table the refractive index of the vapor for either helium light or the green mercury radiation can be calculated for any temperature by means of the formula $n = 1 \pm \frac{m\lambda}{\epsilon}$ (the + or — sign being used according to the direction of the shift), in which m is the fringe shift and ϵ the length of the column (16 cms.).

Curves are given for these optically determined densities in Plate 2.

THE REFRACTION AND DISPERSION OF SODIUM VAPOR OF GREAT DENSITY.

A knowledge of the absolute value of the refractive index of the vapor, and its dispersion, enables us to compile a table of the refractive-indices for all wave-lengths, for vapors of varying density. This has been done for the very dense vapor obtained by heating a vacuum tube containing the metal to the temperature of 644° C. A column of the vapor at this temperature 8 cms. in length examined by transmitted light has a distinct blue color, as a result of the channelled absorption spectrum. The values are given in the following table, and will be spoken of in future as " observed values " to distinguish them from values calculated from the dispersion formula. It must be remembered that sodium vapor as dense as that with which we are dealing in the present case has an absorption band at the D lines broad enough to completely cut out everything down to and even below the helium line, at least for all thicknesses with which it is possible to work. On this account we are obliged to calculate the refractive indices within this region from observations made with a less dense vapor, a method which in the present case is probably allowable within certain limits. A thin enough sheet of the vapor would probably transmit light within this region with a velocity indicated by the calculated indices. The question of selective reflection at the surface and refusal to transmit the radiation will be considered presently.

In the following table the wave-lengths are given in the first column, the fringe displacements in comparison with helium light in the second column. These values, with the exception of those in the extreme red, blue-violet, and ultra-violet, were obtained with the interferometer.

In the third column are given the actual fringe shifts which would be found for a layer of vapor 8 cms. thick (effective thickness 16 cms.), and in the fourth column the refractive indices. The indices calculated from the dispersion formula are given in the fifth column, for the sake of comparison with the observed values. More will be said of them in the next section.

λ	Relative Shift.	Total Shift.	Ref. Index Obs.	Ref. Index Calcul.
7500	2	25	1.000117	
6310	4	50	1.000197	
6200	6	75	1.000291	1.000285
6137	7	88	1.000335	
6055	11	138	1.000523	1.00052
6013	14	175	1.000658	
5977	20	250	1.000934	
5960	25	313	1.001164	
5942	33	413	1.001532	
5916	60	750	1.002972	
5875	100	1250	0.995410	0.9958
5867	67	834	0.996929	0.99692
5858	50	625	0.997711	
5850	40	500	0.998172	0.99815
5843	33	413	0.998492	
5827	25	313	0.998862	
5807	20	250	0.999093	
5750	11	138	0.999505	
5700	9	113	0.999599	
5650	7.4	92	0.999650	
5460	4	50	0.9998294	0.999829
5400	3.6	45	0.9998481	
5300	2.9	36	0.9998807	0.999885
4500	1.4	17.5	0.9999508	0.999965
3610	1.09	13.6	0.9999698	
3270	0.9	11.4	0.9999768	0.999987
2260	0.7	8.7	0.9999877	0.999995

Refractive Indices in the Vicinity of the D Lines.

λ	n Cal.	n Obs.
5875	0.9958	0.9954
5882	0.9890	0.9908
5885	0.9830	0.9860
58866	0.9750	0.9770
58884	0.9450	0.9443
58896	0.697	0.614
5991	1.0046	
5904	1.0092	
5901	1.0138	
58994	1.0184	
58976	1.0557	
5897	1.094	
58964	1.386	

For $\lambda = \propto$ the value of n is calculated to be 1.0000275, while for $\lambda = 4167$ it is 0.999975, the deviations being the same in both cases, but in opposite directions.

Application of the Results to the Dispersion Formula.

The simplest form of the dispersion formula, developed from electromagnetic considerations for a medium with a single absorption band, is

$$n = 1 + \frac{m\,\lambda^2}{\lambda^2 - \lambda_m{}^2},$$

in which m is a constant, λ the wave-length of the light employed, and λ_m the wave-length at the centre of the absorption band.

The vapor of sodium has, of course, a pair of very close absorption bands (the D lines), which are chiefly effective in modifying the refractivity of the medium. As I have said, the ultra-violet bands affect the dispersion in their immediate vicinity, but their effect can be neglected in comparison with the stronger band, except for wave-lengths very close to them.

The first question to consider is whether we are justified in considering the D lines as a single absorption band in the case of the very dense vapor. If we consider the medium as having a single band, and assign to λ_m the value 5893 (a point midway between the D lines), we shall find that the observed and calculated values of the refractive index agree very closely, up to within a distance of the band about equal to the distance of the D_3 line of helium. If we attempt to go closer than this we immediately find discrepancies, which become larger as we approach the D lines.

The constant m in the above formula can be determined from a single observation of the refractivity. It was determined from two values, namely, the refractivity for the green line of mercury ($\lambda = 546$) and that for $\lambda = 585$, the latter wave-length being quite close to the D lines. Since the refractivity has widely different values for these two wave-lengths, we should expect the difference between the two calculated values to be a maximum in this case, in the event of the dispersion being incorrectly represented by the formula. The values for m found in the two cases were 0.000056 and 0.000054, a surprisingly close agreement. The mean value $m = 0.00055$ was taken, and the indices for a number of wave-lengths calculated. Some of these values are given in the table of refractive indices, and they will be found to agree very closely with the observed values. The values calculated between the helium line and the D lines are given in the second part of the table.

It will be noticed that the greatest discrepancies are found in the ultra-violet, the observed values indicating a greater deviation (negative, since the index is less than unity) than we should expect theoretically. In other words, we should expect the deviated spectrum to be more nearly in coincidence with the undeviated than was found in the photographs taken through the slotted slide. I am inclined to attribute the discrepancy to the presence of hydrogen in the tube. The dispersion tube was exhausted and sealed off from the pump before introduction into the combustion furnace. A small amount of hydrogen is always liberated from the sodium as soon as it is heated, and without doubt the tube contained a small amount of this gas during the experiment. At the time it was not expected that this would give any trouble, but observations appear to indicate the contrary. A tube containing air, or any ordinary gas, acts like a prism with its thin edge down, when the floor of the tube is strongly heated (compare with mirage). This is owing to the rarefaction of the gas along the heated floor. Now the sodium prism is turned in exactly the opposite way, the denser portions being close to the floor. But on the blue side of the sodium absorption band the index of refraction is less than unity, consequently the oppositely oriented prisms (hydrogen and sodium vapor) deviate in the same direction, and the effect of the hydrogen prism will be greatest in the ultra-violet for obvious reasons. The result will be that slightly too great deviations will be found in this region, which is precisely what we require to explain the discrepancies between the observed and the calculated values.

Unfortunately sufficient time did not remain for the repetition of the experiment with a tube permanently in connection with the pump, but the omission is not serious, for I feel certain that the explanation given above is the correct one.

An inspection of the formula shows us that, according as we are on the red or blue side of the absorption band, the refractive index is given by adding to or subtracting from unity the value of the constant $m =$ 0.000055 multiplied by the fraction $\frac{\lambda^2}{\lambda^2 - \lambda_m^2}$. In the case of all other substances showing anomalous dispersion, aniline dyes for example, to the dispersion of which a formula has been applied, the value of this fraction never exceeds 3 or 4, owing to the impossibility of applying it to wave-lengths very close to the centre of the band. For example, in the case of the dispersion of nitroso-dimethyl-aniline, with its strong absorption band at $\lambda = 43$, we cannot obtain accurate data nearer than $\lambda = 50$. In this case $\frac{\lambda^2}{\lambda^2 - \lambda_m^2} = 3.9$.

In the case of sodium vapor the value of the fraction may be several hundred or even thousand. In the case of $\lambda = 5882$ the fraction is 367, and yet the observed and calculated values agree closely. For $\lambda =$ 58884 the fraction is 1940; and for $\lambda = 5889.6$ we have a value as high as 3944. The product of these very large numbers and the small fraction 0.000055 give, however, values of the index which are in close agreement with the observed values.

Discrepancies occur in the immediate vicinity of the D lines which can be explained in the following way. To get values in any way consistent with the observed values it was necessary to assign to λ_m the value of the D_2 line, the mean value 5893 being too far removed from the wave-lengths in question to give the requisite steepness to the curve. The calculated values, therefore, apply to a medium with a single band at D_2 and with a constant $m = 0.00055$. This gives us a pretty good approximation to the observed curve, but the latter is due to the combined effects of the bands D_1 and D_2, the presence of the D_1 band tending to make the observed curve flatter than the calculated. A more correct approximation could be obtained by assigning to λ_m a value intermediate between D_2 and 5893. The proper method of procedure would, of course, be to make use of two members in the dispersion formula, one for D_1 and the other for D_2 thus:

$$n^2 = 1 + \frac{m\lambda^2}{\lambda^2 - \lambda_m{}^2} + \frac{m'\lambda^2}{\lambda^2 - \lambda_{m'}{}^2}.$$

If m and m_1 were each assigned the same value obtained by dividing our original value by 2, in all probability a very close approximation would be obtained in the region in question. This has not been done for two reasons. In the first place it does not appear as if much would be learned by the procedure, and in the second place m and m' are not equal, as is shown by the stronger dispersion near D_2, and until the relative-values have been determined we are not in a position to write the two-member formula accurately. It is doubtful whether anything new would come out of such a determination, and it was on that account not attempted.

Another matter of considerable interest is the question of the indices represented by the square root of a negative quantity in the immediate vicinity of the absorption band on the blue side. Lord Kelvin interprets this as indicating that no light of such wave-lengths enters the medium; in other words, it is metallically reflected.

It is in this way that he has explained the apparent greater broadening of the D lines on the more refrangible side in some of Becquerel's photographs. In the case with which we are dealing the second term of our original formula does not become less than unity until we reach wave-length 58898, which we get by equating $\frac{m\lambda^2}{\lambda^2 - \lambda_m{}^2}$ to unity and solving for λ.

This shows us that, even with a vapor so dense that both D lines run together and broaden out into a wide band, we do not get values of the index which are imaginary until we are within 0.2 of an Ångstrom unit of the D line, or in other words until we are within a distance of D equal to $\frac{1}{30}$ of the distance between D_1 and D_2.

In the case of the comparatively rare vapor employed by Becquerel we should have to approach much closer than this to get the imaginary values. This makes it appear certain that the greater broadening on the more refrangible side, if it exists, must be assigned to some other cause than imaginary values of the refractive index.

The medium is exceptionally interesting in that its dispersion can be represented throughout the entire range of wave-lengths without taking the extinction coefficient into account, as is always necessary in the case of solids and liquids when in the vicinity of the absorption band.

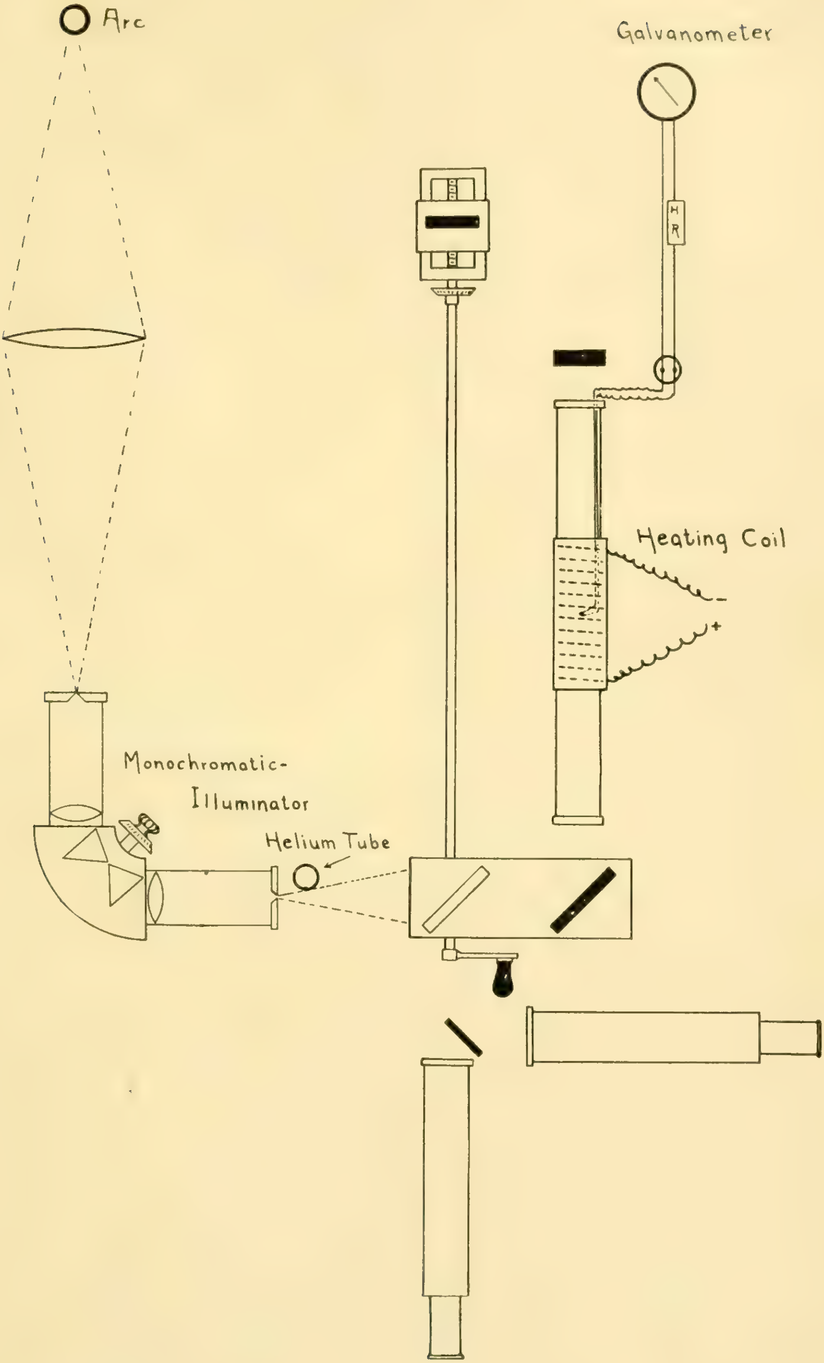
The Question of the Selective Reflection of Sodium Vapor.

The vapor of sodium should exhibit strong selective reflection in the vicinity of the D lines, just as the aniline dyes have metallic reflecting power for light of certain wave-lengths. For obvious reasons the vapor must be very dense if the phenomenon is to be detected experimentally. It must moreover terminate abruptly in a flat surface. This can only be accompanied by confining it in closed vessels made of some transparent material, since in a vacuum tube we have a gradual transition from dense to rare vapor at the free surface. I feel justified in speaking of the free surface of a gas in a vacuum tube in this particular case. A more precise definition of the apparent surface may be an isothermal surface on one side of which we have sodium vapor and on the other sodium fog. From such a surface we should hardly expect to get any trace of reflection, which requires that the transition from dense to rare be sudden. If we confine the vapor in glass bulbs we at once encounter many difficulties. The glass is at once attacked and discolored, and we have the reflection from the glass surfaces. An attempt was however made to observe the phenomenon in the following way. A small amount of the metal was sealed up in a hard glass bulb, highly exhausted. The bulb was mounted in a small air bath of sheet iron which could be rapidly raised to a red heat.

The reflection of the filament of a Nernst lamp by the inner and outer surfaces of the bulb was observed through a large direct vision prism. The two minute points of light were seen drawn out into spectra, and it was hoped that any trace of selective reflection in the vicinity of the D lines would manifest itself by a change in the relative intensities of the two spectra at the point in question. The bulb was heated very rapidly, but no conclusive observation made. The glass discolored so rapidly that the densest vapor could not be studied. In one case it was thought that a slight brightening in the yellow occurred, and the subject will be further investigated. Possibly by employing light polarized in such a plane as to be refused reflection by glass surfaces we may be able to get the desired results. The phenomenon is doubtless connected intimately with the fluorescence of the vapor. In studying this phenomenon with very dense vapors I was forced to the conclusion that the illumination of the vapor with light of the wave-length of the D lines did not provoke the fluorescence. Wiedemann and Schmidt observed a bright band in the fluorescence spectrum in the case of vapor confined in glass bulbs which appeared to coincide with the D lines. The same appearance I afterwards observed independently, but on continuing the study of the subject was forced to refer the existence of this band in the yellow to sodium in the flame which heated the bulb. This I have since found was a mistake, for on repeating the experiments with exhausted glass bulbs I have succeeded in stirring up a faint fluorescence with approximately monochromatic light from the illuminator already referred to, of wave-length equal to that of the D lines. The failure to observe it in the case of the experiments made last year by Mr. Moore and myself was due to the fact that this yellow light was removed from the incident beam by the sodium vapor before the light rays met at the focus. This work is still in progress, and the fluorescent spectra given by the vapor when illuminated with monochromatic light of various wave-lengths have been photographed.

The investigations recorded in the present paper have been made possible by a very generous grant from the Rumford Fund, and I wish to express to the trustees of the fund my appreciation of and thanks for the aid which I have received.

I wish also to express my appreciation of the very faithful work done by my Assistant Mr. A. H. Pfund, who has worked with me and made many valuable suggestions, and my thanks to the board of trustees of the Carnegie Institution for the means placed at my disposal, by which I have been able to secure his services.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–
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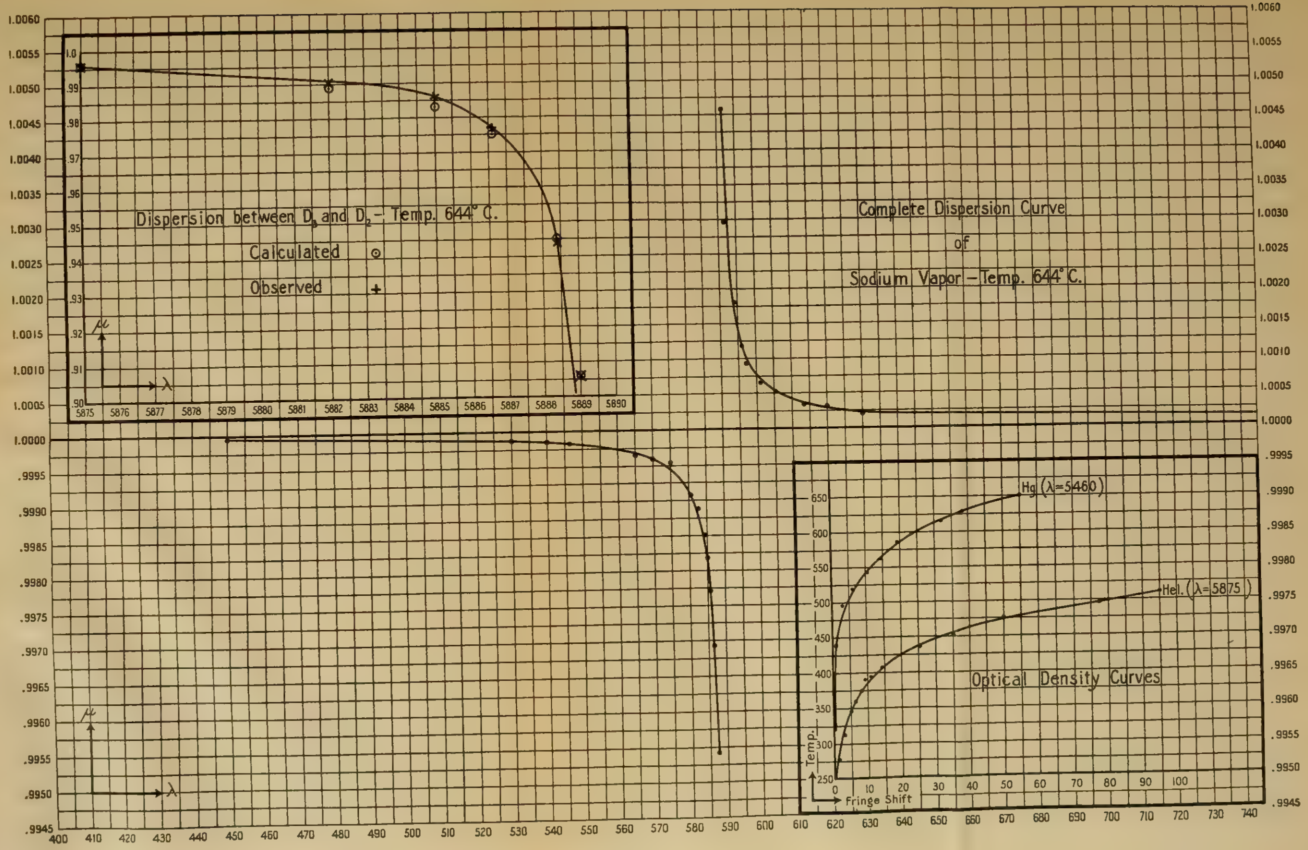
Helium Tube


Dispersion between D$_3$ and D$_2$ – Temp. 644° C.
Calculated ⊙
Observed +
μ
λ
1.0
.99
.98
.97
.96
.95
.94
.93
.92
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Complete Dispersion Curve of Sodium Vapor – Temp. 644° C.
1.0060
1.0055
1.0050
1.0045
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1.0035
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710
720
730
740
Optical Density Curves
Hg (λ=5460)
Hel. (λ=5875)
Temp.
650
600
550
500
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400
350
300
250
Fringe Shift
0
10
20
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40
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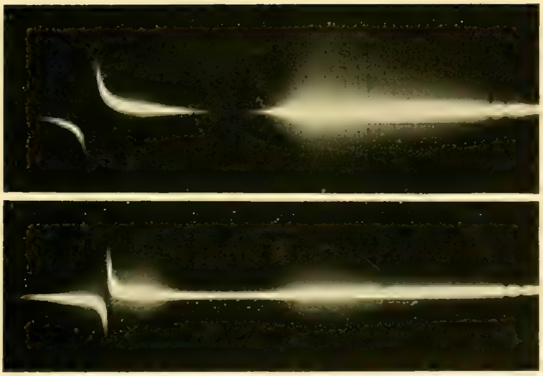
100



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CONTRIBUTIONS FROM THE RESEARCH LABORATORY OF
PHYSICAL CHEMISTRY OF THE MASSACHUSETTS
INSTITUTE OF TECHNOLOGY. — No. 2.

THE ELECTRICAL CONDUCTIVITY OF VERY DILUTE HYDROCHLORIC AND NITRIC ACID SOLUTIONS.

By H. M. Goodwin and R. Haskell.

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By H. M. Goodwin and R. Haskell.

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I. Purpose of the Investigation.

It is well known that there is great difficulty in obtaining an exact value for the equivalent conductivity of completely dissociated acids, owing to the large effect of the impurities of the water on the conductivity of even strong acids in very dilute solutions. These give rise to a maximum value of the conductivity at a dilution of from 500 to 2000 liters, and to a marked decrease in its value at greater dilutions. Yet a knowledge of the theoretical limiting value is of great importance, since upon it is based the assumed value of the equivalent conductivity of the hydrogen ion and the calculated degrees of dissociation of acids in general.

Kohlrausch† in 1898 derived values for the conductivity of strong acids at extreme dilution by adding to the observed value at 0.001 normal a quantity equal to the product of the observed increase between 0.001 and 0.00001 normal in the case of neutral salts by the ratio of the

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* The following investigation has been made possible through financial aid extended by the Trustees of the Wm. E. Hale Research Fund.

† Wied. Ann., **66**, 794 (1898).

increase between 0.1 and 0.001 normal in the case of the acids to the corresponding increase in the case of neutral salts, the conductivity of the water having been first subtracted in all cases. By subtracting from these values the conductivity of the negative ions, which had been derived from the conductivity and transference values of neutral diionic salts, he arrived at the value 318 for the equivalent conductivity of the hydrogen ion at 18° C. — a value, however, which he regards only as a rough estimate. Three years later * he deduced also the temperature change of this conductivity, expressing it by the equation:

$$\Lambda_t = 318\,[1 + 0.0154\,(t - 18) - 0.000033\,(t - 18)^2].$$

Quite recently Noyes and Sammet † have deduced the equivalent conductivity of the hydrogen ion by an independent method; namely, by combination of the transference number of moderately dilute hydrochloric acid with the equivalent conductivity of the chlorine ion derived from transference determinations and conductivity measurements of potassium chloride. The value thus obtained is 330 at 18° or 3.8 per cent greater than that of Kohlrausch. The authors consider that this divergence cannot be due to experimental errors, but that it must arise either from an error in the extrapolated values of the conductivity of acids at extreme dilution, or from some real defect in the theoretical considerations from which the identity of the results of the two methods is predicted.

As it seemed possible to eliminate to a great extent the effect of the impurities in the water by appropriate measurements of the conductivity of dilute acids and a treatment of the data by certain methods to be described below, we undertook an investigation in this direction, both with nitric and with hydrochloric acid, and desire now to present the results which have been obtained. It may be stated in advance that these results are in substantial agreement with the extrapolated value of Kohlrausch.

II. Preparation of the Solutions.

In the preparation of the solutions, water was used which had a specific conductance of $0.8 - 1.5 \times 10^{-6}$ reciprocal ohms and which had been purified by redistillation of ordinary distilled water to which alkaline permanganate was added, the first and last portions of the distillate

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* Sitzungsberichte der Preuss. Akad. d. Wissenschaften, 1901, 1026.

† Journ. Amer. Chem. Soc., **24**, 965 (1902); **25**, 165 (1903). Zeitschr. phys. Chem., **43**, 49 (1903).

being rejected. The acid solutions were made by adding to definite portions of this water weighed portions of a standard acid of strength about 0.01-normal. The standard hydrochloric acid solution was obtained by absorbing in water hydrogen chloride gas made from twice-precipitated sodium chloride and pure sulphuric acid. Its strength was determined gravimetrically by precipitating with silver nitrate and weighing the silver chloride in Gooch crucibles. The several determinations made showed an average deviation from the mean value of less than 0.05 per cent. A standard nitric acid solution approximately 0.01 normal was made by diluting chemically pure acid through which air had been drawn to remove any lower oxides of nitrogen dissolved in it. This was standardized against barium hydroxide solution, whose strength had been determined by two independent methods. Five 0.01 normal solutions of potassium chloride were used to obtain the cell-constant; these were made from a sample of the salt which had been twice precipitated with pure hydrochloric acid and then crystallized from water, dried, and ignited to incipient fusion. A 0.01 normal sodium chloride solution made from salt prepared in the same way was used as a check.

In all cases, for the sake of greater convenience and accuracy, the content of the solution was determined by weighing, and not by volume measurement; but the values thus obtained of the number of milli-equivalents of solute in 1000 grams of water were invariably (both in the calculation of the cell-constant and of the specific and equivalent conductivity of the acid solutions) considered to be numerically identical with the concentration in milli-equivalents per liter of solution. That no appreciable error (not even one as great as 0.05 per cent) is introduced by this assumption will be readily seen, since none of the solutions were stronger than 0.01 normal.

III. The Conductivity Apparatus and Method of Measurement.

The conductance of the solutions was measured by the Wheatstone slide-wire bridge arrangement in the form recommended by Kohlrausch, using an alternating current produced by an induction coil and a telephone as an indicator of zero potential. The bridge consisted of a marble drum on which was wound five meters of manganine wire. Five or six settings were made for each solution, so as to eliminate as far as possible any personal error in determining the exact point corresponding to the minimum sound in the telephone. The average deviation of the separate values from the mean was less than 0.1 per cent except in the case of the extreme dilutions, where it amounted in a few cases to 0.3 per cent. The probable error of the mean was always less than 0.1 per cent except in the case of water. Both the slide-wire and the rheostat used were carefully calibrated, and the corrections were found in no case to exceed 0.1 per cent. The resistance vessel for holding the solution whose conductance was to be measured was a platinum cylinder 15 cm. high and 5 cm. in diameter. Into this was inserted, so as to rest on the bottom of the cylinder, an inverted glass bell 4.4 cm. in diameter attached to a glass tube sealed near the bottom, as shown in Figure 1.

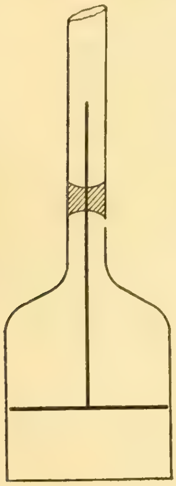
FIGURE 1.

Into the bell an unplatinized but mechanically roughened platinum disk was sealed by means of a stiff platinum wire. The electrode was not platinized, so as to reduce as far as possible absorption effects. It had an area of 15 sq. cm., and the distance between its lower surface and the bottom of the platinum cylinder was 1.9 cm. There was enough space along the side of this disk to allow the liquid to pass, and near the top of the bell a large hole was blown so that raising and lowering the glass plunger gave an efficient means of stirring. It was proved that the "cell constant" of the apparatus varied less than 0.05 per cent with different settings of the plunger in the vessel. The use of a platinum vessel made it possible to collect the water at a little below the boiling temperature as it came off fresh from the still, whereby better water could be obtained, as there was less chance for the absorption of carbon dioxide. The plunger passed through a vulcanite cap which fitted the top of the cylinder, and thus kept the solution from much contact with the air. The cylinder was immersed almost to the top in a thermostat kept always within 0.02° of 18°. The temperature was measured by a thermometer that had been standarized and corrected to the scale of an air thermometer. The room was also kept nearly at the same temperature to prevent condensation at the top of the cell. Contacts were made by mercury.

The solutions were made by adding successively a number of portions of standard acid (approximately 0.01 normal) from a weighed Beckman pipette to a weighed quantity (about 180 grams) of water in the vessel. The weight of the added portion could easily be determined to less than 0.5 milligram, or 0.05 per cent in the case of the smallest portion. In this way a series of solutions, usually varying in concentration from about 0.0001 normal up to 0.001 or 0.002 normal, were prepared, and their conductance measured.

Several experiments were made to determine whether the conductivity of dilute acid solutions changed with time upon standing in the covered conductivity vessel in the thermostat. The change upon standing over night amounted to only 0.1 per cent of the conductivity of a 0.0002 normal solution.

In order to confirm the accuracy of the method, similar series of measurements were made with potassium chloride solutions at the same dilutions as in the case of the acids. The results, which are communicated in the Section V, agree within 0.1 per cent with those obtained by Kohlrausch and Maltby, in their very accurate determinations published in 1900.*

IV. METHODS OF CORRECTING FOR IMPURITIES IN WATER AT GREAT DILUTION.

We have employed two methods for eliminating the influence of the impurities in the water on the conductivity of the acid solutions. Both of these involve the principle that *the total effect of the acid and of the impurities in the water on the conductivity of each other is already produced when a relatively small quantity of acid has been added to the water, after which further additions of acid remain unaffected by the impurities.*

In the first method of computation this principle is applied in a direct manner by simply subtracting the conductivity of a very dilute acid solution from that of a more concentrated one, and regarding the ratio of this difference to the difference in the respective concentrations as the equivalent conductivity of the acid at the higher concentration; that is, if κ_1 and κ_2 are the conductivities at the concentrations C_1 and C_2 respectively, C_1 being a very small concentration, it is assumed that $\frac{\kappa_2 - \kappa_1}{C_2 - C_1} = \Lambda_{C_2}$, the equivalent conductivity at the concentration C_2.†

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* Wissensch. Abhandlungen d. phys.-technischen Reichsanstalt, **3**, 157–227 (1900). Zeitschr. phys. Chem., **36**, 750 (1901).

† In the calculation of Λ_{C_2} it is necessary to apply a slight correction to the observed conductance (κ_1'), since when the second portion of acid is added the

Thus, the underlying idea is the preparation, by the addition of a little acid, of a sample of solvent the impurities in which shall have no appreciable influence on the conductivity of subsequently added portions of acid. Theoretically, it seems highly probable that this must be practicable; for any basic constituents of the water like ammonium or other alkaline hydroxides or carbonates would be completely neutralized, and any weakly acid constituent like carbonic acid which Kohlrausch* has shown to be undoubtedly the chief source of impurity, would have its dissociation reduced nearly to zero, as soon as the concentration of the strong acid added reached a value as great as, or a few times greater than, the concentration of the impurity. It is true that even though the impurities may have been thus rendered harmless, the increase in conductivity produced by the added portion of acid will not be strictly identical with the conductivity which it would have if it were added to pure water; for the acid originally present will be somewhat reduced in dissociation, and will itself reduce the dissociation of the added acid: but this effect is known to be very small at high dilutions, and the last-mentioned part of it is eliminated by regarding the calculated equivalent conductivity as corresponding to the final concentration rather than to the difference in concentration.†

In regard to the choice of the initial concentration, it may be pointed out that this must be taken so large that the maximum effect of the impurities in the water and of the acid on the conductivity of each other shall be attained, and that this condition is shown to be fulfilled when the equivalent conductivities calculated from two successive values of the initial concentration are in substantial agreement. The fact that

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volume of the solution is somewhat increased, whereby the observed conductance and concentration would be decreased in the same proportion. Thus if m_w, m_1 and m_2 are the weights of water, of the first portion of added acid of a concentration C_0, and of the second portion respectively, then, placing volumes proportional to weights, which at the dilutions in question produces no error, we have:

$$\kappa_1 = \kappa'_1 \frac{m_w + m_1}{m_w + m_1 + m_2}. \quad \text{Also} \quad C_2 - C_1 = \frac{C_0 m_2}{m_w + m_1 + m_2}.$$

* Wissensch. Abhandlungen d. phys.-tech. Reichsanstalt, **3**, 193 (1900).

† For example, it is readily calculated from Kohlrausch's mean values (Wied. Ann., **66**, 794, 1898) of the change of the conductivity at 18° of the ions of neutral salts with the dilution that the mean error in their equivalent conductivity would be 0.42 per cent at 0.0002 normal and 0.25 per cent at 0.0005 normal if computed by the method here described, taking 0.0001 as the initial concentration. In the case of the strong acids, whose conductivity changes much less rapidly with dilution, the error from this source would be about one-half as great.

such an agreement is actually found to exist even with relatively small values of the initial concentration is evidence of the correctness of the principle on which the computation is based.

The second method of computation is based on the assumption that the true values of the equivalent conductivity increase regularly to a maximum at the highest dilution, and that the decrease in the observed values from the maximum obtained at a moderate dilution is due to a *constant* error in the specific conductance arising from the effect of the impurities in the water. Therefore it is justifiable to add to the observed values of the specific conductance such a quantity as will cause the maximum in the so-corrected equivalent conductivity values to occur at the lowest concentration at which the error just referred to has attained its constant value. The method of procedure is best illustrated by a reference to the accompanying diagram (Figure 2). In this the curve AB represents the "observed" values of the equivalent conductivity Λ, of the acid computed from the results of an actual series of experiments (first series, in nitric acid, page 410) by subtracting from the observed specific conductivity of the solution κ_s the observed conductivity of the water κ_w, and dividing the difference by the concentration c; that is, $\Lambda_{Obs} = (\kappa_s - \kappa_w) / c$. Now the error in the value Λ at A must be at least as great as the ordinate AC, C being equal to the

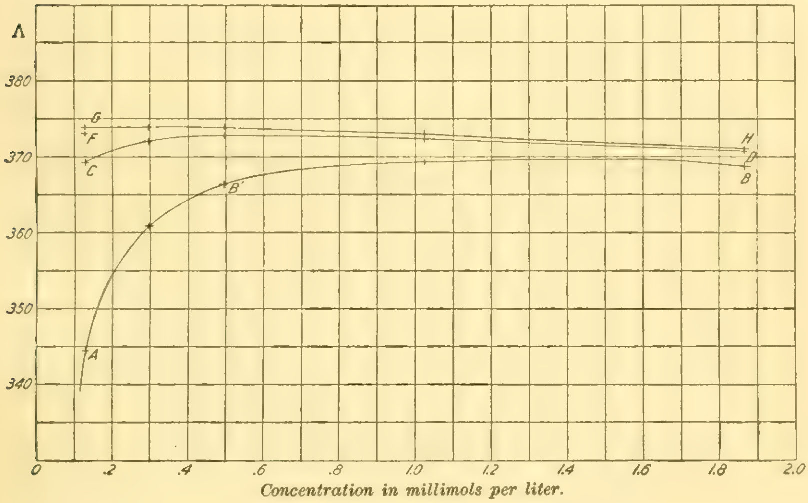

FIGURE 2.

maximum ordinate of AB. If we therefore increase its value by this amount and increase the observed value of Λ of every other point as B′ by this amount multiplied by the inverse ratio of the concentrations corresponding to B′ and A respectively, a new series of values corresponding to the curve CD is obtained. The value C is still seen to be in error by an amount as great as CF (F being equal again to the maximum ordinate of the new curve CD); the other values on this curve are in error by an amount less in proportion to the greater concentration to which they correspond: we therefore increase all the values in the same way as before, producing a third curve, and this process is repeated till the maximum point on the resulting curve lies on the ordinate through ACF and the curve has the form of GH. This method may also be described algebraically as follows. The true specific conductivity κ_c, of the acid (that which it would exhibit in pure water at any concentration c), is equal to the observed conductivity of the solution κ_s, diminished by that of the water κ_w, and increased by a quantity δ, constant at all concentrations and equal to the reduction in conductivity produced by mutual influence of the acid and impurities; that is, $\kappa_c = \kappa_s - \kappa_w + \delta$, or expressed in terms of equivalent conductivities, $\Lambda_c = \Lambda_{0bs} + \frac{\delta}{C}$ where Λ_c is the true equivalent conductivity of the acid and Λ_{0bs} is the "observed" equivalent conductivity as above defined. The value of δ is then determined from this equation by a series of approximations as described above until finally $\frac{\delta}{C} = 0$ and Λ_c coincides with Λ_{0bs}.

It should be understood, however, that this method gives only a minimum value for the equivalent conductivity at the highest dilution, for it is only in case the Mass-Action Law equation holds that the conductivity curve approaches asymptotically to a maximum with increasing dilution. If the empirical formula of Kohlrausch $\Lambda_0 - \Lambda_c = K\sqrt{C}$ is applicable, the upward slope of the curve must continually increase with decreasing concentration. In fact, Kohlrausch found that the behavior of neutral salts at very low concentrations (0.001 — 0.0001 normal) was intermediate between that required by these two formulae. The values extrapolated with the help of the cube-root formula will therefore give in all probability a maximum limit for the conductivity at the greatest dilution, and the true value will lie between it and that derived by the method just described.

In applying these methods to our results, from 0.06 to 0.1 milliequivalents per liter was taken as the standard initial concentration, although some measurements and computations have also been made at greater dilutions. The reasons for this selection are: first, that the resistance measurements, owing to a sharper minimum, are considerably more accurate (the average deviation of the separate readings from the mean being less than 0.1 per cent) than in the more dilute solution; and secondly, that the computed equivalent conductivities differ when the initial concentration chosen is varied from less than 0.03 to 0.06 millinormal, but that they no longer do so to an important extent when it is varied from 0.05 to 0.01 millinormal, showing that in the 0.03 millinormal solution the maximum influence of the water has not yet been produced.

It will be seen from the data given in the next section that the water used in different series of experiments had a conductivity varying from 0.8 to 1.6×10^{-6}, and consequently, the observed equivalent conductivity at the higher dilutions also exhibit considerable variations. In fact, the latter exist even where different samples of water with the same conductivity were used, showing that the character of the impurities varies and exerts a specific influence. Nevertheless, the corrected values of the equivalent conductivity are substantially identical — a fact which furnishes strong evidence of the correctness of the principle underlying the computations.

The above methods only serve to extend the series of reliable values of the equivalent conductivity of acids from about 0.002 normal down to 0.0002 or 0.0001 normal. To what extent the values at the latter concentration differ from those at infinite dilution can only be estimated. Probably the best basis for this estimate is the assumption of Kohlrausch which is most briefly expressed by the relation:

$$\frac{(\Lambda_\infty - \Lambda_{4000}) \text{ for acids}}{(\Lambda_{4000} - \Lambda_{100}) \text{ for acids}} = \frac{(\Lambda_\infty - \Lambda_{4000}) \text{ for salts}}{(\Lambda_{4000} - \Lambda_{100}) \text{ for salts}} = \frac{0.9}{3.0}{}^*$$

This method of extrapolation has been applied to our results. Since according to our measurements with hydrochloric acid described in Sections V. and VI. $\Lambda_{4000} - \Lambda_{100} = 377.6 - 369.3 = 8.3$, the so-determined value of $(\Lambda_\infty - \Lambda_{4000})$ for acids is only 2.5 units (or 0.7 per cent).

V. The Conductivity Data.

The experimental data and the values calculated therefrom will now be presented.

The temperature was in all cases 18.00° ± 0.02°. The standard solution of hydrochloric acid used in all the experiments was 0.009074* normal; that of nitric acid was 0.010145 † normal. The cell constant or capacity of the resistance vessel (that is, the ratio of the specific conductance of any solution to its actual conductance as measured in the vessel) was 0.11659.‡

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* Kohlrausch, Wied. Ann., **66**, 794 (1898).

The following tables contain the results of the separate series of experiments. As headings are given, the weight in grams (m_w) of water taken, its actual conductance (L_w) multiplied by 10^6 when measured in the vessel, and its specific conductance (κ_w) multiplied by 10^6. The first column gives the number of the portions of standard acid or potassium chloride solution which were added; the second gives the total weight of the added portions; the third, the concentration (c) of the resulting solution in milliequivalents per liter (see Section II); the fourth, the conductance in reciprocal ohms measured in the resistance vessel and multiplied by 10^6; the fifth, the equivalent conductivity of the acid calculated by subtracting from the specific conductance of the solution (κ_s) that of the water and dividing the difference by the concentration i. e. $(\kappa_s - \kappa_w)/c$. In the sixth and seventh columns are given the corrected values of the equivalent conductivity computed by the first method described in Section IV, taking for the initial concentration that corresponding to the first and second added portions, respectively. In the eighth column are given the corrected values of the equivalent conductivity calculated by the second method described in Section IV. (In the case of the potassium chloride series these are replaced by the equivalent conductivities obtained by Kohlrausch and Maltby.)

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* This result was based upon four determinations, of which the data were as follows:

Grams HCl solution taken	346.27	328.60	347.70	356.38
Grams AgCl obtained	0.4506	0.4277	0.4521	0.4623
Equivalents per 1000 grams sol.	0.009076	0.009078	0.009068	0.009047

The last on account of its great deviation was omitted in taking the mean.

† This solution was obtained by dilution (by weight), of a 0.05 normal solution which has been standardized against a 0.1 normal solution of recrystallized barium hydroxide, the titration values showing an average deviation of 0.03 per cent from their mean. The barium hydroxide was standardized by neutralizing it with nitric acid, evaporating, drying at 170°, and weighing as $Ba(NO_3)_2$, and also precipitating it with sulphuric acid, filtering out, igniting, and weighing the $BaSO_4$. The average deviation from the mean of the four determinations by those two methods was 0.05 per cent.

‡ This was the mean derived from measurements with five distinct 0.01 normal potassium chloride solutions containing 0.7455 grams KCl weighed in air, in 1000 grams water: the average deviation of the five values from the mean was 0.03 per cent.

HYDROCHLORIC ACID.

$m_w = 163.73$; $10^6\, L_w = 11.13$; $10^6\, \kappa_w = 1.30$.

Portion No.	Total Weight of Added Portions.	Concentration in Milliequivalents per Liter.	Observed Conductance $\times 10^6$.	Equivalent Conductivity.			
				Water Subtracted.	By First Method.		By Second Method.
					I.	II.	
1	0.2248	0.0125	34.82	221.8	. . .	. . .	420.6
2	2.0305	0.1114	349.34	354.6	371.3	. . .	376.9
3	5.0528	0.2717	868.42	367.9	374.8	376.8	376.9
4	8.2065	0.4341	1390.9	371.3	375.7	377.8	376.9
5	12.9987	0.6689	2144.8	372.6	375.4	376.1	376.2
6	19.190	0.9574	3061.0	371.4	375.4	375.7	374.0
$m_w = 170.96$; $10^6\, L_w = 11.95$; $10^6\, \kappa_w = 1.37$.							
1	1.3360	0.07036	221.3	346.8	. . .	. . .	379.6
2	5.4563	0.2806	904.3	370.6	378.5	. . .	376.4
3	9.6226	0.4835	1557.9	372.7	377.0	. . .	376.3
4	12.032	0.5966	1923.8	373.6	377.0	. . .	376.4
5	17.928	0.8612	2775.5	374.1	376.4	. . .	375.9
6	37.166	1.6204	5214.8	374.3	375.4	. . .	375.3
$m_w = 193.83$; $10^6\, L_w = 9.41$; $10^6\, \kappa_w = 1.10$.							
1	0.5642	0.02634	76.56	297.1	. . .	. . .	382.8
2	1.2787	0.05947	183.82	341.8	377.2	. . .	379.5
3	2.4332	0.1125	354.3	357.4	375.6	374.4	377.5
4	5.0564	0.2307	737.5	367.9	376.8	376.8	377.6
5	7.3384	0.3311	1062.2	370.7	377.0	376.9	377.3
6	11.162	0.4942	1588.8	372.6	376.7	376.7	377.0
7	15.184	0.6593	2119.6	373.1	376.1	376.2	376.3
8	35.180	1.3942	4489.5	374.6	376.0	375.9	376.2
$m_w = 184.65$; $10^6\, L_w = 7.44$; $10^6\, \kappa_w = 0.868$.							
1	2.0218	0.09828	313.9	363.5	. . .	. . .	378.7
2	3.8508	0.1854	597.8	371.3	379.8	. . .	379.4
3	5.7913	0 2760	889.8	372.8	377.7	375.7	378.1
4	8.1604	0.3841	1238.7	373.7	377.1	376.0	377.5
5	11.665	0.5392	1741.3	374.9	377.3	376.8	377.6
6	18.636	0.8319	2678.1	374.3	375.6	375.1	376.1
$m_w = 184.65$; $10^6\, L_w = 7.29$; $10^6\, \kappa_w = 0.854$.							
1	0.9254	0.04526	141.28	345.1	. . .	. . .	371.6
2	2.5137	0.1219	392.35	368.3	381.9	. . .	378.3
3	5.4093	0.2583	835.20	373.7	379.8	378.4	378.4
4	7.8974	0.3722	1201.8	374.2	378.1	376.9	377.4
5	11.571	0.5351	1728.0	374.9	377.5	376.8	377.2
6	15.715	0.7117	2298.5	375.3	377.2	376.7	376.9
7	19.915	0.8834	2848.5	374.9	376.5	375.9	376.2
8	40.055	1.6176	5208.0	374.8	375.6	375.4	375.5

Nitric Acid.

$m_w = 181.38$; $10^6\ L_w = 13.35$; $10^6\ \kappa_w = 1.59$.

Portion No.	Total Weight of Added Portions.	Concentration in Milliequivalents per Liter.	Observed Conductance × 10^6.	Equivalent Conductivity.			
				Water Subtracted.	By First Method.		By Second Method.
					I.	II.	
1	0.8480	0.0472	132.1	293.0	. . .	. . .	374.6
2	2.3534	0.1299	397.0	344.1	373.1	. . .	373.9
3	5.4994	0.2985	937.7	360.9	373.4	373.6	373.9
4	9.3059	0.4951	1568.1	366.1	373.5	373.6	373.8
5	20.477	1.0291	3273.2	369.3	372.6	372.6	373.0
6	40.940	1.8679	5922.0	368.8	371.4	371.3	370.8
$m_w = 175.49$; $10^6\ L_w = 13.05$; $10^6\ \kappa_w = 1.52$.							
1	1.0421	0.0599	183.7	332.0	. . .	. . .	374.4
2	3.2765	0.1860	588.9	361.0	374.7	. . .	374.7
3	6.5193	0.3634	1159.7	367.9	374.7	374.8	374.7
4	9.0405	0.4972	1586.0	368.8	373.8	373.5	373.9
5	14.931	0.7955	2538.4	370.1	373.0	372.7	373.2
6	20.373	1.0553	3368.2	370.6	372.8	372.6	373.0
$m_w = 169.23$; $10^6\ L_w = 12.03$; $10^6\ \kappa_w = 1.40$.							
1	1.8262	0.1083	337.8	350.6	. . .	. . .	375.2
2	4.1137	0.2465	782.5	364.4	375.1	. . .	375.2
3	7.0453	0.4110	1306.2	367.9	373.9	. . .	374.4
4	12.1903	0.6868	2190.6	369.8	373.2	. . .	373.7
5	19.894	1.0720	3421.2	370.8	372.9	. . .	373.4
6	40.299	1.9550	6229.7	370.2	371.8	. . .	371.6
$m_w = 183.75$; $10^6\ L_w = 12.64$; $10^6\ \kappa_w = 1.48$.							
1	1.6063	0.0879	277.1	350.7	. . .	. . .	372.1
2	3.7972	0.2054	656.0	365.2	373.9	. . .	374.1
3	5.8714	0.3141	1005.6	369.5	375.3	374.8	374.2
4	8.8942	0.4684	1500.0	370.2	374.6	374.1	374.1
5	14.166	0.7261	2325.0	369.6	373.9	373.6	372.2
6	20.282	1.0085	3224.0	371.2	373.1	372.8	372.8
7	40.360	1.8270	5827.7	371.0	371.9	371.5	372.0
Potassium Chloride.							
$m_w = 185.22$; $10^6\ L_w = 0.862$; $10^6\ \kappa_w = 1.006$.							
Conc. Stand. Solution = 0.009990.							(Kohlrausch and Maltby.)
1	1.2147	0.0651	80.66	129.02	. . .	. . .	129.16
2	3.5447	0.1876	215.62	128.64	128.52	. . .	128.81
3	9.9054	0.5072	565.55	128.06	127.99	. . .	128.10
4	20.108	0.9784	1078.2	127.45	127.46	. . .	127.39

In connection with the above data, attention is called to the close agreement in the case of potassium chloride between our values of the equivalent conductivity, given in column five marked "water subtracted," with those of Kohlrausch and Maltby; also to the fact that the application of the first correction method to this neutral salt causes a change of less than 0.1 per cent in the values of the equivalent conductivity. It will be seen, on the other hand, that, in the case of the acids, both methods of computation raise the equivalent conductivity on an average by about 2 per cent in 0.25 millinormal, and by about 0.5 per cent in millinormal solution. Although the values in column five vary quite widely in the different series owing to differences in the quality of the water used, yet when corrected by Method I they are in substantial agreement in all cases where the concentration corresponding to the first added portion exceeds 0.05 millinormal. The agreement between the results obtained by the two methods is considered on page 413.

The mean of the "water-subtracted" values at the concentration millinormal (374.0 for hydrochloric acid and 371.4 for nitric acid) are nearly one per cent lower than the corresponding values earlier obtained by Kohlrausch* (377 and 375 respectively). This fact led us to make a new series of measurements on the conductivity of hydrochloric acid at higher concentrations. These were made at 18° in a U-shaped apparatus with a new sample of acid prepared and standardized as before.† The dilutions of the stock solution (0.13747 normal) were made by weight and reduced to volume-concentrations at 18°, by means of the specific gravities of water and of the stock solution. On account of the wide range of concentrations measured it was found necessary to use two resistance vessels. The cell constants or capacities of these, determined by five 0.01 normal potassium chloride solutions, were 2.5417 and 8.5370. The average deviation of the separate values from the mean was less than 0.03 per cent in each case.

The results are contained in the following table. Values corresponding to the round concentrations are summarized in Section VII (page 415).

The values are seen to be only slightly (0.1–0.4 per cent) lower than

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* Kohlrausch and Holborn, Leitvermogen der Electrolyte, p. 160.

† The analytical data were as follows:

Grams HCl solution taken . . .	112.34	106.58	91.31
Grams AgCl obtained	2.2121	2.0988	1.7994
Equivalents per 1000 grams sol. .	0.13734	0.13734	0.13744

HYDROCHLORIC ACID.

$10^6 L_w = 0.34$. $10^6 \kappa_w = 0.85$. Cell constant = 2.542.

Concentration in Equivalents per Liter.	Actual Conductance × 10^6.	Equivalent Conductivity.*	Equivalent Conductivity (interpolated from Kohlrausch).
0.001917	270.12	375.0	376+
0.001937	285.87	375.1	376+
0.004744	697.25	373.1	373+
0.005525	809.79	372.5	372+
0.010124	1470.8	369.2	370
0.01147	1661.8	368.2	369+
0.02714	3878.6	363.4	365
0.03137	4468.9	362.1	364−
$10^6 L_w = 0.10$. $10^6 \kappa_w = 0.85$. Cell constant = 8.537.			
0.13747	5600.9	347.8	347+

those of Kohlrausch. Noyes and Sammet † found in $\frac{1}{58}$ and $\frac{1}{252}$ normal solution values 0.5–06 per cent less than Kohlrausch's. The above values were plotted against the cube root of the concentration, and between 0.005 and 0.137 normal were found to lie upon a straight line, the greatest deviation from it being less than 0.1 per cent.

VI. THE CORRECTED VALUES OF THE EQUIVALENT CONDUCTIVITY.

In the following table are summarized the equivalent conductivity values of the dilute solutions computed by both methods from all the series of experiments. The values are interpolated for round concentrations intermediate between those involved in the experiments. In the case of the first method the values computed from the initial concentration corresponding to the first added portion were used, except in the cases where this was less than 0.05 millinormal.

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* In these values no account is taken of the effect of the water, the correction for which is negligible except in the first two cases in which the concentration is approximately only 0.002 normal. For this dilution the correction is +0.2 unit, making the corrected values 375.2 and 375.3 respectively. This correction is taken as one-half that computed from data on p. 409 for a 0.001 normal solution.

† Journ. Amer. Chem. Soc., **24**, 964 (1902).

HYDROCHLORIC ACID.

Concentration Milliequivalents.	*First Method.* Separate Series.					Mean.	*Second Method.* Separate Series.					Mean.
0.10	. . .	. . .	. . .	. . .	. . .	. . .	376.9	379.1	377.5	378.8	378.3	378.1
0.25	376.8	378.5	376.8	377.7	378.4	377.6	376.9	376.9	377.5	378.2	378.4	377.6
0.50	377.4	377.0	376.7	377.3	376.8	377.0	376.7	376.4	377.0	377.5	377.3	377.0
0.75	376.1	376.7	376.2	375.6	376.5	376.2	375.5	376.1	376.3	376.5	376.7	376.0
1.00	375.7	376.2	376.0	. . .	375.8	375.9	373.7	375.8	376.2	375.6	376.1	375.5
1.50	. . .	375.6	375.9	. . .	375.5	375.7	. . .	375.4	376.1	. . .	375.6	375.6
NITRIC ACID.												
0.10	. . .	. . .	. . .	. . .	. . .	. . .	374.0	374.6	375.2	374.1	. . .	374.3
0.25	373.6	374.7	375.1	374.5	. . .	374.5	373.9	374.7	375.2	374.1	. . .	374.3
0.50	373.6	373.8	373.7	374.5	. . .	373.9	373.8	373.9	374.2	373.8	. . .	373.9
0.75	373.1	373.0	373.1	373.8	. . .	373.3	373.4	373.3	373.6	373.4	. . .	373.4
1.00	372.6	372.8	372.9	373.1	. . .	372.9	373.1	372.9	373.4	372.8	. . .	373.0
2.00	371.1	. . .	371.8	371.7	. . .	371.5	370.4	. . .	371.5	371.9	. . .	371.3

It will be seen that the results of the two methods of calculation agree almost completely in the case of both acids. The equivalent conductivity for 0.25 millinormal hydrochloric acid is 377.6, and for nitric acid of the same strength is 374.4. The difference, 3.2, agrees well with the difference, 3.6, between the conductivities of the Cl and NO_3 ions as derived by Kohlrausch from measurements with neutral salts. It is also of interest to note that the conductivity value extrapolated for 0.25 millinormal hydrochloric acid from those between 0.005 and 0.137 normal under the assumption of the validity of the cube-root formula ($\Lambda_0 - \Lambda_c = K C^{\frac{1}{3}}$) is 381 or only about 0.9 per cent greater than our corrected value.

It remains only to apply the estimated correction of +2.5 units, discussed at the end of Section IV, in order to obtain the most probable values of the equivalent conductivity at infinite dilution. The values so obtained are 380.1 for hydrochloric acid, and 377.0 for nitric acid. The values recently derived by Kohlrausch * are 383.4 and 379.8 respectively, which differ from ours by less than 1 per cent. Unless the change with the concentration at very high dilutions follows a law radically different from that prevailing at moderate dilutions, it seems certain that these values are substantially correct. The much higher value 395.5 for hydrochloric acid derived by Noyes and Sammet from transference experiments apparently arises from some considerable error the nature of which still remains to be explained.

VII. Summary.

In this article have been described measurements of the electrical conductivity at 18° of very dilute (0.0001 to 0.002 normal) hydrochloric and nitric acid solutions made by adding to a known weight of water successive portions of 0.01 normal acid. To eliminate the effect of the impurities of the water, two methods of computation of the equivalent conductivity have been used. In both of these, the assumption is involved that the total effect of the acid and impurities on the conductance of each other is produced when a relatively small quantity of acid has been added to the water. The experiments themselves furnish evidence of the correctness of this assumption. In the first method, the increase of the specific conductance over this initial value divided by the increase of concentration is regarded as the equivalent conductivity at the higher concentration. In the second method, such a quantity, constant at every concentration, is added to the observed values of the specific conductance as will cause the maximum in the equivalent conductivity values calculated therefrom to occur at the lowest concentration at which the total influence of the acid and impurities on each other has been produced. In spite of considerable variations in the observed conductivities due to the use of different samples of water, the corrected values of the equivalent conductivity derived from different experiments and those computed by the two methods agree well with each other. New determinations were also made at higher concentrations with hydrochloric acid. The final values of the equivalent conductivity (Λ) at various round concentrations (C) in equivalents per liter are as follows:

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* Sitzungsber. d. konigl. preuss. Akad., 1901, 1031.

Concentration in Mols.	Λ_{HCl}.	Λ_{HNO_3}.	Concentration in Mols.	Λ_{HCl}.
0	380.1	377.0	0.005	372.6
0.00025	377.6	374.5	0.010	369.3
0.00050	377.0	373.9	0.020	365.5
0.00075	376.2	373.3	0.030	362.5
0.00100	375.9	372.9	0.05	358.4
0.00150	375.6		0.100	351.4
0.00200	375.3			

The values for zero concentration or infinite dilution are extrapolated from those for 0.00025 normal under the assumption that the change between these two limits of concentration stands in the same ratio to that between 0.00025 and 0.01 normal as in the case of neutral salts; this ratio having been already derived by Kohlrausch. These values are less than one per cent lower than those of Kohlrausch and are believed to be substantially correct. That derived by Noyes and Sammet from transference experiments for hydrochloric acid (395.5), on the other hand, is, for some reason as yet unknown, four per cent higher.

In conclusion the authors desire to express their appreciation to the trustees of the William E. Hale Research Fund for the grant from that fund which enabled this research to be carried out. Also to Professor Noyes for many valuable suggestions received from him in the course of the work.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF IODINE.

By Gregory Paul Baxter.

A REVISION OF THE ATOMIC WEIGHT OF IODINE.

By Gregory Paul Baxter.

Presented by T. W. Richards, Oct. 12, 1904. Received August 6, 1904.

The atomic weight of iodine has been for some time considered one of the best determined of chemical constants, owing to the extremely concordant results of Stas and Marignac, who both deduced the value 126.85 (O = 16.000) from syntheses of silver iodide. Within the last two years, however, two series of determinations, one by Ladenburg and one by Scott, have yielded results over one tenth of a unit higher than the above. Ladenburg's method consisted in heating silver iodide in a current of chlorine until all the iodine had been displaced, and gave the result 126.96, while Scott, like Stas and Marignac, synthesized silver iodide and obtained in two analyses the values 126.96 and 126.98.* The anomaly in the atomic weights of iodine and tellurium has always been of the greatest interest, and has led to a large number of investigations upon the atomic weight of tellurium during the last few years, which have shown conclusively that this value lies in the neighborhood of 127.6. The doubt thrown upon the atomic weight of iodine by the work of Ladenburg and Scott makes it imperative to subject iodine to further careful investigation.

PART I.

The Ratio of Silver to Silver Iodide.

In choosing a compound for an atomic weight determination, two of the most important points to be considered are the stability of the substance under various conditions, and the certainty with which the atomic weights are known of those elements, besides the one under examination, which compose it. In the case of iodine, few of its compounds are stable, and the only one which satisfies both the above conditions is silver iodide. Accordingly synthesis of silver iodide from a weighed amount of silver was selected for the first method of investigation.

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* Very recently, since the work described in this paper was completed, Köthner and Aeuer have published a preliminary notice of experiments involving syntheses of silver iodide as well as a repetition of Ladenburg's work, from which they conclude that the atomic weight of iodine cannot be lower than 126.963. Details of their work are not given. See Ber. d. d. chem. Gesell., **37**, 2536.

Purification of Materials. — The problem of purification of materials was a comparatively simple one. Silver was prepared as follows: To a solution of silver nitrate which had been made strongly acid with nitric acid, there was added a large excess of concentrated hydrochloric acid. After the precipitate had been washed many times with pure water, it was reduced to metallic silver by means of a solution of sodium hydroxide and invert sugar. The silver was washed well with water and fused with a blowpipe on charcoal. Surface impurities were removed by scrubbing the buttons with sand, and boiling them, first with dilute hydrochloric acid, and then with ammonia. Next, the whole of the process just described was repeated, but with reagents which had been purified with the greatest care. Water and acids were freshly distilled, the sodium hydroxide was electrolyzed in a platinum dish until free from iron, and the final fusion was carried out in a porcelain crucible lined with the purest lime. This lining was made by the ignition of a mixture of reprecipitated calcic carbonate and recrystallized calcic nitrate. After the buttons had been cleansed as before, the silver was deposited electrolytically by making one button the cathode, the other buttons in succession the anode, in a cell containing a strong solution of silver nitrate made by dissolving one of the buttons in pure nitric acid. Connection was made with the battery by means of a platinum wire, which, however, did not dip into the solution. The crystals of silver were washed, first with water, then with ammonia, and again several times with water. Finally they were dried and fused in a current of hydrogen on a boat of pure lime contained in a porcelain tube. The hydrogen was generated electrolytically from a cell composed of zinc amalgam, hydrochloric acid, and platinum, and was purified from hydrochloric acid and dried by passing over beads moistened with caustic potash solution and then over solid potassic hydroxide. The silver was cut into fragments weighing from three to five grams each by means of a clean chisel and anvil, and the iron adhering to the surface was removed by warming the pieces repeatedly with fresh portions of dilute nitric acid, until iron could no longer be detected in the acid. The fragments were washed thoroughly with the purest water, then with ammonia, and again with water as before. They were allowed to dry in the air, and finally the last traces of moisture were eliminated by heating in a vacuum, since silver which is heated to a high temperature in the air has been shown by Stas to absorb considerable oxygen.* Silver prepared in a similar fashion by investigators in this laboratory has always been found to be pure,† and since two different samples were used in the following work with identical results, there can be no doubt of the purity of that employed here. Furthermore, one synthesis of silver chloride, carried out with a portion of this silver by Wells in the investigation of Richards and Wells upon the atomic weight of chlorine, yielded the value 35.467, which is identical with the average of other similar determinations made with his own silver.

The chief impurities in commercial iodine are halogens of lower atomic weight and iodide of cyanogen. These were removed by dissolving iodine in a strong solution of half its weight of potassic iodide, and distilling the greater portion of the iodine from a retort into a flask cooled with cold water. The iodine thus obtained was next converted into hydriodic acid by covering it with considerable water and passing through the solution a stream of sulphuretted hydrogen. This gas was generated by the action of dilute sulphuric acid upon ferrous sulphide, and was purified by bubbling through three gas washing bottles containing water and through two towers filled with beads moistened with water. The reaction between the sulphuretted hydrogen and the iodine results chiefly in the formation of hydriodic acid and the precipitation of sulphur, although a small quantity of sulphuric acid is always produced. Iodide of cyanogen with sulphuretted hydrogen yields hydrocyanic acid, hydriodic acid, and sulphur.‡ The solution was first boiled for a short time until the sulphur had clotted together, and the sulphur was removed by filtration. The clear solution was then boiled for several hours in order to eliminate the hydrocyanic acid. § Finally the hydriodic acid was partially converted into iodine by distilling the solution with potassic permanganate. This permanganate must also have had the effect of oxidizing any organic impurities. Since in the reaction three eighths of the iodine remain in the form of iodides, the resulting iodine was thus subjected to a second distillation from an iodide. The product was again converted into hydriodic acid by means of hydrogen sulphide, and this again into iodine by recrystallized potassic permanganate which was free from even a trace of chlorine. Since in the case of bromine it has already been shown that two distillations from a pure bromide are sufficient to free this element from chlorine,* three distillations of iodine from a solution of an iodide should insure complete elimination of chlorine and bromine even if the original substance were very impure. The purified iodine was again converted into hydriodic acid by hydrogen sulphide, and this into ammonium iodide by an excess of freshly distilled ammonia. This material was used in the analyses made with Sample I.

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* Œuvres Complètes, **3**, 125.

† Proc. Amer. Acad., **28**, 22; **29**, 64; **31**, 173; **39**, 249.

‡ Dammer, Handb. d. anorg. Chem., II, **1**, 432.

§ Richards and Singer, Amer. Chem. Jour., **27**, 205.

It was suggested by Professor T. W. Richards that an undiscovered element resembling the halogens, but of a higher atomic weight, might exist, and an attempt to detect the existence of such an element was carried out as follows: One pound of iodine was purified as described above, but the process of converting the iodine into hydriodic acid by hydrogen sulphide, and the hydriodic acid into iodine by potassic permanganate, was repeated four times. The iodine obtained in the last distillation with permanganate, amounting in all to about fifty grams, was set free in four fractions by introducing the permanganate in portions of about two grams each. An element belonging to the halogen family, and of higher atomic weight than iodine, should be more easily set free than iodine, hence the first of the four fractions should have contained nearly all of such an element which occurred in the pound of iodine. Each one of the fractions was once distilled with pure water, and was then converted into ammonium iodide, as in the case of Sample I. The first fraction was Fraction 1 of Sample II, the fourth was Fraction 4 of Sample II. No difference in appearance between these samples and Sample I or even the original crude iodine could be detected.

All reagents used in the course of the purification or in the analyses were carefully purified. Acids and ammonia were redistilled, usually with the use of a platinum condenser. The water was distilled twice, once from alkaline permanganate, once from a solution containing a trace of sulphuric acid. In both distillations block tin condensers were used.

Method of Analysis. — In order to convert a known weight of silver into silver iodide the following method was employed. The silver was dissolved in nitric acid in an Erlenmeyer flask, into the neck of which a column of glass bulbs was ground.† The acid consisted of the constant boiling distillate diluted with an equal volume of water. Solution was effected by warming so gently that almost no gas was evolved. When the silver was completely dissolved, the solution was diluted with an equal volume of water and was heated until all nitrous acid was destroyed and all oxides of nitrogen were expelled. It was then transferred to one of the precipitating flasks. These were Erlenmeyer flasks of one and two liters capacity, into the necks of which glass stoppers had been carefully ground.* Into the solution of silver nitrate, which had been diluted until not more than one gram of silver was contained in one hundred cubic centimeters, and to which an excess of ammonia had been added, a slight excess of the solution of pure ammonium iodide, which also had been diluted until it was not stronger than one per cent, was poured. The flask was stoppered and shaken for some time and was allowed to stand for about a day. Then a very slight excess of nitric acid was introduced, the flask was again shaken for some time, and was allowed to stand for one or two days until the solution above the precipitate seemed perfectly clear. The precipitate was washed by decantation with one per cent nitric acid at least eight times, and was transferred with water to a weighed Gooch crucible through which the washings had been poured. Finally the crucible was heated in an electric air-bath for several hours, first at 100–110°, then at 200° C.

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* Proc. Amer. Acad., **28**, 17; **31**, 165; **33**, 106; **39**, 249.

† Richards, Proc. Amer. Acad., **25**, 198.

Although silver iodide is very slightly sensitive to diffused daylight, the operations of precipitation and filtration were performed in a large cupboard lighted with orange light. When taken out of the dark room, the flask was enveloped in several thicknesses of black cloth.

The tendency of precipitated silver iodide to pass into a milky condition when washed with pure water is very marked. It has been found by Stas and others that if the water is previously heated to about 60° C, and plenty of time is allowed for the precipitate to settle at that temperature, this tendency can be overcome. When washed with warm water, however, silver iodide adheres to the flask to such an extent that it is impossible entirely to remove the precipitate even by vigorous rubbing. The conditions which were found most favorable for preserving the precipitate in such a state that it could be easily and rapidly manipulated were as follows. In the first place precipitation was carried out in ammoniacal solution, because the silver iodide clotted together much more rapidly than in acid solution, owing probably to the greater solubility of silver iodide in the alkaline solution. A large excess of nitric acid also coagulated the precipitate, but since the acid caused the separation of a large amount of iodine, it could not be employed in the present case. Dilute nitric acid, on the other hand, proved to be a satisfactory medium for washing the silver iodide, for it was found that the precipitate could be washed indefinitely with even one per cent nitric acid without passing into the colloidal state. The washings obtained in this way, when treated with an iodide, gave no trace of opalescence even on long standing, showing that silver iodide is insoluble in the dilute acid. When silver iodide which has been washed with nitric acid only is dried at a high temperature, a loss of iodine takes place, owing to action on the silver iodide by the nitric acid. This was evident from the dark color of the iodide. In order to avoid this difficulty, the precipitate was finally transferred to the Gooch crucible with pure water, sometimes after one rinsing with water. The nitric acid was so completely displaced by this means that the precipitate did not darken even on fusion. Unfortunately, even this slight washing with water in most cases caused some of the precipitate to pass through the crucible in a colloidal condition, consequently these last washings were collected separately. In the earlier analyses the silver iodide in this colloidal solution was determined by first boiling the solution with a small quantity of ammonium iodide until the silver iodide was coagulated, and then collecting the precipitate upon a small filter paper, together with the asbestos shreds and any silver iodide which were contained in the remainder of the filtrate and washings. As a rule this operation was performed only after the wash waters had stood for two or three days, in order that insoluble matter might have time to settle. The bottom of each flask was rubbed gently with a rubber-tipped stirring rod to detach adhering particles. Finally the filter paper was burned in a weighed porcelain crucible at as low a temperature as possible, for if a high temperature is employed, a loss of silver iodide by volatilization occurs. The ash was treated with a drop of nitric acid, and, after warming, a drop of ammonium iodide solution was added. The excess of acid and ammonium salt was driven off and the crucible was weighed.

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* Richards, Proc. Amer. Acad., **28**, 24.

In these earlier analyses it was found necessary to detach small particles of adhering silver iodide from the neck of the precipitating flask by rubbing with a rubber-tipped rod. Later a better method was devised for collecting this small amount of precipitate together with that contained in the colloidal washings. First the flask was rinsed with a small quantity of a solution of potassium cyanide, and this solution was poured into the colloidal washings. Then the solution was evaporated to small bulk and was electrolyzed in a weighed platinum crucible, which was heated to 130° in an electric oven and was finally reweighed. The film was dissolved in dilute nitric acid and the solution was precipitated with an excess of ammonium iodide. This precipitate of silver iodide, if over 0.0005 gram in weight, was collected on a small weighed Gooch crucible, and the filtrate, together with the original filtrate and wash waters, was passed through a small filter repeatedly until clear. If the precipitate weighed less than the above quantity, it was collected wholly upon a filter. These filters were of course ignited and treated as before. The weight of the silver iodide and asbestos was used in computing the results, the weight of electrolyzed silver serving merely as a check upon the weight of silver iodide.

In order to drive off the last traces of moisture from the silver iodide it was fused in a porcelain crucible. The bulk of the precipitate, freed as completely as possible from asbestos, was transferred to a clean crucible, which was then weighed with its cover. The crucible and cover were placed inside a large porcelain crucible, and were heated until fusion took place. A temperature much above the melting point of silver iodide was avoided, since this substance is distinctly volatile at higher temperatures. The loss in weight was then determined. The fused salt when cold was light yellow in color, with no trace of darkening, showing that no appreciable loss of iodine had taken place.

The density of pure silver iodide was found by displacement of water. Three determinations were made with material which had been fused in a porcelain crucible and cooled by pouring the fused mass upon a cold tile. The solidified salt was broken into small fragments and was introduced into a weighed pycnometer for solids, which was then reweighed. Water sufficient to cover the salt was introduced into the pycnometer and the system was placed in a vacuum desiccator which was kept exhausted until it seemed probable that all air had been extracted from the solid. Then the pycnometer was immersed in a bath at 25° C. and exactly filled with water. Finally the system was weighed. Two more determinations were carried out with material which had been cast into sticks by pouring the fused salt into a porcelain boat. These sticks were used in an elongated form of pycnometer for solids.* The average of the last two experiments is undoubtedly more accurate than that of the first three, since it was extremely difficult to pump out the air from the more porous material used in the first experiments.

The vacuum correction for silver iodide, when weighed with brass weights, calculated from this value for its specific gravity, is +0.000071 gram for every apparent gram of salt. A vacuum correction of −0.000031 gram was applied to every apparent gram of silver.

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* Baxter and Hines, Amer. Chem. Jour., **31**, 222.

Weight of Silver Iodide in Vacuum. grams.	Weight of Water displaced in Vacuum. grams.	Density of Silver Iodide. 25°/4°.
15.5871	2.7590	5.650
14.5141	2.5652	5.658
16.3685	2.8853	5.673
	Average	5.660
21.5876	3.8017	5.678
16.7274	2.9505	5.669
	Average	5.674 *

The platinum-plated brass weights were twice carefully standardized to hundredths of a milligram, and the two sets of corrections agreed to within two hundredths of a milligram in every case. Two short-armed Troemner balances, sensitive to a fiftieth of a milligram with a load of fifty grams, were used in the work. All weighings were made by substitution. In the case of the Gooch crucible a platinum tare, and in the case of the porcelain crucibles, porcelain tares were used.

In the following table are given the results of all the syntheses which were completed without accident. The atomic weight of silver is assumed to be 107.930 in the calculations.

The results tabulated in the preliminary series were undoubtedly less accurate than those of the final series, since it was in these analyses that experience in handling silver iodide was gained. Hence these analyses are collected by themselves. In the final series the rejection of Analysis 12 seems justifiable, since two other analyses made with the same material

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* The values for the density of silver iodide found by other experimenters are as follows:

Boullay, Ann. de Chim. et de Phys., (2) **43**, 266 (1830)	5.614
Karsten, Jour. für Chem. und Phys., **65**, 417 (1832)	5.026
Filhol, Ann. de Chim. et de Phys., (3) **21**, 417 (1847)	5.500
Schiff, Ann. der Chem. und Pharm., **108**, 21 (1858)	5.35
H. St. Clair Deville, Comptes Rendus, **64**, 325 (1867)	5.687 at 0°
Schroeder, Ann. der Chem. und Pharm., **192**, 295 (1878) . .	5.650 to 5.718

THE ATOMIC WEIGHT OF IODINE.

No. of Analysis.	Sample of Iodine.	Weight of Silver in Vacuum.	Weight of Silver Iodide in Vacuum.	Loss on Fusion.	Weight of Silver from Filtrate.	Wt. Silver Iodide and Asbestos fr. Filtrate.	Corrected Weight Silver Iodide in Vacuum.	Atomic Weight of Iodine.
Preliminary Series. Ag : AgI.								
		grams.	grams.	grams.	grams.	grams.	grams.	
1	I	5.23123	11.38079	0.00030	. . .	0.00482	11.38531	126.970
2	I	3.57039	7.77026	not fused.	. . .	0.00007	7.77033	126.961
3	I	4.60798	10.02751	0.00047	. . .	0.00100	10.02804	126.951
4	I	4.52467	9 84838	0.00036	. . .	0.00020	9.84822	126.986
5	I	4.66256	10.14605	0.00062	. . .	0.00048	10.14591	126.930
Average								126.960
Final Series. Ag : AgI.								
6	I	4.77244	10.38668	0.00020	0.00033	0.00050	10.38698	126.975
7	I	4.82882	10.50876	0.00035	0.00053	0.00140	10.50981	126.977
8	II,1	4.04262	8.79718	0.00058	. . .	0.00095	8.79755	126.947
9	II,1	1.64711	3.58503	0.00020	0.00015	0.00032	3.58515	126.994
10	II,2	4.86054	10.57440	0.00122	0.0025	lost.	10.57318	126 972
11	II,2	4.83482	10.52213	0.00044	0.00032	0.00072	10.52241	126.967
12	II,3	4.97120	10 81696	0.00025	0.00052	0.00129	10.81800	126.940
13	II,3	3.53858	7.70139	0.00038	0.00017	0.00035	7.70136	126.969
14	II,3	3.89693	8.48181	0.00026	0.00012	0.00032	8.48187	126.985
15	II,4	5.33031	11.59785	0.00034	0.00185	0.00360	11.60111	126.973
16	II,4	5.08748	11.07231	0.00035	0.00017	0.00063	11.07259	126.973
Average								126.970
Average, rejecting Analysis 12								126.973
Average of analyses with Sample I								126.976
Average of analyses with Sample II, Fraction 1 . .								126.971
“ “ “ “ 2 . .								126.970
“ “ “ “ 3 . .								126.977
“ “ “ “ 4 . .								126.973

yielded results in close agreement and considerably higher than that of Analysis 12.

From an examination of this table two important deductions may be drawn. First, there can be little doubt of the identity of the different samples of iodine; for the extreme averages of the results from the various samples differ by only seven thousandths of a unit. The variation in the values from the analysis of Sample II, Fraction 1, was undoubtedly due in the case of Analysis 8 to lack of experience in manipulating silver iodide, for this analysis was the first one to be completed in the final series, and in the case of Analysis 9 to the small quantity of available material. One may conclude without hesitation that, in the material examined in this investigation at least, no new halogen existed.

The second important deduction is that the atomic weight of iodine is evidently very slightly greater than 126.973; for most of the experimental errors, such as loss of silver iodide, or loss of iodine by the silver iodide, would have caused the result to be too low.

PART II.

The Ratio of Silver to Iodine.

The ratio of silver to iodine was next investigated. The problem of obtaining iodine in a *dry* state presented most difficulties in this portion of the research, and was finally solved as follows: Iodine which had been prepared in the same way as Sample I, was first freed from the greater part of the water which it contained by exposure to concentrated sulphuric acid, which had been boiled to remove every trace of hydrochloric acid. It was then sublimed from a crystallizing dish to the bottom of a glass dish half filled with water, which covered the crystallizing dish. In this way the greater part of the "included" moisture must have been eliminated. The sublimed crystals were finally again sublimed in a current of pure dry air from a porcelain boat contained in a hard glass tube into the weighing tube. This weighing tube was about ten centimeters long and fifteen millimeters in diameter, and was drawn down to less than half this diameter at both ends. Glass stoppers were ground into both ends of the tube. The air was purified and dried by passing over beads moistened with a solution of silver nitrate, then over sodic carbonate, and finally over three feet of beads moistened with concentrated sulphuric acid, all in an apparatus made entirely of glass and connected with the sublimation tube by means of a ground glass joint. During the final sublimation of the iodine, the end of the hard glass tube, which had been drawn down to small diameter, was inserted into one end of the weighing tube. After the weighing tube had been filled, the glass stoppers were inserted, the tube was carefully wiped with a slightly damp "chemically clean" cloth, and was allowed to stand in a desiccator for some time. It was then weighed. Needless to say the weighing tube had been originally treated as above before being weighed empty. Both weighings were made with a counterpoise exactly similar to the weighing tube.

In all but one of the experiments the weighing tube was broken during the solution of the iodine, so that it was impossible to weigh the tube after the experiment, and determine how much the glass was attacked by the warm iodine. In one experiment, however, a loss in weight of 0.00032 gram was found. Whether or not this change was accompanied by the evolution of a gas, the loss in weight must have been at least partially due to combination of the iodine with the alkaline metals of the glass, with the formation of soluble iodides which were dissolved by the solution of sulphurous acid. Hence the error must have been considerably less than three tenths of a milligram, if, as was necessarily the case owing to breakage of the tubes, the first weight of the tube was used in determining the weight of the iodine.

Immediately after being weighed, in order to avoid loss by volatilization, the iodine was converted into hydriodic acid by means of pure sulphurous acid. This acid was made by heating sulphuric acid with metallic copper and collecting the sulphurous oxide in water, and then distilling the sulphurous oxide from the solution into the purest water. During this distillation any trace of halogen acid which might have had its source in either copper or sulphuric acid must have been completely eliminated, for it must have been almost wholly in the ionized condition.

A considerable quantity of this sulphurous acid was poured into one of the precipitating flasks, and the weighing tube, containing the iodine, was introduced, after the lower stopper had been allowed to drop out of the tube into the flask. The sulphurous acid immediately sealed the open end of the tube so that no iodine vapor could escape. The other stopper was then removed by means of a platinum wire, the wire was rinsed into the flask, and the flask was quickly closed by means of its glass stopper. Solution of the iodine in the sulphurous acid was hastened by gently agitating the flask. Any iodine vapor which escaped from the tube must have been instantly converted into hydriotic acid. After sufficient time had been allowed for every trace of this hydriodic acid to be absorbed by the solution, the flask was opened and the solution was transferred to another precipitating flask, and a slight excess of the purest ammonia was added. When iodine is dissolved in a large excess of sulphurous acid the solution becomes colored yellow, owing to the formation of an iodide of sulphur,* and upon standing, this solution may deposit sulphur. But since the solution was made alkaline with ammonia as soon as the iodine was dissolved, with the complete disappearance of the color and without the formation of the slightest trace of a precipitate, no danger was to be feared from this source.

From the weight of the iodine, the weight of silver necessary exactly to combine with it was calculated. This silver was weighed out and dissolved in nitric acid as described previously in this paper. The solution was diluted until not stronger than one per cent and was added slowly to the already dilute solution of ammonic iodide in the precipitating flask. The flask was shaken for some time, and was then made acid with a considerable excess of nitric acid. Long-continued shaking, followed by several days' standing, yielded a clear supernatant solution. Twenty-five cubic centimeter portions of this solution were pipetted into nephelometer tubes and were tested with cubic centimeter portions of hundredth normal silver nitrate and hydriodic acid solutions in a nephelometer.† If an excess of either iodine or silver was present, the deficiency of the other was made up in the remaining solution by addition of standard silver nitrate or hydriodic acid until the exact end-point was reached. It was never necessary to add more than one tenth of a milligram of either iodine or silver, so that the liquid removed from the flask for the tests could be neglected. This end-point is very sharp in the case of silver and pure iodine; for so little silver iodide is dissolved that the two nephelometer tubes remain almost absolutely clear. An excess of a tenth of a milligram of silver in a liter of solution is easily detected. This almost complete lack of opalescence in the nephelometer tubes is strong evidence of the absence of even a trace of either chlorine or bromine in the iodine, for silver chloride or bromide, on account of their greater solubility, would have produced much more marked precipitates.

The specific gravity of solid iodine was assumed to be 4.933, ‡ hence a vacuum correction of +0.000102 was applied to every apparent gram of iodine.

As was to be expected, the average of this series is slightly higher than that of the previous one, and undoubtedly represents very closely

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* Dammer, Handb. d. anorg. Chem., **1**, 557.

† Richards and Wells, Amer. Chem. Jour., **31**, 235.

‡ Ladenburg, Ber. d. d. Chem., Gesell., **35**, 1256.

THE ATOMIC WEIGHT OF IODINE.

Ag : I.

Number of Analysis.	Weight of Silver in Vacuum	Weight of Iodine in Vacuum	Atomic Weight of Iodine.
	grams.	grams.	
17	5.54444	6.52288	126.977
18	6.27838	7.38647	126.979
19	4.57992	5.38814	126.976
	Average	. . . .	126.977

the true value of the atomic weight of iodine. Nevertheless, the investigation was not allowed to rest at this point.

PART III.

The Ratio of Silver Iodide to Silver Chloride.

In any atomic weight investigation it is extremely desirable to obtain the value sought by reference to as many different well-known atomic weights as possible. The method of heating silver iodide in a current of chlorine, which has already been used by Berzelius and Dumas as well as by Ladenburg, furnishes the ratio between iodine and chlorine, and seemed capable of yielding trustworthy results; for since the silver halides fuse at a comparatively low temperature, there is no possibility of inclusion of silver iodide by the silver chloride formed in the reaction, and hence there is certainty that the reaction will be complete.

First, silver iodide was prepared by precipitating an ammoniacal solution of the purest ammonium iodide (Sample I) with a solution of recrystallized silver nitrate. In this operation a slight excess of ammonium iodide was used. The precipitate was well washed with one per cent nitric acid, rinsed with water, and was collected on a Gooch crucible with the use of a disk of filter paper instead of an asbestos mat. In this way contamination of the precipitate with asbestos shreds was avoided. The silver iodide was finally dried in an air-bath at about 100° C for at least eight hours. After removal of the filter paper, those portions of the precipitate which had come in contact with the filter paper were cut away with a clean knife. Next the substance was fused in a weighed crucible protected from the flame by a very large crucible. While the silver iodide was fused, a small quantity of the purest iodine was placed upon the lower side of a second crucible cover, and this cover was substituted for the one which had been weighed with the crucible. The iodine immediately vaporized and the silver iodide was thus fused in an atmosphere containing iodine vapor. Finally, the cover was removed, so that the uncombined iodine escaped from the crucible, and the salt was kept fused, covered with the original crucible cover, until it was certain that all excess of iodine had been eliminated. That no excess of iodine was retained by the silver iodide was readily shown in one experiment with eighteen grams of salt by reheating the salt to its fusing point and reweighing. A loss in weight of only 0.00003 gram resulted.

After the silver iodide had been weighed it was heated in a current of chlorine. This gas was generated by dropping concentrated hydrochloric acid upon manganese dioxide, and it was purified and dried by bubbling through water and passing through a three-foot tube filled with beads moistened with concentrated sulphuric acid. Traces of bromine or iodine in the chlorine would have been no disadvantage, and it is inconceivable that it should have contained fluorine. In order to prevent spattering of the fused salt from the crucible, a perforated porcelain disk, which fitted the crucible half-way between the bottom and the top, was placed in the crucible. This disk was always weighed with the crucible. The chlorine was conducted into the crucible by means of a small hard glass tube, which passed just through the perforated cover of a Rose crucible. The apparatus for generating chlorine was constructed wholly of glass with the exception of the joint between this hard glass tube and the drying tube. The chlorine did not come in contact with the rubber tube used in making the connection tight, however, for the hard glass tube telescoped into the drying tube for some distance and the joint was sealed with concentrated sulphuric acid. In order to avoid volatilization of the silver salts, the heat applied was very gentle and only sufficient to fuse the silver chloride. Even at this temperature the iodine was rapidly replaced. Heating in chlorine was continued some time after the color of the iodine vapor had ceased to be visible. Then the Rose cover was replaced by the ordinary cover and the silver chloride was kept fused for several minutes, with occasional lifting of the cover, so as to drive off any chlorine which might have been dissolved by the fused salt. Although fused silver chloride, when cooled in chlorine, dissolves this gas very appreciably, no evidence was found that any remained in the solidified salt when it was heated for a short time in air. In one experiment eighteen grams of the chloride after the usual treatment were re-fused in air with a loss in weight of only 0.00002 gram. The same chloride, when fused and cooled in chlorine, gained five milligrams.

It was soon discovered that a porcelain crucible, when used for the conversion of silver iodide into silver chloride, gradually gained in weight. In the first experiment this gain was over three milligrams, and in several subsequent experiments with the same crucible, amounted to slightly less than a milligram in each case. This gain did not take place to the slightest extent when the crucible was heated alone in chlorine, nor after the reaction was complete, for the weight of the crucible and silver chloride very soon became constant. Probably it was due partially to solution of the silver salt in the glaze, as was shown by slight discoloration on the bottom of the crucible. Possibly, however, it was caused by the attacking of the glaze by the "nascent" chlorine and iodine set free in the process, perhaps with evolution of oxygen. At any rate too great an uncertainty existed as to the weight of the crucible at the end of the experiment. Accordingly quartz crucibles were next employed for the same purpose. These crucibles behaved in an ideal fashion, for they remained practically constant in weight through the process. It was only necessary to rotate the crucibles during the solidification of the fused salts in order that the salt might solidify in a thin layer on the sides of the crucible. Neglect to do this almost invariably resulted in the cracking of the crucible.

After the first weighing of the crucible, it was again heated in chlorine for an hour, and again cooled and weighed. In no case did a loss in weight of more than a tenth of a milligram take place, showing both that the iodine was completely replaced, and that no silver chloride had volatilized.

Four determinations (Analyses 20 to 23) were completed with quartz crucibles in the manner described. Two more determinations (Analyses 24 and 25) were made by heating silver iodide first in a current of carbon dioxide and bromine, and then in chlorine. The weight of the silver bromide was obtained, but although the results for the atomic weight of iodine calculated from the weight of the bromide agreed closely with those obtained by the other methods, on account of some uncertainty as to the purity of the bromine, these results are withheld until the experiments can be repeated. The only possible impurity in the bromine, chlorine, would not affect the final weight of the silver chloride.

At the time these experiments were performed, the atomic weight of chlorine was thought to be 35.456. The atomic weight of iodine calculated from the ratio between silver iodide and silver chlorine, using this value for chlorine, was very close to 126.96 in every case, which is considerably lower than the results obtained from the other two methods. At about this time, however, Richards and Wells, working upon the atomic weight of sodium in this laboratory, found that the atomic weight of chlorine is in reality 35.467.* In the following table the results are calculated upon this basis.

The specific gravity of silver chloride was assumed to be 5.62, as recently determined by Richards and Stull in this laboratory, and accordingly a vacuum correction of +0.000073 was applied to every apparent gram of salt.

THE ATOMIC WEIGHT OF IODINE.

AgI : AgCl.

Ag = 107.930 Cl = 35.467

Number of Analysis.	Weight of Silver Iodide in Vacuum.	Weight of Silver Chloride in Vacuum.	Atomic Weight of Iodine.
	grams.	grams.	
20	9.26860	5.65787	126.980
21	6.72061	4.10258	126.975
22	11.31825	6.90910	126.978
23	10.07029	6.14753	126.970
24	13.65457	8.33535	126.975
25	17.35528	10.59454	126.974
	Average		126.975
	From Ratio Ag : AgI		126.973
	From Ratio Ag : I		126.977
	Average from all three ratios . .		126 975

The close agreement of the average of the last series with the averages of the first two series leaves no doubt that the atomic weight of iodine is very close to the final average of all three series, 126.975. Incidentally this agreement is additional evidence in favor of Richards and Wells's new value for the atomic weight of chlorine, 35.467. Further work is under way upon the atomic weight of iodine, with the purpose of determining the ratio between iodine and silver iodide, and the ratio between all the halogens as described upon page 433 of this paper.

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* This investigation will soon be published.

In discussing the bearing of this investigation upon the results of earlier work by other chemists, the experiments of Millon upon silver and potassium iodates * and of Berzelius † and Dumas,‡ who converted silver iodide into silver chloride, may be disregarded, since at the time the analyses were made, quantitative analysis was in its infancy. Marignac's § value for the atomic weight of iodine, 126.85, obtained from titration of weighed amounts of silver with potassic iodide, and from syntheses of silver iodide from weighed quantities of silver, may be accounted for by the supposition that the iodine used in the experiments was not pure. To explain Stas's ‖ low value, 126.85, is a difficult matter. His iodine was purified by two different methods, i. e. by once precipitating or distilling the iodine from a strong solution of potassic iodide, and by precipitation of nitrogen iodide. A third sample was purified by both methods. Since the material purified by each of the two methods gave identical results with that purified by both methods, it is inconceivable that either method of purification should not have been effective. Impurity in the silver or loss of silver iodide are improbable causes of the discrepancy, for the weight of the silver iodide produced was equal to the sum of the weights of the silver and iodine employed. Richards and Wells have recently shown, however, that Stas was not infallible, and in fact was capable of making serious mistakes, such that his value for the atomic weight of sodium was two tenths of a per cent too high, and that for the atomic weight of chlorine three hundredths of a per cent too low, so that it is not at all surprising to find that here also his work was faulty in some undiscovered particular.

Ladenburg's result,¶ when calculated from the true value of the atomic weight of chlorine, becomes 126.978, which agrees very closely with the value deduced in this paper. His determinations were affected by several small errors, so that the close agreement is somewhat the result of chance. In the first place, porcelain crucibles, as has been pointed out before, gain in weight when used for the conversion of silver iodide into silver chloride, so that the weight of the silver chloride is somewhat uncertain. Fur-

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* Ann. de Chem. et de Phys. (3) **9**, 400 (1843).

† Ibid., (2) **40**, 430 (1829).

‡ Ann. Chem. Pharm., **113**, 28 (1860).

§ Berzelius' Lehrbuch, 5th Ed. **3**, 1196.

‖ Œuvres Complètes, **1**, 548.

¶ Ber. d. d. Chem. Gesell., **35**, 2275.

thermore Ladenburg did not fuse the silver iodide before weighing it. Although Stas states that silver iodide may be completely dried without fusion, his experiments show a loss in weight on fusion of two thousandths of a per cent, while the average loss on fusion as given on page 427 is about four thousandths of a per cent. Moreover, Ladenburg's method of purifying silver iodide, by washing the precipitated salt with ammonia, could hardly be expected to remove last traces of silver chloride and silver cyanide "included" by the precipitate. These errors are all so small that they would not affect the second place of decimals. However, Ladenburg's, Scott's,* and Köthner and Aeuer's work all afford confirmation that the atomic weight of iodine is undoubtedly much higher than has been usually supposed.

The results of the research are, then, as follows:

1. The atomic weight of iodine is found to be 126.975 (O = 16.000).

2. Richards and Wells's value for the atomic weight of chlorine, 35.467, is confirmed.

3. The existence of an element of the halogen family of higher atomic weight than iodine is shown to be improbable.

4. The specific gravity of pure fused silver iodide is found to be 5.674 at 25° referred to water at 4°.

I am indebted to the Cyrus M. Warren Fund for Research in Harvard University for platinum vessels, quartz crucibles, and balance, all indispensable in the research.

Chemical Laboratory of Harvard College,
Cambridge, Mass., August 6, 1904.

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* Proc. Chem. Soc., **18**, 112.

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SOME ADDITIONS TO THE ARC SPECTRA OF THE ALKALI METALS.

By F. A. Saunders.

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SOME ADDITIONS TO THE ARC SPECTRA OF THE ALKALI METALS.

BY F. A. SAUNDERS.

Presented by Charles R. Cross. Received September 8, 1904.

LENARD * discovered a new series and several other lines in the arc spectrum of sodium. He removed the slit of a spectroscope and in its place focussed a real image of the arc. With suitable dispersion he observed that different parts of the arc gave different "lines," and the new ones which he discovered were emitted by the hottest vapor, near the positive pole. Konen and Hagenbach † succeeded in photographing many of these lines and finding others in the lithium spectrum which are apparently emitted under similar circumstances. The writer, also, hoped to obtain photographs of such lines, using the usual slit, if his spectrograph were designed to give very bright spectra free from astigmatism, and if an image of the proper part of the arc were cast on the slit. The attempt was made with all the elements of the lithium family and the results, which were partially successful, are given below.

The essential feature of the apparatus used was a Rowland concave grating of about 10 cm. width and 305 cm. radius, ruled with lines of somewhat unusual length on a parabolic surface. The ruled surface had an area of 5 by 8 cm. The grating was mounted with the slit close to it on a solid iron casting bolted to a brick wall. An arm, supported from this casting and constructed of heavy gas-pipe, carried the camera at its end and could be turned about a point immediately below the grating. The table carrying the grating turned with this arm. The light from the slit fell on a parabolic mirror, similar to the one on which the grating was ruled, placed at such a distance that the reflected light formed a parallel beam. This then fell upon the grating and the spectra were formed about 150 cm. from the grating along a curve which was nearly a circle of 75 cm. radius. The incident and reflected beams at the mirror made an angle of 3 degrees with one another, while the angle between the axis of the grating and the beam incident on it

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* Ann. der Phys., **11**, 636 (1903). † Phys. Zeitschr., **4**, 592, 801 (1903).

varied from 12 to 21 degrees, depending on which order was being photographed. The axis of the grating was adjusted to meet the photographic film at its middle point. By rotation of the arm, this relation remained unchanged, but the focal distance changed with the angle. The camera was accordingly made adjustable along the normal to the grating, and by careful trial, three positions were obtained at which the camera could readily be set and clamped, so as to obtain spectra in good focus whose middle points lay near 3900, 5900, or 7800 tenth-meters. The whole mounting was enclosed by a wooden structure with doubly curtained door, which the observer could enter or leave during an exposure without fogging the film.

With this arrangement the spectra were almost entirely free from astigmatism for a space of 25 cm., corresponding to nearly 2700 t.-m. in the first order. Economy of light was insured by the short focus of the mounting and the large size both of grating and mirror.

The spectra were photographed on films 35 cm. long and usually 2.5 cm. wide. These were mounted in a holder which bound the film all along both edges and forced it to take such a curvature that the spectra were in focus along its whole length. This holder could be moved vertically by measurable amounts by a couple of large screws, so that several spectra could be photographed above one another on the same film. Each film took in over 3800 t.-m. at once, with good definition over almost the whole length, if the adjustments were correctly made. The scale of the photographs was nearly 11 t.-m. to the millimeter in the first order, and two sharp lines could be seen distinctly separated if they were 0.7 t.-m., or less, apart.

The films used were from the Seed Company and were coated with their 26× emulsion. A few Eastman Non-Curling films were also tried and found very satisfactory for wave-lengths less than 5900. For photographing lines in the red, the Seed films were stained with a simple cyanin bath. The writer is glad to have an opportunity of thanking Mr. R. James Wallace of the Yerkes Observatory for the formula, furnished by him, for this staining process, and recommends it to others interested, as a very simple and efficient way of sensitizing plates as far into the red as 8000 t.-m. Mr. Wallace's formula is as follows: —

Cyanin solution in Alcohol (1 : 500) . . .	5 cc.
Alcohol	30 cc.
Water	60 cc.
Ammonia	10 drops.

The plate should be bathed in this for two minutes and washed for one; it may be used at once, even before drying. Such plates do not keep for many days as a rule, though the writer has used some that were kept under very favorable conditions for a couple of months and found them still fairly good.

The source of light for the present work was usually the carbon arc, using 10 to 15 amperes, direct current. The writer is indebted to the International Acheson Graphite Co. of Niagara Falls, N. Y., for some unusually pure graphite rods with which all these spectra were taken. This graphite by itself gave only half a dozen lines (mostly Ca; no iron) outside of the band spectrum of carbon (which showed the "tails" beautifully), but when it was saturated with a salt solution, several lines of titanium came out, evidently from the graphite. These were not unwelcome, as they were always sharp, and, as their wave-lengths are given in Rowland's table of solar lines, they made excellent standards of measurement. The differences in wave-length between these lines in the sun and in the arc are too small to be worth considering in the present set of measurements. Eye observations showed that the alkali metal spectra were particularly well developed in the arc when the graphites were well saturated with salt solution and were separated by only 2 or 3 mm. With such a source the most successful photographs were taken; the well-known lines of the elements were then very much strengthened and broadened and the newer lines made their appearance. Fairly good photographs were, however, obtained with the arc longer, so that the "flames" were fully formed, if the light were taken from near the terminal. Exposures of three hours' duration were taken in the effort to pick up new lines in the deepest red. If the source of light could have been maintained in its most efficient condition during the whole time, the results might have been more complete, but the task of keeping the image of such lively flames as form these arcs constantly on the slit proved impossible.

A few photographs were also taken of the spark spectra of some of these elements, both with and without self-induction in the spark circuit, but no differences were detected in the relative intensities of any of the lines of the spectra of the spark or arc.

The photographs were usually taken with half the slit exposed directly to the light, and the other half covered with colored glass. In the resulting spectrum, lines in the first and second orders could easily be picked out where these overlap, as the ultra-violet lines were half the length of the others. In the deep red photographs, half the slit was open for a short time and then covered with red glass, the other half being so covered throughout. In this way, images of unknown red lines would form part of the *same* spectrum with known second order lines; no shift could occur to alter their relative positions, as the colored glasses were supported independently and could be changed without affecting the position of the slit in the least. In all cases the beam of light from the condensing (quartz) lens, passing through the slit, filled the mirror and grating completely. The condensing lens was rigidly fixed throughout. The writer, for these reasons, feels certain that the photographs obtained can be relied upon to show the true positions of the lines.

The measurement of the films was accomplished by means of a Gaertner micrometer microscope with a run of 5 cm. graduated to read to 0.005 mm. Its screw was investigated and found to possess no error large enough to be worth considering. A magnifying power of about 15 diameters was used. In measuring an unknown line, in every case measurements were taken on several standard lines lying on both sides of it, and its position was calculated from each of these; 10 settings were made on each line. As a rule, the wave-length of any line, as given, is the average of several such sets of measurements taken from different photographs.

In the following tables of the complete arc spectra of the alkali elements, the writer has given in the first column the series to which the line belongs (P for principal, I for first subordinate, &c.). In the second column are placed various values for the wave-lengths, and, opposite each, in the third and fourth columns, the error as estimated by each observer and the observer's initial letter. The following are the observers quoted: L, Lehmann [*]; Ld, Lenard; K & R, Kayser and Runge; K & H, Konen and Hagenbach; H, Hagenbach [†]; E & H, Exner and Haschek [‡]; L & D, Liveing and Dewar; B, Lecoq de Boisbaudran, and S for the writer. The custom of stating errors seems to vary with different observers. The writer believes that the errors of measurement proper are usually small compared with errors due to wrong interpretation of the photographic image. In his own experience, several settings on a diffuse line may have agreed with one another to less than 0.1 t.-m., while a different observer has made equally concordant measurements leading to a result 0.2 or more away. Where so many lines are broadened or diffuse, as in these spectra, the importance of this class of error will be easily seen. The writer's own estimates of error are not based on variations in his microscope readings; if they were, they would be half or a third as large. He has tried to fix the error at such a value that the chances are extremely small that the measured wave-length will differ from the true one by more than the amount given. These estimates have been formed by the help of test measurements taken on accurately known lines, following the same method as with unknown ones.

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[*] Ann. der. Phys **5**, 638 (1901).

[†] Ann. der Phys., **9**, 729 (1902).

[‡] Wellenlängen-Tabellen (1902).

It is usual to give an estimate of the intensity of each line along with its wave-length. This has not been done in the following tables, as such estimates have usually, especially for the greater wave-lengths, depended more on the sensitiveness of the photographic plate for each vibration than on the real intensity in the source of light. The lines of a series, of course, decrease in intensity with decrease in wave-length, those of the principal series more rapidly than the others. The first subordinate series is stronger than the second, and the new series lines are the faintest of all. Quantitative measurements of the real intensities of spectrum lines are much to be desired, but the writer does not know of any that are applicable to these spectra.

LITHIUM.

The writer was not aware until after this work was done that Konen and Hagenbach had already found the lines at 6240 and 4148, which form a new series (with 4636) in the lithium spectrum; he gives his values for the wave-lengths of these and a few other lines in the hope that they may be of value, especially as some of them differ from the values already given by considerable amounts. The line at 4148, such as it is, is visible in Figure 6, Plate 1, of Kayser's Handbuch, Vol. II, immediately to the right of 4132.44 (which by a misprint is numbered 4273 in the figure).

The "line" at 4602 deserves especial mention. Kayser[*] regards this as a line heavily reversed and much broadened toward the red. Hagenbach regards it as a pair of lines, the weaker one being constantly reversed, the heavier occasionally: the separation of the two amounts to over a tenth-meter. He is, apparently, ready to believe that all the lines in this spectrum are pairs with so great a separation. It is easy to obtain photographs of the strong line at 6708, and others, in which the lines are

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[*] Handbuch, 2, 366; also Plate II., Figure 5.

LITHIUM.

Series.	Wave-length.	Error.	Observer.	Series.	Wave-length.	Error.	Observer.
II	8127.34	0.27	L	I	4132.44	0.2	K & R
	8127.0	0.3	S	II	3985.94	0.2	"
P	6708.2	0.2	K & R	III	3924	. . .	K & H
III	6240.8	. . .	K & H		3921.8	. . .	K & H
	6240.3	0.4	S	I	3915.2	0.2	K & R
I	6103.77	0.03	K & R	II	3838.3	3.0	"
II	4972.11	0.1	"	I	3794.9	5.0	"
III	4636.14	. . .	H	I	3718.9	5.0	"
	4636.04	. . .	K & H	I	3670.6	5.0	"
	4636.3	0.4	S	P	3232.77	0.03	"
I	4602.37	0.1	K & R	P	2741.39	0.03	"
	4603.04	0.01	H	P	2562.60	0.03	"
	4602.00	. . .	H	P	2475.13	0.1	"
	4603.2	0.2	S	P	2425.55	0.1	"
	4601.6	0.2	S	P	2394.54	0.2	"
II	4273.44	0.2	K & R	P	2373.9	. . .	L & D
III	4149.1	. . .	K & H	P	2359.4	. . .	"
	4148.2	1.0	S				

sharp enough to show a doubling, if the components were even closer than a tenth-meter, and no such doubling has been observed. The curious reversals obtained by Hagenbach certainly call for an explanation, but it is difficult to adopt the one given by him in view of the absence of doubling in those lines where it would be most easily detected. The writer took a set of six photographs of the 4602 group on a film, using the second order spectrum (5.5 t.-m. to the mm.) and varying the exposures and amounts of vapor in the arc so as to furnish wide differences among the set. In these photographs the points of maximum density in the image on either side of the "reversal" remain in constant positions, even though the amount of vapor in the arc is small, in which case they are separated from each other by an absolutely clear space on the negative. The conclusion seems unavoidable from these images that we have here to deal with no reversal at all, but with two lines; a strong one at 4603.2, much broadened towards the red, and a weaker one at 4601.6, broadened toward the violet, neither of them being ordinarily reversed. If this view be adopted, the spectrum of lithium shows another analogy to that of sodium, for, in the latter spectrum, immediately beside the pair in the first subordinate series which is homologous to Li 4602 lies a faint pair broadened toward the violet in a similar manner.

The 4602 group presents the same aspect in the spark spectrum as in the arc.

SODIUM.

In the spectrum of sodium, the new series of Lenard has been successfully photographed and measured; a new term in the red has been added and a faint haze at 4372 was detected on one photograph which is doubtless the sixth member of the series. The pair at 4472 was so diffuse that the lines could not be seen separately; the setting was made on the middle of it. The writer's measurements differ considerably from those of Lehmann and of Konen and Hagenbach on several lines; a repetition of the measurements led to the same values. Konen and Hagenbach give a line at 4973 which does not appear with certainty on any of the present photographs; nor does there seem to be any companion to 4660. The pair at 7410 is exceedingly faint, and may possibly not belong to sodium. It has, however, approximately the same separation as the Lenard series pair near by, and, like that pair, the line of greater wave-length is slightly the stronger. If this is a member of another series, it would seem likely that the other lines are too faint to have been observed, unless, possibly, it could be grouped with the pair at 5670 and the lines at 4975 and 4660. An extremely faint group was observed at 8210, but it was impossible to determine just what it was; this was the greatest wave-length which was photographed on these films.

The lines 5100, 4820, and 4730 mentioned by Lenard could not be found on the photographs, nor were eye observations with a plane grating any more successful.

It may be worth noting that the wave-number differences for Lenard's series (III) are as follows: 14.72, 14.77, 18.2, and 18.7. As the first

SODIUM.

Series.	Wave-length.	Error.	Observer.	Series.	Wave-length.	Error.	Observer.
I	8194.76	0.2	L		4820		Ld
	8196.1	0.4	S	II	4752.19	0.15	K & R
I	8184.33	0.2	L	II	4748.36	0.15	"
	8184.5	0.4	S		4730	. . .	Ld
	7418.3	0.4	S	I	4669.4	0.5	K & R
	7410.0	0.4	S	I	4665.2	0.5	"
III	7377.4	0.4	S		4660	. . .	K & H
III	7369.4	0.4	S		4660.2	0 5	S
II	6161.15	0.1	K & R	III	4633.1	. . .	K & H
II	6154.62	0.1	"		4629.5	1.0	S
P	5896.16	. . .		III	4629.4	. . .	K & H
P	5890.19	. . .			4625.5	1.0	S
I	5688.26	0.15	"	II	4546.03	0.2	K & R
I	5682.90	0.15	"	II	4542.75	0.2	"
	5675.92	0.15	"	I	4500.0	1.0	"
	5670.40	0.15	"	I	4494.3	1.0	"
III	5531.7	. . .	K & H	III	4470	. . .	Ld
	5532 7	0.4	S		4472 5	2.0	S
III	5527.1	. . .	K & H	II	4423.7	. . .	L & D
	5528.2	0.4	S	II	4420.2	. . .	"
II	5153.72	0.1	K & R	I	4393.7	. . .	"
II	5149.19	0.1	"	I	4390.7	. . .	"
	5100	. . .	Ld	III	4372	5.0	S
I	4983.53	0.2	K & R	II	4343.7	. . .	L & D
I	4979.30	0.2	"	I	4325.7	. . .	"
	4976.1	. . .	K & H	P	3303.07	0 03	K & R
	4975.0	0.4	S	P	3302.47	0.03	"
	4973.0	. . .	K & H	P	2852.91	0.05	"
III	4913.5	. . .	K & H	P	2680 46	0.1	"
	4918.4	1.0	S	P	2593.98	0.1	"
III	4910.1	. . .	K & H	P	2543.85	0.1	"
	4914.0	1.0	S	P	2512.23	0.2	"

POTASSIUM.

Series.	Wave-length.	Error.	Observer.	Series.	Wave-length	Error.	Observer.
	7931.8	0.5	S	II	4863.8	. . .	L & D
P	7699 3	5.0	K & R		4864.5	0.8	S
	7697	. . .	R		4862	. . .	R
	7701.92	0.52	L	I	4856.8	. . .	L & D
	7699.08	0.3	S		4856.8	0.8	S
P	7665 6	5.0	K & R		4857	. . .	R
	7664	. . .	R	II	4850.8	. . .	L & D
	7668.54	0.52	L		4851.0	0.8	S
	7664.91	0.3	S		4829	. . .	R
I	6966.3	0.4	S		4808.8	. . .	L & D
II	6938.8	0.5	K & R	I	4803.8	. . .	"
	6939	. . .	R		4803	. . .	R
	6939.5	0.4	S	II	4801	. . .	R
II	6911.2	0.5	K & R		4796.8	. . .	L & D
	6913	. . .	R		4798	. . .	R
	6911 8	0.4	S		4788.8	. . .	L & D
I	5832 23	0.05	K & R		4767	. . .	R
I	5812.54	0.05	"	I	4759.8	. . .	L & D
II	5802.01	0.05	"		4760	. . .	R
II	5782.67	0.05	"		4642.35	0 3	R
I	5359.88	0.15	"		4642.5	0.3	S
I	5343.35	0.15	"		4638.6	. . .	R
II	5340 08	0.15	"	P	4047.36	0.03	K & R
II	5323.55	0.15	"	P	4044.29	0 03	"
I	5112.68	0.2	"	P	3447.49	0.03	"
II	5099.64	0.2	"	P	3446.49	0 03	"
I	5097.75	0.2	"	P	3217.76	0.03	"
II	5084.49	0.2	"	P	3217.27	0.03	"
I	4965.5	1.0	"	P	3102.37	0.1	"
II	4956 8	1.0	"	P	3102.15	0.1	"
I	4952.2	1.0	"	P	3034.94	0.1	"
II	4943.1	1 0	"	P	2992.33	0.15	"
I	4870.8	. . .	L & D	P	2963.36	0.2	"
	4871.3	0 8	S	P	2942.8	1.0	"
	4870	. . .	R				

two of these are probably accurate to one part in 80, while the two last are much less accurate, it seems fair to say that these differences are probably much smaller than the value for the usual series (17.2).

Many unsuccessful attempts have been made to find a simple formula which would express Lenard's series. The formulae of Kayser and Runge, Rydberg, Fowler and Shaw,* and Ritz,† and modifications of these, have been tried. Unless the formula contained four adjustable constants, it could not be made to fit the observations with any degree of precision. As almost any series, if not too accurately known, could be represented by a four-constant formula, no results of this work are given.

In the spectrum of potassium, two new lines were found, one in almost the position predicted for it.‡ It is one of the hitherto missing first pair of the first subordinate series, which is, for some obscure reason, very faint. (Ritz has given reasons for believing this to be the first rather than the second subordinate series, as Kayser and Runge classified it.) The other member of this pair was not found, owing to the broadening of the line at 6939, near which it doubtless lies. The other new line (7931.8) was so faint that its companion, if it is a member of a pair, could not be seen. The first term of the principal series was excellently photographed on several films and the writer feels considerable confidence in the value of the wave-lengths given for these lines. They are recorded to the second place of decimals, as the difference between the values could be determined more accurately than the values themselves. The best measurements were taken on photographs where the potassium was present as an impurity and the lines were fine and sharp, though measurements on heavily reversed images of these lines gave concordant results.

The line at 4642 seems to be outside the series formation, and, along with the faintness of the first subordinate pair, offers a very odd peculiarity in this spectrum.

RUBIDIUM.

Three new lines were found in the spectrum of rubidium. One is a line at 3158 (not seen as a pair) belonging to the principal series; another, a very diffuse and faint line at 5171, which, with 5234, forms a pair in the second subordinate series. The third is a line at 7759.5, which is a companion to the first subordinate series line 7757.9. The

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* Astrophys. J., **18**, 21 (1903).

† Ann. der Phys., **12**, 264 (1903).

‡ See Ritz, Ann. der Phys., **12**, 264 (1903).

RUBIDIUM.

Series.	Wave length	Error.	Observer.	Series.	Wave-length.	Error.	Observer.
	8513.26	0.26	L	I	5259.8	. . .	B
P	7950.46	0.32	L		5260.51	. . .	R
	7947.6	0.5	S		5260.5	0.4	S
P	7805.98	0.54	L	II	5234.6	. . .	R
	7799	. . .	R		5234.0	0.7	S
	7800.2	0.5	S	I	5194.8	. . .	B
satell.	7759.5	0.5	S		5195.76	. . .	R
I	7753.58	0.54	L		5195.9	0.5	S
	7757.9	0.5	S		5165.35	. . .	R
I	7626.66	0.32	L	II	5171	2.0	S
	7619.2	0.3	S		5161.8	. . .	B
II	7406.19	0.25	L	I	5151.20	. . .	R
	7408.5	0.4	S		5150.8	0.5	S
II	7277.01	0.25	L	II	5132	. . .	R
	7280.3	0.3	S		5133 5	0.8	S
	6306.8	. . .	R	I	5085.8	. . .	B
I	6298.7	0.2	K & R		5089.5	. . .	R
	6298.8	0.3	S		5088.6	0.6	S
I	6206.7	0.2	K & R	I	5076.3	. . .	R
	6206.7	0.3	S		5075.7	0.6	S
II	6159 8	0.2	K & R		5037	. . .	R
	6160.0	0.3	S	I	5021.8	. . .	B
II	6071.2	0.2	K & R		5023	. . .	R
	6071.1	0.3	S	I	5017	. . .	R
I	5724 41	0.15	K & R	I	4983	. . .	R
II	5654.22	0.15	"		4967	. . .	R
I	5648.18	0.15	"	P	4215.72	0.03	K & R
II	5579.3	. . .	R	P	4201.98	0.03	"
	5579.4	0.4	S	P	3591.74	0.05	"
I	5431.83	0.15	K & R	P	3587.23	0.05	"
	5431.9	0.4	S	P	3351.03	0.05	"
II	5391.3	. . .	R	P	3348.86	0.05	"
	5391.2	0.4	S	P	3229.26	. . .	R
I	5362.94	0.2	K & R	P	3228.17	. . .	R
	5363.1	0.4	S	P	3158.7	0.3	S
II	5322.83	. . .	R				
	5323 1	0.5	S				

CAESIUM.

Series.	Wave-length.	Error.	Observer.	Series.	Wave-length.	Error.	Observer.
I	9211.86	0.7	L	satell.	6217.6	. . .	R
IV ?	9171.88	0.7	L		6217.6	0.3	S
P	8949.92	0.76	L	I	6213.4	0.5	K & R
I	8766.10	0.32	L		6213.1	0.3	S
P	8527.72	0.32	L	II	6034.43	. . .	R
III	8080.02	0.48	L		6034.8	0.3	S
III	8019.62	0.48	L	I	6010.59	. . .	R
	8007.1	0.5	S	satell.	5847.86	. . .	R
II	7944.7	0.3	S	I	5845.31	. . .	R
II	7616.58	0.44	L	II	5839.33	. . .	R
	7609.7	0.3	S	II	5746.37	. . .	R
IV	7280.5	1.0	S	I	5664.14	. . .	R
IV	7227.46	0.44	L	I	5635.44	. . .	R
	7228.8	1.0	S	II	5574.4	. . .	R
satell.	6984	. . .	R	II	5568.9	. . .	R
	6983.8	0.3	S	I	5503.1	. . .	R
I	6973.9	5.0	K & R	I	5466.1	. . .	R
	6973.1	0.3	S	I	5414.4	. . .	R
III	6869	. . .	R	II	5407.5	. . .	R
	6872.6	1.0	S	I	5351	. . .	R
III	6829	. . .	R	I	5341.15	. . .	R
	6826.9	1.0	S	I	5304	. . .	R
I	6723.6	5.0	K & R	I	5256.96	. . .	R
	6723.7	0.2	S		5209	. . .	R
IV	6630	. . .	R	I	5199	. . .	R
	6630.5	1.0	S	I	5154	. . .	R
IV	6588.0	1.0	S	P	4593.34	0.05	K & R
II	6590	. . .	R	P	4555.44	0.05	"
	6587.3	0.3	S	P	3888.83	0.1	"
III	6472	. . .	R	P	3876.73	0.1	"
	6475	2.0	S	P	3617.08	0.3	"
III	6433	. . .	R	P	3611.84	0.2	"
	6434	2.0	S	P	3477.25	. . .	R
IV	6359	3.0	S	P	3398.40	. . .	R
II	6354	. . .	R	P	3348.72	. . .	R
	6355.3	0.3	S	P	3314	. . .	R
IV	6325	3.0	S	P	3287	. . .	R

results of the measurements in the deep red differ from those of Lehmann to a marked degree. As a check on the present methods, the writer took a set of measurements of several other and better known lines in this spectrum. The results, which are included in the table, agree reasonably well with those of Kayser and Runge. As a further check, the wave-number differences for the pairs were calculated, using the writer's values for the wave-lengths. The results were: —

P series.	Sub. series I.	Sub. series II.
237.8	234.7	237.7
	235.6	237.7
	235.8	237.1
	236.2	237.3
	236.3	232.8
	233.5	

The constancy of these numbers is satisfactory, with the exception of the short wave-length ends of the series, where the measurements do not pretend to be accurate. A similar table calculated from Lehmann's results shows a much wider divergence.

Ramage has noted a line at 6306; nothing was seen at this place except a ghost of the strong line at 6299. He has also given a sharp line at 5165; at this point on the writer's photographs appeared a sharp line which was the head of a carbon band.

Caesium.

In the caesium spectrum, five new lines were found. The one at 7944.7 was predicted by Ritz and is a member of the second subordinate series, along with 7609.7.

Two new series can be arranged from the odd lines in this spectrum, which might be called the third and fourth subordinate series, as they evidently belong to this class.

III.	IV.
8080	?
8019	9171 ?
6872.6	7280.5
6826.9	7228.8
6475	6630.5
6434	6588.0
	6359
	6325

The wave-number differences of these pairs are approximately constant; they are: 96, 97.5, 97.7, 98.5, 98.4, and 80. The last pair, being on the verge of invisibility, is very inaccurately measured. These differences are much less than for the usual series, being about 0.177 as much. The line of greater wave-length in the pairs has slightly greater intensity. All the lines in these series are so exceedingly diffuse that their positions cannot be measured with much accuracy; it did not seem worth while on this account to attempt to fit a formula to them. From an inspection of their positions, it seems likely that they are not so directly connected with the first and second subordinate series as in the case of Lenard's series in the sodium spectrum. They appear to run together to a common end, which lies somewhat nearer the red than the end of the other series. It seems most unlikely from their appearance that they can be due to any impurity, as in that case they might fairly be expected to be sharp when faint; they are, however, always diffuse. These series are faint, and of approximately equal intensity.

It does not appear to have been previously noticed that the heavier line of each pair in the first subordinate series is accompanied by a satellite (lines all observed by Ramage) on the red side, forming a "secondary" series, using Rydberg's term. The writer has calculated the wave-number differences of the pairs in the caesium spectrum, using Lehmann's data for the two extreme pairs and his own for most of the rest. The results are as follows:

P series.	Sub. series I.	Sub. series II.
553.2	552.0	554.1
	531.9	554.2
	542.3	553.7
	547.2	554.6
	549.7	

It will be noticed that the values for the first subordinate series are lower than the others, and have an evident drift, with the exception of the first, which is possibly in error. As the first member of each pair is accompanied by a satellite (none have yet been observed for the first and last lines), the wave-number differences were calculated between the satellite and the line of shorter wave-length. The result is as follows:

Wave number differences. Calculated from main line.	Calculated from satellite.
531.9	553.7
542.3	553.9
547.2	554.7

It will be seen at once that the differences calculated from the satellites agree beautifully with those of the second subordinate series. Apparently, then, the wave-number difference for the first subordinate series, as usually calculated, should vary slightly, increasing with decreasing wave-length. A glance at the table of differences for rubidium shows the same effect in that spectrum also, and there, too, if we use the satellite at 7759.5 (the only one yet observed) with the line 7619.2, we obtain a difference of 237.3 instead of 234.7, thus bringing this value into agreement with those for the second subordinate series. This principle may be used to calculate the positions of the other satellites in the rubidium spectrum. They should be at 9213.6 (using Lehmann's line 8766.10 for the calculation), 6299.6, and 5725.1, thus being within very short distances of their parent lines. As the latter are broadened toward the greater wave-lengths they cover up the satellites, so that these have not yet been observed. The satellites in the spectra of lithium, sodium, and potassium are also, probably, too close to their parent lines to be distinguishable.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 155.

THE EFFECT OF HEAT ON THE COLOR CHANGES IN THE SKIN OF ANOLIS CAROLINENSIS Cuv.

By G. H. Parker and S. A. Starratt.

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By G. H. Parker and S. A. Starratt.

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I. Introduction.

About a year ago Carlton (: **03**, p. 264) published the statements that, when a brown Anolis is put in the dark it invariably turns green in about twenty-five minutes, and when a green one is exposed to light it almost invariably turns brown in about four minutes. In attempting to ascertain what sense organs were concerned with these color changes, we repeated Carlton's experiments to determine independently the rate of change, and obtained results that were often far from uniform. Marked differences in the time required for the color changes on different days and at different times on the same day made the experiments very unsatisfactory. The change from brown to green was often slower at the beginning than after the experiments had been carried on for a few hours. This led us to suspect that one might apply to Anolis Pouchet's ('**76**, p. 73) conclusion concerning turbots: that exercise shortened the period for color changes. In fact this seemed to be confirmed by the following observations: On the morning of December 28 a set of animals was found that remained brown in the dark for three hours. Subjecting them alternately to the dark and to the light in a box provided with an electric lamp soon brought them to a state at which they would change from brown to green. This they did in an average time of 19.80 minutes, the last four records giving an average of 13 minutes. Thus it seemed that exercise played an important part in determining the rate of change.

Nevertheless, since all our experiments had been carried on with more or less change of temperature, it was necessary to check this before any sound conclusions could be drawn as to other factors. On December 30, with the temperature in the box varying from 22° C. just before the experiments were begun to 27° at the close of them, the average time of twenty records at various intermediate temperatures was 15.58 minutes, while at 27° it was 12 minutes. Therefore, notwithstanding the opinion of many investigators that temperature plays little or no part in the color changes of lizards and other lower vertebrates, we determined to test the influence of heat on Anolis, and the results obtained are given on the following pages.

II. Apparatus.

The apparatus used consisted of a wooden, upright, constant-temperature box, 70 cm. high, 35 cm. deep, and 40 cm. wide. The front of this box was in the form of a tight-fitting door, near the middle of which was a small glass window so placed that observations could be made from the outside without opening the box to the air. This window was provided with a light-proof shutter.

The interior of the box was lined with black cloth, and on the bottom was placed a cylindrical metal water-chamber 10 cm. high and 32 cm. in diameter. This held about 8 litres of water, which could be kept at constant temperature by a slow flow through an inlet and outlet connected with a water supply outside the box. On the inside of the back of the box, 45 cm. above the top of the water-chamber, was placed an electric incandescent lamp, which could be controlled from the outside by a key. About 10 cm. below this was inserted in the box a wooden cross partition carrying an oblong flat-bottomed glass dish. This contained distilled water 3 cm. deep, and served as a heat screen when the lamp was lighted. As the air of the small upper chamber which contained the lamp soon became heated when the lamp was lighted, a ventilating system was provided. This consisted of a lower inlet pipe and an upper outlet pipe, both of which had elbows in them to prevent the entrance of light. When the lamp was lighted a temperature anywhere between 10° and 50° C., and constant to within half a degree or so, could be easily maintained at the level of the top of the water-chamber. This was shown by a thermometer inserted through the wall of the box at this point.

It was found on trial that the nervous states of the lizards had much to do with the rate of the color changes, and to get uniform records it was necessary to keep them as nearly in one state in this respect as possible. Since the lizards chase one another when together, and thus introduce disturbing factors, each one was placed in a small oblong box painted black inside and resting directly on the top of the water-chamber; each box was covered by a plate of glass. When the light was turned on, the rays, after passing through the heat screen and the glass top of the small box, fell upon the lizard with an intensity of about 115 candle-metres. The top of the water-chamber was large enough to allow six lizards to be experimented upon at once, and that number was usually employed.

III. EXPERIMENTS.

From what has already been stated by Carlton (:**03**) it is evident that in testing the effect of temperature changes on Anolis it is necessary to exclude light, or at least to control it so that its influence will not enter into the final result. The constant-temperature box, already described, was planned with this in view.

Our first experiments were designed to determine the rate of change from brown to green at different temperatures. For this purpose six lizards were placed, one in each of the six small boxes on the water-chamber, and after the whole apparatus had come to the requisite temperature the rate of change was ascertained in the following way: The incandescent lamp was lighted and the lizards were kept under its influence until they were thoroughly brown. Then the light was turned out, and the length of time needed for the change to green was recorded for each individual. To ascertain when this had occurred the box was from time to time momentarily illuminated by the lamp and an observation was made through the small glass window. As a rule, four such determinations on each of six animals were made for each temperature, so that in the following table the average time for a given temperature is based on twenty-four records.

Table I shows the average rates at which these lizards changed from brown to green at temperatures ranging from 10° C. to 45° C.

At 10° the animals remained brown in the dark as well as in the light. Hence at this temperature heat was the controlling factor.

At 20° the animals slowly changed to green. Since at this temperature they remained brown in the light, it is evident that the change to green is not dependent upon heat, but upon the absence of light. That the heat, however, had some influence at this stage is seen from the fact that the change to green was very slow, the heat at 20° almost counteracting the change that takes place as the result of darkness.

At 25° the animals changed more quickly to green than at 20°, showing that at a higher temperature the heat had less retarding effect. The same was true, but still more noticeably, at 30°.

At 35° the heat checked the coming on of the green more than at 30° or even 25°, though the condition at 35° was followed by stages (40° and 45°) at which the animals were persistently greenish even in the light. Why retardation should occur at 35° is not clear, for one would expect a still more rapid rate than at 30°. However, subsequent tests of this invariably showed that the rate was persistently slower at 35° than at 30°, and hence we cannot conclude that this record was due to faulty observation. Possibly the increase of temperature which between 20° and 30° may have acted on the melanophores of the skin in one way, e. g., directly, may at 35° have influenced them in another way, e. g., indirectly through nerves. But to this question we have no definite answer.

TABLE I.

RATE OF CHANGE OF ANOLIS FROM BROWN TO GREEN AT DIFFERENT TEMPERATURES.

Temperatures in Degrees Centigrade.	Average Time in Minutes to change from Brown to Green.	
	In the Dark.	In the Light.
10	Remained brown.	
20	19.66	Remained brown
25	13.23	“ “ “
30	10.93	“ “ “
35	15.48	“ “ “
40	Greenish gray.	
45	Greenish gray to green.	

At 40° and 45° the animals became greenish and remained persistently so. As this happened not only in the dark but also in the light, where we might have expected the animals to have remained brown, it follows that at these temperatures heat was again a controlling stimulus preponderating over light.

Since at 40° and 45° the animals were persistently green, or at least greenish, it was impossible by the method we adopted for using the constant-temperature box to get records of rates comparable with those for the temperatures 20° to 35°. As a means of ascertaining roughly what the rate was at high temperatures four animals were put in a muslin-covered cage, and after they had become thoroughly brown in the light, the cage and they were quickly transferred to the dark, constant-temperature box at 54°. In six minutes two of the lizards were green and two yellowish-green. In a second trial with the box at 58° the change to green was accomplished in a little over four minutes. In both these trials the necessity of opening the box to the outer air and the conditions under which the lizards were exposed to the heat inside made it impossible to state exactly what the temperature was that induced the change. It must, however, be evident that at these higher temperatures the rate is more rapid than at those recorded in Table I, and this is what was to be expected.

TABLE II.

RATE OF CHANGE OF ANOLIS FROM GREEN TO BROWN AT DIFFERENT TEMPERATURES.

Temperatures in Degrees Centigrade.	Average Times in Minutes to change from Green to Brown.	
	In the Dark.	In the Light.
10	Brown.	
20	Remained green	4 23
25	" " "	3.52
30	" " "	3.13
35	" " "	2.80
40	Remained greenish gray.	
45	Remained greenish gray to green.	

The effects of heat on the change from green to brown has also been worked out, as in the case of the opposite change, and the results are given in Table II.

At 10° the animals became brown, and, as in the former records at this temperature, they remained so, irrespective of illumination. Their rate of change at this temperature could not therefore easily be determined. Green animals, however, when introduced into the dark, constant-temperature box changed to brown in about 18 minutes, but this record, which was obtained only after opening the box and handling the animals, can be regarded as only approximate. Its slowness is doubtless due to the circumstance that it represents a conflict between two opposing conditions: dark, inducing the green state, and low temperature, inducing the brown one; for when green lizards were introduced into the lighted box at 10° they changed to brown in 2.55 minutes.

From 20° to 35° the green animals when in the dark remained green, a condition determined not by temperature but by the absence of light, for when they were exposed to light at these temperatures they changed to brown at ever-increasing rates from 4.23 seconds at 20° to 2.80 seconds at 35°. Although heat between 20° and 35° is no more the controlling factor in the change from green to brown than it is in the reverse change (Table I), its influence is seen in the increased rate of the change, characteristic of the range from 20° to 35° under uniform illumination.

At 40° and 45° green animals, as might have been expected, remained green or greenish gray, irrespective of the illumination.

From these two sets of records it is evident that at a low temperature (10°) the lizards change from green to brown but not the reverse, and remain brown irrespective of illumination (115 candle-metres). At high temperatures (40° and 45°) they turn from brown to green but not the reverse, and remain green or greenish irrespective of illumination (115 candle-metres). Between these two extremes, at both of which heat is the controlling factor, there is a range (20° to 35°) through which light is the controlling factor, though heat here is not without its influence. This is indicated by the differences of rate in the changes from brown to green (Table I) and green to brown (Table II), accompanying the differences of temperature.

Thus it is well established that the rate of the color changes in Anolis is always more or less influenced by temperature, and the discrepancies between our early results and those of Carlton undoubtedly depended chiefly upon this factor, for Carlton's experiments were carried on in a room that throughout the winter retained a rather high uniform temperature (18° to 20°), whereas ours were begun in one whose temperature was subject to much variation.

Before turning to a general discussion of our results comment should be made on the bearing of these conclusions on certain conditions observed in Anolis under natural surroundings. Judging from their reactions to light as recorded by Carlton (:**03**), we should expect these animals in their native haunts to be brown in the daytime and green at night. We have never had the opportunity of studying Anolis in the field, but through the kindness of Dr. Caswell Grave we have been informed that in the neighborhood of Beaufort, N. C., on a hot, sunny day about as many green animals may be taken in bright sunshine as brown ones. This is in all probability a mixed reaction, in that some of the animals are more sensitive to light than to heat and hence turn brown in bright sunlight, and others are more sensitive to heat than to light and hence turn green under like conditions.

IV. Discussion of Results.

The brown state in Anolis, which corresponds to the dark gray or black state of many fishes, amphibians, and other reptiles, is dependent upon the accumulation of pigment in the fine peripheral branches of the melanophore cells of the derma. The green state, on the other hand, is possible only after this pigment has withdrawn into the body of the melanophore cell. We may therefore conclude that a low temperature, such as 10°, induces a migration of the pigment toward the periphery and a high one (40° or 45°) a migration toward the centre.

Thus far very few records of the migration of the pigment of chromatophores in relation to temperature have been published, but such as have appeared support the conclusions just stated, and suggest for them a much broader application. Thus it seems clear that in the African chameleon the melanophores draw in their pigment and the skin becomes light when the animal is subjected to heat, for Brücke, though he ('**51**, p. 804) declared that dark heat rays had no effect in changing the color of the skin, stated ('**52**, p. 26) that when the animal was put in an atmosphere that warmed its skin to 30° it became lighter colored. Moreover this observation was confirmed by Keller ('**95**, p. 129), who observed that when a chameleon was subjected to heat at 30° to 38° it took on a lighter tint than usual and retained this even under strong illumination, where it would be expected to become dark.

Biedermann ('**92**, p. 487), who experimented on the chromatophores of frogs and tree-toads, fully confirmed the earlier observations of Hering ('**69**, p. 53) to the effect that in cold water the pigment of the melanophores passed out into the cell processes, producing a dark skin, and in warm water it passed back into the body of the cell, leaving the skin light. Similar results have been more recently obtained on larval salamanders by Fischel ('**96**), and these have been confirmed by Flemming ('**97**^{a}, **97**^{b}).

Thus all recorded cases support the generalization that a high temperature induces a migration of the pigment into the body of the cell, and a low one the reverse. Wiedersheim (see Hoffmann, '**90**, p. 1353), therefore, was not without reason in suspecting that the dark skin of Phrynosoma on cool, cloudy days, and its pale skin on sunny days might be due to temperature differences, and Keller's ('**95**, p. 132) criticism of this explanation is at least premature. It must, however, be admitted that, as Thilenius ('**97**, p. 536) has pointed out, light is usually more effective than heat as a chromatophore stimulus, though to deny heat any part in the determination of color changes is, as our results show, certainly erroneous.

The relations of heat to light in these respects are not without interest. Thus while heat seems in all cases to follow the simple rule that high temperature induces a central migration and low temperature a peripheral migration of the melanophore pigment, light sometimes follows this rule and at other times does not. For in Veranus, Uromastix, and Agama (Thilenius, '**97**), a strong light acts like a high temperature and inducing central migration, and a dim light or no light acts like a low temperature and inducing peripheral migration. On the other hand, in the African chameleon (Brücke, '**52**; Keller, '**95**, etc.), and in Anolis (Carlton, :**03**) a strong light acts like a low temperature, inducing a peripheral migration, and a dim light or no light acts like a high temperature, inducing a central migration. In other words, in the first group of animals mentioned light and heat act together; in the second, they are antagonistic. Whether there are animals which react to heat in the reverse way, — high temperature causing in them a peripheral migration of pigment, — as some do to light, remains still to be made out, but from our present knowledge it seems probable that this is not true, and furthermore that animals are much more differentiated toward light than toward heat, even though from a physical standpoint these two stimuli have much in common.

V. SUMMARY.

1. At 10° C. Anolis carolinensis becomes brown and remains so, irrespective of illumination (115 candle-metres). At this temperature heat is the controlling factor.

2. At 40° and 45° C. Anolis becomes green or greenish-gray, and remains so irrespective of illumination (115 candle-metres). Here also heat is the controlling factor.

3. At intermediate temperatures, 20° to 35°, light (115 candle-metres) and dark are controlling factors, but the effect of heat is still evident over this range in that it may influence the rate of the color changes.

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A PROBLEM IN STATICS AND ITS RELATION TO CERTAIN ALGEBRAIC INVARIANTS.

By Maxime Bôcher.

A PROBLEM IN STATICS AND ITS RELATION TO CERTAIN ALGEBRAIC INVARIANTS.

By Maxime Bôcher.

Presented May 11, 1904. Received November 16, 1904.

It is the object of the present paper to show how a certain problem in the equilibrium of particles, when treated by the use of complex quantities, can, by the introduction of homogeneous variables, be brought into connection with the theory of algebraic invariants. A very special case of this problem was considered by Gauss and F. Lucas, while another case, much more general in some respects, was made use of by Stieltjes in his discussion of polynomial solutions of certain homogeneous linear differential equations of the second order. Nowhere, however, so far as I know, has the mechanical interpretation been carried so far as I carry it here, nor has the connection with the subject of algebraic invariants been pointed out. The following pages are intended to be suggestive rather than exhaustive, only a few of the simpler applications of the method being taken up.

§ 1. The Field of Force.

Given a number of fixed particles $P_1, P_2, \ldots P_n$ in a plane, with masses $m_1, m_2, \ldots m_n$; let us suppose each of them repels with a force equal to its mass divided by the distance. We do not exclude the possibility of some of the particles having negative masses, in which case these particles will attract instead of repelling.

To obtain the field of force in the plane due to these centres of force, let us take the plane as the plane of complex numbers, and determine the positions of the particles $P_1, \ldots P_n$ in the ordinary way by means of the complex numbers $e_1, \ldots e_n$. Then, if any point P in the plane is determined by the complex number x, the force at P, due to the centres of force at $P_1, \ldots P_n$, is given both in magnitude and in direction by the complex quantity:

$$(1) \qquad K\left(\frac{m_1}{x - e_1} + \frac{m_2}{x - e_2} + \ldots + \frac{m_n}{x - e_n}\right).$$

where the symbol K is used to indicate that we are to take the conjugate of the complex quantity which follows it.*

This problem may be extended in a useful manner by the following considerations.

Let us take first the special case of two particles whose masses are the negatives of each other. Let the positions of the particles be determined by the complex quantities e_1 and e_2, and let their masses be respectively m and $-m$. Then the force at any point x in the plane is by (1):

$$m\,K\frac{e_1 - e_2}{(x - e_1)\,(x - e_2)}.$$

By considering first the angle and then the absolute value of this complex quantity we obtain at once the two results: †

I. The lines of force of two particles P_1 and P_2 whose masses are the negatives of each other are the circles through P_1 and P_2, the force at any point being directed away from the particle of positive and towards that of negative mass.

II. The intensity of the force at a point P is given by the formula:

$$\frac{m\cdot\overline{P_1\,P_2}}{\overline{P_1\,P}\cdot\overline{P_2\,P}}.$$

The first of these statements shows that if through P_1 and P_2 we pass any spherical surface, the force is tangential to the spherical surface at all its points. The same will, therefore, be true if on any spherical surface we have any number of pairs of particles P_1, P_2; P_1', P_2'; . . ., provided that the masses of the two particles of every pair are the negatives of each other. Now it is evident that on a spherical surface any system of particles $P_1, P_2, \ldots P_n$ *the sum of whose masses is zero* is equivalent to such a system of pairs of particles. For let the masses of $P_1, P_2, \ldots P_n$ be $m_1, m_2, \ldots m_n$ respectively $(m_1 + m_2 + \ldots + m_n = 0)$, and let us at an arbitrarily chosen point P_0 of the spherical surface place n coincident

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* If, in particular, $m_1 = m_2 = \ldots = m_n = 1$, and if we let

$$f(x) = (x - e_1)(x - e_2)\ \ldots\ (x - e_n),$$

the field of force becomes $K\left(\frac{f'(x)}{f(x)}\right)$,

and the points of no force are given by the roots of $f'(x) = 0$. This theorem was stated by Gauss (Werke, 3, 112) and afterwards rediscovered by F. Lucas (C. R., 1868).

† These results are very well known, and can easily be proved by other methods.

particles of masses $-m_1, -m_2, \dots -m_n$. Since the sum of the masses of these new particles is zero, they do not affect the field of force; and now we have our pairs of particles P_1, P_0, with masses $m_1, -m_1$; P_2, P_0, with masses $m_2, -m_2$, etc. We have thus proved the theorem:

*If a number of particles lie on a spherical surface, and if the sum of their masses is zero, the force produced by them at any point on the spherical surface is tangential to that surface.**

It is this spherical field of force we now wish to study. For this purpose let P_0 be any point on the spherical surface, and project the sphere stereographically from P_0 onto the diametral plane perpendicular to the diameter through P_0. Call the projections of $P_1, \dots P_n$ on the plane $p_1, \dots p_n$, and let us consider the plane field of force due to particles of masses $m_1, \dots m_n$ situated at these points.† We will prove the following two theorems:

I. *The direction of the force at any point* P *of the spherical field is the stereographic projection of the direction of the force at the corresponding point* p *of the plane field.*

Thus the lines of force on the sphere are the projections of the lines of force in the plane.

II. *The intensity of the force at any point* P *of the spherical field is equal to the intensity at the corresponding point* p *of the plane field multiplied by* $\frac{1}{2}(1 + \mathrm{r}^2)$, *where* r *is the distance from the centre of the sphere to* p, *and the radius of the sphere is taken as the unit of length.*

We will first prove these two theorems for the special case in which there are only two particles situated at P_i and P_0 with masses m_i and $-m_i$ respectively. Since the circle $P_i P P_0$ is a line of force on the sphere, the force acting away from P_i or towards it according as m_i is positive or negative, and since the projection of this circle is the straight line $p_i p$, and this is a line of force in the plane, theorem I is obviously true in this case.

Since the intensity of the force at p is

$$f = \frac{|m_i|}{p_i p},$$

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* The same reasoning shows that if a number of particles lie on a circle, and if the sum of their masses is zero, the force produced by them at any point on the circle is tangential to the circle. It also admits of immediate application to spherical multiplicities in space of n dimensions.

† If one of the points P_i lies at P_0, the corresponding point p_i will be at infinity, and the particle in question in the plane is simply to be omitted.

and, by a formula obtained above, the intensity of the force at P is

$$F = \frac{|\, m_i \,|\, \overline{P_0 P_i}}{\overline{P_0 P} \cdot \overline{P_i P}},$$

we see that

$$F = \frac{\overline{p_i p} \cdot \overline{P_0 P_i}}{\overline{P_0 P} \cdot \overline{P_i P}} \cdot f.$$

Now it is well known that the stereographic projection of the sphere on the plane is equivalent to an inversion with regard to a sphere having P_0 as centre and radius $\sqrt{2}$. Accordingly:

$$P_0 P = \frac{2}{P_0 p} = \frac{2}{\sqrt{1 + r^2}}.$$

Moreover, by a familiar property of inversion, the triangles $P_0 P_i P$ and $P_0 p p_i$ are similar, so that:

$$\frac{p_i p}{P_i P} = \frac{P_0 p}{P_0 P_i}.$$

Substituting these values we find

$$F = \frac{1 + r^2}{2} f,$$

as was to be proved.

Having thus established theorems I and II in the special case of two particles, one of which lies at P_0, the proof in the general case follows by replacing the n particles $P_1, \ldots P_n$ as we did above, by the *n pairs* of particles P_1, P_0; P_2, P_0; $\ldots$ P_n, P_0. We thus get n forces acting at P and n corresponding forces acting at p. Moreover each force at P acts in the direction which is the stereographic projection of the direction of the corresponding force at p; and accordingly, since angles are not changed by stereographic projection, the figure formed by the *directions* of the forces at P is congruent with the figure formed by the directions of the forces at p. Moreover every force at P is $\frac{1}{2}(1 + r^2)$ times the corresponding forces at p. Accordingly the forces at P are represented by n vectors, which form a figure similar (in the geometrical sense of the word) to that formed by the n vectors which represent the forces at p. If we now complete these figures by constructing the resultant in each case, the figures will clearly remain similar. Accordingly the directions of these resultants, since they make equal angles with corresponding components, are the stereographic projections of each other; and the intensities of the resultants are in the same ratio as the intensities of the components. Thus theorems I and II are proved.

It has long been a familiar fact that one and the same plane vector field gives the force due to n particles $p_1, p_2, \ldots p_n$ in a plane, of masses $m_1, m_2, \ldots m_n$; and the steady flow in a plane conducting lamina due to electrical sources at $p_1, \ldots p_n$ of intensities $m_1, \ldots m_n$. On the other hand it is well known that the vector field on a spherical surface which represents the steady flow of electricity on the surface due to n sources $P_1, \ldots P_n$ with intensities $m_1, \ldots m_n$ $(m_1 + \ldots + m_n = 0)$ may be obtained by projecting stereographically onto a diametral plane and considering the flow due to sources at $p_1, \ldots p_n$ (the projections of $P_1, \ldots P_n$) of intensities $m_1, \ldots m_n$. If the length of each vector in this last mentioned plane field be multiplied by $\frac{1}{2}(1 + r^2)$, — the radius of the sphere being taken as unit of length, — and the field thus modified be projected back onto the sphere, we obtain the desired flow on the sphere. Comparing these facts with theorems I and II we see that *one and the same vector field on the spherical surface represents the flow in the electrical problem just mentioned and the force in the mechanical problem considered before.*

§ 2. Relation to Algebraic Invariants.

Let us now, in the discussion of the field of force, introduce homogeneous variables by the formulae:

$$x = \frac{x_1}{x_2}, \qquad e_i = \frac{e_i'}{e_i''}.$$

The plane field (1) then becomes:

$$K\left[x_2 \sum_1^n \frac{m_i e_i''}{e_i'' x_1 - e_i' x_2}\right].$$

In this formula we may, without real loss of generality, assume that $m_1 + m_2 + \ldots + m_n = 0$. For if this were not the case we could regard our formula as the special case of the corresponding formula where n is replaced by $n + 1$, $e_{n+1}'' = 0$, and m_{n+1} is so chosen that $m_1 + \ldots + m_{n+1} = 0$. This expression for the field of force, when reduced to a common denominator, becomes, when we take account of the relation $m_1 + m_2 + \ldots + m_n = 0$ which we now assume to hold,

$$(2) \qquad K\left[x_2^{\,2}\,\frac{\phi\,(x_1,\,x_2)}{(e_1''\,x_1 - e_1'\,x_2)\;.\;.\;.\;(e_n''\,x_1 - e_n'\,x_2)}\right],$$

where ϕ is a binary form of degree $n - 2$. We will now prove that ϕ is a covariant of weight 1 of the system of linear forms which stand in the denominator of (2).

For this purpose let us notice that in the case $n = 2$, ϕ reduces at once to $m_1\,(e_1'\,e_2'' - e_2'\,e_1'')$ which is an invariant of weight 1. If then we regard the particle e_n of mass m_n as consisting of $n - 1$ coincident particles of masses $- m_1, - m_2, \ . \ . \ . - m_{n-1}$, we are led to write (2) in the form

$$K\left[x_2^{\,2}\sum_{i=1}^{i=n-1}\frac{m_i\,(e_i'\,e_n'' - e_n'\,e_i'')}{(e_i''\,x_1 - e_i'\,x_2)\,(e_n''\,x_1 - e_n'\,x_2)}\right].$$

Each term under the sign of summation being a covariant of weight 1, the same is true of their aggregate, which is merely the fraction in (2) of which ϕ is the numerator. The denominator of this fraction being a covariant of weight zero, it follows that its numerator ϕ is a covariant of weight 1, as was to be proved.

The form ϕ obviously vanishes at every point in the field where the intensity of the force is zero. Besides these points of equilibrium ϕ cannot vanish anywhere except perhaps at the points e_i where the denominator of (2) vanishes. If a particle e_i whose mass is different from zero does not coincide with any of the other particles, ϕ will not vanish there; for if it did, formula (2) would yield a force whose intensity remains finite as we approach e_i. If, however, k of the particles whose total mass is not zero coincide at the point e_i, the form ϕ must contain the linear factor $e_i''\,x_1 - e_i'\,x_2$ exactly $k - 1$ times in order that (2) should yield a force which becomes infinite to the first order at e_i. Such a point, in spite of the vanishing of ϕ, is, of course, not a point of equilibrium. We shall, however, speak of it as a point of *pseudo-equilibrium*, in justification of which term we may notice that if we let k distinct particles whose total mass is not zero coincide at e_i, $k - 1$ points of true equilibrium fall together at this point.

It remains then only to consider a point at which k of the particles whose total mass is zero coincide. At such a point we have a force of finite intensity and, accordingly, ϕ must have at least a k-fold root there. If the multiplicity is exactly k, we do not have equilibrium, but such a point we shall again call a point of pseudo-equilibrium. If the multiplicity of the root of ϕ is greater than k we have true equilibrium.

We have so far regarded ϕ as an integral rational covariant of weight 1 of the system of linear forms

$$(3) \qquad e_i'' x_1 - e_i' x_2 \qquad (i = 1, 2, \ldots n)$$

which involves n real parameters $m_1, \ldots m_n$ connected by the relation $m_1 + m_2 + \ldots + m_n = 0$.*

Instead, however, of determining the positions of the particles by the vanishing of the linear forms (3), as we have practically been doing, we may, if we prefer, build up forms of higher degree $f_1, f_2, \ldots f_k$ by multiplying the linear forms (3) together in groups, and then regard the positions of the particles as determined by the vanishing of these forms f. The form ϕ will still be a covariant of the system of forms f, but it will in general be an *irrational* covariant of this system. If, however, the particles corrresponding to the form f_i all have the same mass, ϕ is rational and symmetric in the roots of f_i and therefore rational in the coefficients of f_i. Thus we obtain the result:

Given a system of binary forms $\mathrm{f}_1, \mathrm{f}_2, \ldots \mathrm{f}_k$ *of degrees* $\mathrm{p}_1, \mathrm{p}_2, \ldots \mathrm{p}_k$ *respectively, and* k *real quantities* $\mathrm{m}_1, \mathrm{m}_2, \ldots \mathrm{m}_k$ *subject to the condition* $\mathrm{p}_1 \mathrm{m}_1 + \ldots + \mathrm{p}_k \mathrm{m}_k = 0$; *and suppose that at each of the* p_i *points determined in the complex plane or on the complex sphere by the equation* $\mathrm{f}_i = 0$ *are placed equal particles each of mass* m_i *which repel with a force which varies directly as the mass and inversely as the distance; then the positions of equilibrium in this plane or spherical field of force are determined as the roots of a certain integral rational covariant* ϕ *of the system of forms* f.

Besides vanishing at the points of equilibrium, ϕ *vanishes only at the points of pseudo-equilibrium described above. Such points can occur only at the multiple roots of a form* f_i *or at a common root of two of these forms.*

The simplest case is when $k = 2$. Here $p_1 m_1 + p_2 m_2 = 0$, and therefore, since only the ratio of the masses m_1 and m_2 is important, we may without real loss of generality assume that $m_1 = p_2$, $m_2 = -p_1$. If we let:

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* It would be possible to consider the more general case in which the quantities m_i are complex, the force having then not merely an attractive or repulsive component, but also a component at right angles to this. Thus if the quantities m_i are pure imaginaries, we have, in the plane, the electromagnetic field due to the steady flow of electric currents through long straight wires which cut the plane at right angles.

$$f_1 = (e_1'' x_1 - e_1' x_2) \ . \ . \ . \ (e_{p_1}'' x_1 - e_{p_1}\ x_2),$$

$$f_2 = (e_{p_1+1}'' x_1 - e_{p_1+1}' x_2) \ . \ . \ . \ (e_{p_1+p_2}'' x_1 - e_{p_1+p_2}' x_2),$$

the field of force is

$$K\left[x_2\left(p_2 \sum_{i=1}^{i=p_1} \frac{e_i''}{e_i'' x_1 - e_i' x_2} - p_1 \sum_{i=p_1+1}^{i=p_1+p_2} \frac{e_i''}{e_i'' x_1 - e_i' x_2}\right)\right] = K\left[x_2\left(p_2 \frac{\frac{\partial f_1}{\partial x_1}}{f_1} - p_1 \frac{\frac{\partial f_2}{\partial x_1}}{f_2}\right)\right].$$

Accordingly

$$\phi = \frac{1}{x_2}\left(p_2 f_2 \frac{\partial f_1}{\partial x_1} - p_1 f_1 \frac{\partial f_2}{\partial x_1}\right),$$

which reduces, when we apply Euler's theorem for homogeneous functions, to the Jacobian of f_1 and f_2. Hence

The vanishing of the Jacobian of two binary forms f_1 *and* f_2 *of degrees* p_1 *and* p_2 *respectively determines the points of equilibrium in the field of force due to* p_1 *particles of mass* p_2 *situated at the roots of* f_1, *and* p_2 *particles of mass* $-p_1$ *situated at the roots of* f_2.*

It is easy now to express the covariant ϕ which we have in the general case integrally and rationally in terms of the ground-forms and their Jacobians. For this purpose let us write as the field of force:

$$K\left[x_2 \sum_{i=1}^{i=k} m_i \frac{\frac{\partial f_i}{\partial x_1}}{f_i}\right] = K\left[\frac{x_2}{p_k} \sum_{i=1}^{i=k-1} m_i \left(p_k \frac{\frac{\partial f_i}{\partial x_1}}{f_i} - p_i \frac{\frac{\partial f_k}{\partial x_1}}{f_k}\right)\right]$$

$$= K\left[\frac{x_2^{\,2}}{p_k} \sum_{i=1}^{i=k-1} \frac{m_i J_{ik}}{f_i f_k}\right],$$

where J_{ik} denotes the Jacobian of f_i and f_k. Accordingly:

$$\phi = \sum_{i=1}^{i=k-1} \left(\frac{m_i}{p_k} J_{ik} \cdot f_1 \ . \ . \ . \ f_{i-1} f_{i+1} \ . \ . \ . \ f_{k-1}\right),$$

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* If one of the two ground-forms is linear, the theorem may be stated thus:

If ϕ is the first polar of the point (y_1, y_2) with regard to a binary form f of degree p, the vanishing of ϕ determines the points of equilibrium in the field of force due to p unit particles situated at the roots of f, and one particle of mass $-p$ at, (y_1, y_2).

The special case of this where $y_2 = 0$ leads us back to Gauss's theorem referred to near the beginning of § 1.

an expression from which the lack of symmetry could be removed by well known and obvious methods.*

The mechanical significance we have thus attached to the vanishing of certain covariants makes it sometimes possible by direct mechanical intuition to obtain information concerning the position of the roots of these covariants. The following theorem serves as a starting point:

A point (in the plane or on the sphere) cannot be a position of true equilibrium if it is possible to draw a circle through it upon which not all the particles lie, and which completely separates the attractive particles which do not lie on it from the repulsive particles which do not lie on it.

This is at once obvious when we consider the spherical field of force, for there will clearly be in this case a component at the point in question perpendicular to the circle.

This principle enables us in many cases, when we know the positions of the roots of the ground-forms, to find regions in the plane in which no root of the covariant ϕ lies.

Let us look at the case of two ground-forms f_1 and f_2; and let us suppose that there are two regions S_1 and S_2 on the sphere, bounded by circles C_1 and C_2 respectively, which do not overlap, and such that all the roots of f_1 lie within or on the boundary of S_1 and all the roots of f_2 lie within or on the boundary of S_2; provided, however, that if C_1 and C_2 have a point or points in common, not all the roots of both f_1 and f_2 shall lie at such common points. In this case the principle just stated shows that the Jacobian of f_1 and f_2 can have no root lying in the region between the circles C_1 and C_2. In other words all the roots of this Jacobian must lie within or on the boundary of S_1 and S_2.

We can in general go further by constructing in S_1 and S_2 curvilinear polygons T_1 and T_2 bounded by arcs of circles such that each side of T_i passes through at least two roots of f_i and, when extended, through at least one root of the other ground-form, and is so situated that the circle of which it forms a part separates from one another all the roots of f_1

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* We thus get as the field of force:

$$K\left[\frac{x_2^2}{k}\sum_{i=1}^{i=k}\sum_{j=1}^{j=k}\frac{m_i J_{ij}}{p_i f_i f_j}\right],$$

and the covariant becomes:

$$\phi=\frac{1}{k}\sum_{i=1}^{i=k}\sum_{j=1}^{j=k}\frac{m_i}{p_i}\frac{f_1\,\cdot\,\cdot\,\cdot f_k}{f_i f_j}J_{ij},$$

where we must remember that $J_{ii}=0$, $J_{ij}=-J_{ji}$.

and f_2 which do not lie upon it. Some of the roots of f_i will form the vertices of T_i, while the others lie within it or on its boundary. It follows by the principle we have already made use of, that the roots of the Jacobian must lie in the polygons T_1 and T_2.

By a slight additional consideration we can even determine how the roots of the Jacobian are distributed between the two regions. For this purpose let us allow the roots of f_1 and f_2 to change, always remaining in S_1 and S_2 respectively, in such a way that the roots of f_i all approach a point a_i within S_i. If this change is a continuous one, the roots of the Jacobian will also change continuously on the complex sphere, and therefore such of these roots as originally lay in S_i will remain there. We may, however, during this process allow the regions S_1 and S_2 to shrink down towards the points a_1 and a_2 respectively, while they always include the roots of f_1 and f_2. Accordingly all the roots of the Jacobian which originally lay in S_i must be approaching a_i as their limit. But we have seen above that when k particles whose total mass is not zero fall together at a point, $k - 1$ positions of true equilibrium coalesce into a position of pseudo-equilibrium. The Jacobian must therefore have had just $p_1 - 1$ roots in S_1 and $p_2 - 1$ roots in S_2. Hence the theorem:

If the roots of a binary form f_1 *of degree* p_1 *lie within or on the boundary of a region* T_1, *and if the roots of a second binary form* f_2 *of degree* p_2 *lie within or on the boundary of a second region* T_2 *which does not overlap or touch the first, and if these two regions are bounded by arcs of circles each one of which circles separates the roots of* f_1 *which do not lie on it from the roots of* f_2 *which do not lie on it; then the Jacobian of* f_1 *and* f_2 *has just* $\mathrm{p}_1 - 1$ *roots in* T_1 *and* $\mathrm{p}_2 - 1$ *roots in* T_2.*

The method of proof here used can be immediately extended to the general case of k ground-forms $f_1, \ldots f_k$. If all the roots of such of these forms as correspond to the positive constants of the set $m_1, m_2, \ldots m_k$ lie in T_1 and all the roots of the other forms lie in T_2, we see in this way that all the roots of the covariant ϕ lie in T_1 or T_2, and that ϕ has in T_i one less root than the ground-forms have there.

The principle we have used so far is not the only one which helps us to locate the roots of ϕ. Let us, for simplicity, consider again merely the case of two ground-forms f_1 and f_2, and let us suppose that these forms are real, so that the roots of each, so far as they are not real, will be conjugate imaginary in pairs. The force at any point on the axis of reals will here be in the direction of this axis, and we have the theorem:

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* The special case in which one of the ground-forms reduces to x_2 gives the following theorem, which is an immediate consequence of Gauss's theorem quoted above, and was first explicitly stated by F. Lucas, Journal de l'École Polytechnique, Cahier 46 (1879), p. 8.

The roots of the derivative of any polynomial in x lie in any convex rectilinear polygon in the complex plane which includes within itself or on its perimeter all the roots of the original polynomial.

If ϕ *is the Jacobian of two real binary forms* f_1 *and* f_2, *then in an interval of the axis of reals bounded by roots of one of these ground-forms and containing no root of either form there lies at least one root of* ϕ.

The interval in question may, it should be noticed, be infinite, extending through the point at infinity if this is not a root of f_1 or f_2, otherwise extending up to this point. Immediate consequences of the last theorem are these:

If all the roots of f_1 *and* f_2 *are real, their Jacobian has a number of real roots at least as great as the difference between the degrees of* f_1 *and* f_2.

If all the roots of f_1 *and* f_2 *are real and distinct, and if all the roots of one of these forms lie in one of the intervals into which the roots of the other form divide the axis of reals, then all the roots of the Jacobian of* f_1 *and* f_2 *are real and distinct, and just one of these roots lies in each of the intervals into which the roots of* f_1 *and* f_2 *divide the axis of reals, except that the two intervals which are bounded at one end by a root of* f_1, *at the other by a root of* f_2, *contain no root of the Jacobian.*

These theorems also can easily be extended to the case of more than two ground-forms.

The proofs of the theorems which we have here deduced from mechanical intuition can readily be thrown, without essentially modifying their character, into purely algebraic form. The mechanical problem must nevertheless be regarded as valuable, for it suggests not only the theorems but also the method of proof.

§ 3. Stieltjes's Generalization.

Up to this point we have been considering the problem of determining the positions in which a single particle free to move in a certain field of force can rest in equilibrium. This problem may be generalized in a fruitful manner by considering with Stieltjes * a number of particles of equal mass which are all free to move in the plane or on the sphere, and are acted upon not merely by the field of force due to the fixed particles, but also by one another, the action here again being repulsion of intensity equal to the product of the masses divided by the distances. We may without loss of generality take the mass of each of the moving particles as unity, in which case the total mass of the fixed and moving particles must evidently be +1 in order that the force acting on each of the moving particles should be tangential to the sphere.* The problem of determining the positions of equilibrium of the system of moving particles is readily seen, by reference to the results already obtained, to have the same invariant character as in the special case already considered. Neither Stieltjes's original treatment of the problem nor my own earlier methods brought out this fact in the analytic work, the positions of equilibrium being obtained as the roots of the polynomial solutions of certain homogeneous linear differential equations of the second order, — equations whose invariant character was in no way evident. I hope to take up this whole subject before long from the point of view here indicated. I content myself here with pointing out, by a simple example, how Stieltjes's problem can be brought into connection with the elementary theory of algebraic invariants.

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* Acta Math., **6** (1885), 323, where, however, only the case in which all fixed and movable particles lie on the axis of reals is considered. The case in which all the particles are free to lie anywhere in the complex plane was taken up for the first time in the book of the present writer entitled, Ueber die Reihenentwickelungen der Potentialtheorie, Leipzig, 1894, p. 215. Cf. also Bull. Amer Math. Soc., March, 1898, p. 256.

If f *is a binary cubic with distinct roots, and if three fixed particles, each of mass* $\frac{1}{3}$ (1 − k) *are situated at the roots of* f, *then the position of equilibrium, if any exists, of* k *movable particles of mass* +1 *is given by the vanishing of a certain covariant* ϕ *of* f, *which in the simplest cases* $\mathrm{k} \leqq 7$ *is:*

$k = 2$ $\quad \phi = H$,
$k = 3$ $\quad \phi = J$,
$k = 4$ there is no position of equilibrium,
$k = 5$ $\quad \phi = H \cdot J$,
$k = 6$ $\quad \phi = \Delta f^2 - 7\, H^3$,
$k = 7$ there is no position of equilibrium.

Here Δ and H denote the discriminant and Hessian of f, and J the Jacobian of f and H, where, however, in order that the formula for $k = 6$ be correct, we must suppose that, f being written:

$$f = a_0\, x_1^3 + 3\, a_1\, x_1^2\, x_2 + 3\, a_2\, x_1\, x_2^2 + a_3\, x_2^3,$$

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* This restriction is not necessary when, as has always been the case heretofore, the *plane* problem only is considered, and no attempt is made to bring out its invariant character.

numerical factors are included in Δ and H so as to make them have integral coefficients without common factors.

This problem of Stieltjes admits of generalization in still another direction by considering not the field of force due to a number of fixed particles which repel according to the law of the inverse distance, but the field of force given in the plane by the formula

$$K[f(x)],$$

where $f(x)$ is any analytic function of the complex variable x.* We should here seek the positions of equilibrium of a group of k unit particles which repel one another with a force equal to the reciprocal of the distance. Denoting the complex quantities which determine the positions of these movable particles by $x_1, x_2, \ldots x_k$, and letting

$$\phi(x) = (x - x_1)(x - x_2) \ldots (x - x_k),$$

the equations of equilibrium are

$$\frac{1}{2}\frac{\phi''(x_i)}{\phi'(x_i)} + f(x_i) = 0. \qquad (i = 1, 2, \ldots k).$$

Let us consider here in more detail the special case in which

$$f(x) = -cx,$$

where c is a positive constant. The force here may be described as attraction towards the axis of imaginaries and repulsion from the axis of reals, the intensity of the force being in both cases directly proportional to the distance from the axis in question, and the factor of proportionality being the same in the two cases. It is clear that the particles cannot be in equilibrium unless they all lie on the axis of reals, and it is equally clear that there is at least one position of equilibrium of this latter sort. In order to find it write the equations of equilibrium in the form

$$\phi''(x_i) - 2cx_i\phi'(x_i) = 0, \qquad (i = 1, 2, \ldots k).$$

These equations show that the polynomials $\phi(x)$ and $\phi''(x) - 2cx\phi'(x)$, which are of the same degree, have the same roots (obviously distinct, since no two particles could coincide in a position of equilibrium), so that they differ only by a constant factor. Accordingly the polynomial ϕ, whose roots determine a position of equilibrium, satisfies a differential equation of the form:

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* My colleague, Professor B. O. Peirce, calls my attention to the fact that this amounts to considering the field of force which has the real part of $\int f(x)\,dx$ as a force function. We are thus dealing with the most general field, which has a force function with continuous first and second partial derivatives and satisfies Laplace's equation.

$$\frac{d^2 y}{d x^2} - 2 c x \frac{dy}{dx} = C y.$$

Substituting for y the polynomial ϕ, we find at once by comparing the coefficients of x^k on both sides that $C = -2ck$. Hence:

The positions of equilibrium of a group of k *unit particles repelling one another inversely as the distance, and situated in the field of force* K (— c x), *are the roots of the polynomials of the* k*th degree which satisfy the differential equation*

$$\frac{d^2y}{dx^2} - 2 c x \frac{dy}{dx} + 2 c k y = 0. \tag{4}$$

The mechanical problem shows that this equation must have at least one polynomial solution of the kth degree, a fact which may be readily verified by substituting in (4) a polynomial with undetermined coefficients. This substitution shows that there is only one polynomial solution.

The polynomials thus obtained are known as Hermite's Polynomials, having been first discussed by this mathematician * from the following point of view. Consider the function e^{-cx^2}. If we differentiate this function k times with regard to x we obtain the original function multiplied by a certain polynomial in x. Among the properties of this function which Hermite develops is the fact that it satisfies the differential equation (4). This shows its identity with our polynomial ϕ. The fact that all the roots of ϕ are real and distinct, which followed from our mechanical problem, follows also at once from Hermite's definition

If we make the transformation

$$z = e^{-\frac{cx^2}{2}} \cdot y,$$

the differential equation (4) takes the form

$$\frac{d^2z}{dx^2} = [c^2 x^2 - c(2k + 1)] z, \tag{5}$$

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* Comptes Rendus, 1864, pp. 93 and 266.

and this is the equation for the functions of the parabolic cylinder in the form most commonly used. That in the special case we have just been considering (the case in which k is a positive integer) this equation has a solution in the form of the product of an exponential function and a polynomial was noticed by K. Baer.* I am not aware, however, that the connection between this fact and Hermite's work on the one hand and Stieltjes's method on the other has ever been pointed out.

The field of force $K(-cx)$ which we have just considered may be regarded as the limit of the field due to two particles of mass m situated at the points e and $-e$ and two particles of mass $-m$ situated at ie and $-ie$, each particle repelling directly as the mass and inversely as the distance, as e and m both become infinite. That is, the field may be regarded as due to a *quadruplet* at infinity. A simpler case, from this point of view, would be that of a *doublet* at infinity which would produce a uniform field, let us say $K\left(-\frac{c}{2}\right)$. Here, however, there is clearly no position of equilibrium. In order to make equilibrium possible, let us introduce into this uniform field a single fixed particle of mass m, repelling inversely as the distance, and situated, say, at the origin. This problem can be carried through precisely as was the one above, and leads us, not to equation (5), but to:

$$4x\frac{d^2z}{dx^2} + 8m\frac{dz}{dx} = [c^2x - 4c(m+k)]z, \tag{6}$$

and we see again that if k is a positive integer, (6) has a solution which is the product of an exponential factor and a polynomial, the roots of this polynomial being the positions of equilibrium in the problem last considered.

It is interesting to note that equation (6), in the special case $m = \frac{1}{4}$, reduces to the form in which the equation for the functions of the parabolic cylinder presents itself when the subject is approached from a broader point of view than is ordinarily done.†

We may, of course, consider doublets, triplets, etc., which do not lie at infinity, and such centres of force may come in in any number alongside of the simple repulsive particles. Moreover the invariant character of these problems can be brought out both by using the sphere instead of the plane, and by using homogeneous variables and throwing the analytical work into invariant form. I hope, as I have already said, to come back in detail to all these questions.

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* Programm, Küstrin, 1883, p. 9.

† Cf. the book of the present writer already referred to. By introducing x^2 in place of x as the independent variable in (5), and $2k$ in place of k, we can pass at once to the special case of equation (6) in which $m = \frac{1}{4}$.

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THE MANDRAGORA OF THE ANCIENTS IN FOLK-LORE AND MEDICINE.

By Charles Brewster Randolph.

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Although references to mandragora[*] are by no means uncommon in later as well as in classical literature, and although the commentators on many of the Greek and Latin authors have made more or less copious notes concerning it, yet with the exception of Schmidel's researches in the latter part of the seventeenth century, no special study of the subject appears to have been undertaken until within the last few years. The articles which have recently appeared in various scientific journals have directed their attention mainly to its significance in folk-lore.[†] The plant figures but slightly in this connection until in comparatively late times. The many wonderful tales about the middle-age *mandragorae* (German *alraune*) had little to do with the real plant, which was practically unknown north of the Alps.

The chief purpose of this paper is to treat the mandragora as it was known to the Greeks and Romans. The rôle which it played in their folk-lore was certainly a subordinate one; they regarded it in general as a medicinal rather than a magic plant. I wish therefore to examine its uses in Greek and Roman medicine, and particularly to state the facts known in regard to its use as an anaesthetic before surgical operations. But inasmuch as the history of its use even in antiquity is involved to a certain extent in the superstitions which afterwards developed so rich a field for students of folk-lore, it may be of interest to give some account of these superstitions as far as they were known to the Greeks and Romans. Although it is not unlikely that the original home of this very considerable body of superstitions was the Orient, it is, I think, beyond controversy that in the folk-lore of the Greeks and Romans (though generally not yet in connection with this particular plant) we may find the intermediate stage, if not the origin, of many of the superstitions about it which were known to Europe in the middle ages. I have therefore begun with a review of the mandragora superstition as known to the Greeks and Romans. Where it has been an easy step I have passed over the bounds of antiquity into the middle ages, hoping to add something here and there to what has already been written about its later history.*

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[*] The Greek μανδραγόρας, *mandragoras* in classical Latin, *mandragora* in mediaeval Latin and subsequently, is often rendered by the English word *mandrake*. I have used the word *mandragora* as noun and adjective throughout this paper particularly to avoid confusion with the American mandrake (May-apple), which is an entirely different plant.

[†] The first effort to collect the scattered notes on the mandragora was made by Schmidel in his dissertation of nineteen pages. The most important recent articles on the subject are by Cohn (see next footnote, p. 488), pp. 285–293; von Luschan, pp. 726–728; Ascherson, pp. 729–738, 892; Beyer, pp. 738–746; Wetzstein, p. 890 f.; Veth, pp. 81–88, 199–205.

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* In the body of this paper I have given English translations of passages quoted from writers of other languages. The full text of the Greek and Latin passages, with the appropriate references, will be found in the Appendix. In the footnotes — which will be interesting, if to any one, chiefly to students of the classics — I have regularly quoted passages in the original language.

Where I have used simply the name in citing the following writers, the reference is to the work here set down:

Ascherson (Paul): Verhandlungen der Berliner Gesellschaft für Anthropologie, Ethnologie und Urgeschichte, 1891. Berlin.

Beyer (R.): Ibid.

Cohn (Ferdinand): Jahresbericht der Schlesischen Gesellschaft für Vaterländische Cultur, **65**, 1887. Breslau.

Dioscorides, Diosc.: Cited by volume and page of Sprengel's edition in Kühn's Medici Graeci. Lipsiae, 1829.

Galen: Cited by volume and page of Kühn's edition in the Medici Graeci. Lipsiae, 1821–1833.

Häser (Heinrich): Lehrbuch der Geschichte der Medicin. Jena, 1875.

Hippocrates, Hipp.: Cited same as Galen; ed. Lipsiae, 1825–1827.

Koebert (Hermann): De Pseudo-Apulei Herbarum Medicaminibus. Baruthi, 1888.

Von Luschan, v. Luschan (Felix): Same as Ascherson.

Meyer (E. H. F.): Geschichte der Botanik. Königsberg, 1854.

Neuburger-Pagel, Neub.-Pag.: Handbuch der Geschichte der Medicin. Jena, 1902–.

Pliny, Plin.: Naturalis Historia.

Schmidel (Johann): Dissertatio de Mandragora. Lipsiae, 1671. This is apparently the work cited by Beyer, p. 744, note 3, under the name of *Thomasius.*

Theophrastus, Theophr.: Historia Plantarum.

Veth (P. J.): Internationales Archiv für Ethnographie, **7**, 1894. Leyden, etc.

Wetzstein (J. G.): Same as Ascherson.

Mention of the plant is more common in Greek than in Latin literature. Outside the medical and botanical writers the almost universal nature of these allusions is to its narcotic powers, which were proverbial. Its earliest uses seem to have been purely medicinal, but observation of its quieting effect when properly administered, together with the fact that when given in too large quantities it sometimes produced serious mental disturbance, seems to have led to the belief that it possessed peculiar power over the mind. Perhaps as a result of this belief, a body of superstitions began to grow up around it; some of these may have been connected with it from their beginning, but many of them were evidently transferred to it later from other plants which shared some of its properties.

A prominent feature of the mandragora superstition at all times has been the story of the peculiar ceremonies gone through with before digging the plant. The use of such preliminaries was apparently very common with the ancient root-diggers (*ῥιζοτόμοι*) and they employed them in the case of various plants. Naturally a weird story of perils incurred in obtaining a plant strengthened belief in its magic powers and added to its commercial value.

Theophrastus, in his History of Plants,* gives an interesting account of this practice of the root-diggers, from which it appears that some of these preliminaries were practical, such as the anointing of exposed parts of the body, or taking care to stand with the wind at one's back when about to dig a plant from which pungent odors arose; others, such as the digging of certain plants only by night, avoiding the sight of certain birds, and the like, are to be regarded as wholly unnecessary and simply the arbitrary inventions of the root-diggers.

It is in this passage of Theophrastus that we first find a digging ceremony mentioned in connection with the mandragora:

> Theophr., 9. 8. 8: Around the mandragora one must make three circles with a sword, and dig looking toward the west. Another person must dance about in a circle and pronounce a great many aphrodisiac formulas.

Pliny gives about the same account:

In presenting this paper I wish to express my thanks to Professor M. H. Morgan, of Harvard University, who suggested the subject to me, and who has aided me with his advice at many stages of its preparation.

* 9. 8. 4–8.

Plin., 25. 148: Those who are about to dig mandragora avoid a wind blowing in their faces; first they make three circles with a sword, and then dig looking toward the west.*

Comparing these two passages, we observe that Theophrastus is silent as to the precaution regarding the wind, while Pliny says nothing about dancing around and repeating aphrodisiac formulas; otherwise the accounts agree. Pliny's account probably goes back ultimately to Theophrastus. We know that he used Theophrastus much in the botanical portions of his work; he cites him at the beginning of the twenty-fifth book, in which this passage appears, and in § 69 of the same book he names him as his authority for a digging story about *centauris*, which goes back to Theophrastus's History of Plants, 9. 8. 7, the very chapter in which the story about digging mandragora is given. However, it is pretty well established that Pliny did not use Theophrastus at first hand, but cited his views and his name from intermediate authors.† Information ultimately derived from Theophrastus appears constantly in the Natural History in a somewhat altered form, sometimes briefer, sometimes expanded.‡ The precautions about the wind appear to have been added to the story between the time of Theophrastus and Pliny. The omission by Pliny of any reference to aphrodisiac formulas is easily explained by his declaration (25. 25) that he will say nothing in his work about aphrodisiacs or magic spells except what may be necessary to refute belief in their efficacy.

It is to be observed that no part of the digging ceremony as described by Theophrastus could be regarded as belonging to the "practical" preliminaries; Pliny alone mentions the precaution regarding the wind. The earliest account, then, of the digging of mandragora shows the ceremony to have been not a matter of expediency but of superstition.

It has been remarked above that many features of the later mandragora superstition were transferred to it from other plants. That such stories easily passed from one plant to another may be inferred from a comparison of the two passages just discussed with Theophrastus's account of digging ceremonies employed in connection with other plants. Three circles are made with a sword before digging *xiris*.[*] Facing the west while digging is not altogether novel, for hellebore was dug facing *east*,[†] and Theophrastus himself observes that the custom of repeating aphrodisiac formulas in digging mandragora is similar to the repetition of curses in sowing *cuminum*.[‡]

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* These words were misunderstood by the mediaeval compiler Bartholomaeus Anglicus, who says in his De Propr. Rerum, 17. 104, citing Pliny as his authority: Tres circulos gladio circumscribunt et post *expectant effodere ad occasum.*

† See Sprengel, De Ratione quae in Hist. Plant. inter Plin. et Theophr. intercedit, Marp., 1890, and Id., Rhein. Mus., **46** (1891), p. 54.

‡ Id., De Ratione, etc., pp. 19–21.

Theophrastus and Pliny are the only classic writers who mention a digging ceremony in connection with mandragora, and we evidently have in their account the form of the story as far as it pertained to this plant up to at least 100 A. D. But by the fifth century it had taken on some new features. In the Juliana Anicia manuscript of Dioscorides — written in that century — appears a miniature which represents the Goddess of Invention, Heuresis, offering the mandragora to Dioscorides and holding a dead[§] dog[||] by a cord.

Here we see transferred to the mandragora the substance of two very similar digging stories told by Josephus and Aelian about two other plants, which, though bearing different names, were probably identical.[¶]

Josephus[**] says that a place called *Baaras*, near the Dead Sea, brings forth a peculiar plant called by the same name. Its color is fiery, and toward evening it sends out a ray like lightning. It is not easy to obtain, for it has a habit of receding from the grasp of persons who wish to pull it. Even when obtained it is sure to cause the death of those who touch it unless handled with the greatest care. Divers expedients are resorted to in order to procure it without danger. One of these is to dig around it until only a very small part of the root remains fast in the ground; a dog is tied to this; when he starts to follow the one who tied him, he pulls the root. But the dog dies immediately, a sacrifice for the person who secures the plant.

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[*] 9. 8. 7. Circles (presumably three) were made about hellebore also: 9. 8. 8.

[†] 9. 8. 8. Lobeck, Aglaophot., 915^{b}, interprets this looking toward the west as an act of reverence to the chthonic gods. What Theophrastus says in the section preceding this about the custom of depositing honey-cakes in the ground as a propitiatory offering (μελιττούτας ἀντεμβάλλειν μισθόν) upon digging *asclepicum* and *xiris* supports this view.

[‡] 9. 8. 8.

[§] According to Cohn (Friedlaender, Sittengesch.^{6}, vol. 1, p. 576), ein *erdrosselter* Hund. But there seems to be no mention anywhere of the *strangulation* of the dog. The story was rather that the dog's death was caused by the terrible cry uttered by the plant when taken from the ground. See below, pp. 493 and 495.

The miniature in this manuscript is reproduced by Lambecius, Comm. de Bibl. Caes. Vind., pars II, tab. ad p. 211. Other interesting figures representing the mandragora and the dog used to pull it are reproduced from manuscripts and Anglo-Saxon printed editions of pseudo-Apuleius, by J. F. Payne, English Medicine in the Anglo-Saxon Times (London, 1904), figures 3–6, at the end of the book.

[||] According to Th. von Heldreich, Nutzpfl. Griechenlands, p. 36 f., the belief exists in Greece to-day that the person who digs a mandragora root must die, and it is therefore only to be pulled with the aid of a dog tied to the upper part of the root. Von Luschan, p. 728, testifies to a present-day belief in Syria regarding the danger attending the digging of the root.

[¶] Langkavel, Botanik der spät. Griech., p. 33, identifies *baaras* and *aglaophotis* as *Paeonia officinalis*, L.

[**] Bell. Iud., 7. 6. 3.

Aelian's* story he relates of the plant *cynospastus* ("dog-dug") or *aglaophotis*. This, he says, is not distinguished from other plants during the day, but at night glows like a star. A sign is fastened to the roots at night by which it may be known the next day. When the diggers come at dawn to get it, they do not venture to pull it up themselves, for it was said that the first person who touched it perished through ignorance of its nature. They therefore take a dog which has been kept without food for several days and tie him with a strong cord to the lower part of the plant. They then throw him pieces of meat. Attracted by the smell the dog springs forward, and thus pulls the plant. He dies at sunrise, and they bury him on the spot with secret rites, considering that he has incurred death for their sake.

It is evident that we have here two versions of the same story, differing somewhat in their details, but in general agreeing. Both plants glow in the night, both are dug at peril of one's life unless certain precautions are taken, in both cases the dog dies after pulling the root, and his death is regarded as a measure of atonement for the act. Furthermore, the uses of the plants are similar: *baaras* is said to be used to "drive out demons," while *aglaophotis* is good for epilepsy, — doubtless the same malady in each case.†

Josephus says that his plant grows near the Dead Sea; Aelian says nothing about the habitat of *aglaophotis*, nor is it possible to infer anything about it from the subject-matter of this book (XIV), for in it he treats of various matters pertaining to many parts of the world, — Libya, India, Italy, Istria. But Pliny gives us a valuable hint as to the source ot the story when he says that *aglaophotis* grows in eastern

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* Nat. Anim., 14. 27.

† Epilepsy, among other diseases, was regarded by the Jews in the time of Josephus as a manifestation of demoniac influence. See Scribner's Dictionary of the Bible, vol. 1, p. 593.

Arabia, and that *the magi use it to conjure up the gods.*[*] It appears, then, that Aelian is telling a story about a magic plant, and that the origin of this story is to be traced to the Orient, just as that of Josephus's is.

The significance of the picture in the Dioscorides manuscript is plain. We have evidence that an Oriental myth formerly associated with another plant had by the fifth century been transferred to the mandragora.

Even in the middle ages the story was amplified somewhat, as appears from the following extract from Schmidel's dissertation [†]:

> The diggers must go forth on the day of Venus [‡] before sunrise, having filled their ears with cotton and sealed them with pitch or wax. They must make three circles with a tripod and affix the sign of the cross to the plant; then dig all about it, until only a single thread of the root remains fast in the earth. After the root has been thus treated, they must tie it with a rope to the tail of a dog, then quickly withdraw, throwing the dog a bit of bread. As he strains to reach the bread he will pull the root, and immediately on hearing its cry will drop dead.

Here we have the story of the dog pulling the root embellished with a few clumsy details. These are probably to be looked upon as the invention of the swindlers who went about Europe in the middle ages selling the images called *mandragorae* or *alraune.* [§] Their object in circulating the story was doubtless the same as that of the ancient root-diggers, to enhance the value of their wares.

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[*] Plin., 24. 160.

[†] See footnotes, pp. 487 and 488. This extract is from § 53.

[‡] Friday.

[§] These images were generally not made from the genuine mandragora, but from bryony and other easily procurable roots. See Beyer, p. 738 ff., and the passage there quoted from Matthiolus, Comm. in Diosc., lib. 4, cap. 76 [61].

Ascherson, p. 736, says that it can hardly be doubted that artificially prepared mandragora images were known in classical times, and he cites the epithets ἀνθρωπόμορφος and *semihomo* (see footnotes below, p. 495) as indicating to him that "art was already helping out nature at that time." But the resemblance of the natural root to the human body must have been striking enough to call forth such epithets, and there could have been no object in so preparing the root unless the images were to be used as fetiches. The use of mandragora root for such a purpose in classical times, however, is entirely foreign to literature, and there is no evidence that this feature of the later superstition had yet become associated with the plant. When the pseudo-Orphic *Lithica* were written (probably about the 4th century after Christ), the talismanic powers later reported of the mandragora were ascribed to *ophites* (see p. 497 below).

The "day of Venus" was doubtless chosen as the day for digging mandragora because it bore the name of the Goddess of Love,[*] since one of the virtues claimed for the mandragora in the middle ages was that it won the owner favor in love.[†] Digging before sunrise was a common practice in ancient times in the case of several other plants.[‡] Instructions to stop the ears followed naturally upon the incorporation into the story of the idea that the plant shrieked when taken from the ground.[§] Affixing a sign to the roots we have already seen in Aelian; here the sign is the sign of the cross, to suit the age in which this version of the story arose. The rest of the story agrees with one or the other of the versions which we have already examined, and calls for no further comment.[‖]

Closely connected with this digging story in later times was the belief that the mandragora grew in a form resembling the human body, and even presenting the peculiarities of sex. This does not appear to be definitely stated anywhere before mediaeval times. But the germs of the superstition are found in the words of several ancient writers.

Both Dioscorides and Pliny refer to a "male" and a "female" species of mandragora.[¶] These terms, which the ancients applied to many plants, have nothing to do with sex, but signify more robust species (i. e., those having larger leaves, roots, etc., and attaining a greater height) and their opposites.

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[*] Friday was never sacred to Venus in the ancient religion. It received the name "day of Venus" when the names of the seven planets were given to the days of the week, which Dio Cassius (37. 18) says was not very long before his time (2d–3d century of our era). See Hare, Philol. Mus., **1** (1832), p. 2 ff.

[†] Levinus Lemnius, Herb. Bibl. Explic., cap. 2.

[‡] See Theophr., 9. 8. 5 (*clumenum*); Colum., De Re Rust., 6. 5. 3 (*consiligo*); Plin., 25. 145 (*anagallis*); Diosc., 1. 349 (*centaureum*).

[§] See below, p. 495.

[‖] Why they should have made circles about the plant with a *tripod* before digging is unintelligible to me, and I can only suggest that Schmidel (who is the only one to cite this custom, so far as I know), or some one before him who read of this ceremony, misinterpreted the phrase relating to this part of it. Ponce de Leon in his Commentary on Physiologus, cap. 4, thus translates the words of Theophrastus, τὸν δ' ἕτερον κύκλῳ περιορχεῖσθαι (9. 8. 8): iubent et alium saltantem *tripudio* circumagi. Probably the words *tripudium* and *tripus* were confused, and terno *tripudii* circumductu should be the reading in Schmidel instead of terno *tripodis* circumductu.

[¶] See below, p. 505.

(Pseudo-) Pythagoras,* according to the interpolator in Dioscorides, called the mandragora "anthropomorphic," † and Columella speaks of it as "half human." ‡ These words without doubt refer to a fancied resemblance of the root to the lower part of the human body. §

Furthermore, Pliny writes about the plant *eryngе* that its root presents the characteristics of either sex. ‖

We know, then, that the mandragora comprised two species, called "male" and "female"; that the lower part of the root was thought to resemble the human form; and that it was claimed for another plant that it possessed the marks of sex. What was more natural, in the case of a plant about which various strange beliefs were already entertained, than that these three features should be combined?

The climax of these superstitions seems to have been reached toward the end of the middle ages. The following passage from Schmidel's dissertation will serve to illustrate the absurd beliefs once entertained by the credulous in regard to this plant, and evidently believed to a considerable extent even in his day. ¶

> § 53: At the foot of the gallows on which a man has been unjustly hanged for theft, it is said that there springs from the urine voided just before death, a plant with broad leaves, a yellow flower, and a root which exactly represents the human form, even to the hair and sexual organs. Some say that this plant was alive under the ground. . . . The Germans call it the *alraun*. To dig it is said to be attended with great danger, for it gives forth such groans when drawn from the earth, that the digger, if he hears them, dies on the spot.

Even in this form the greater part of the story may be traced back to antiquity. The part of it referring to the hanged thief is plainly an adaptation of an ancient fable about a so-called *herb of Prometheus* (*φάρμακον Προμηθεῖον*), which is thus described in the Argonautica of Apollonius Rhodius:

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* Author of a lost work on the properties of plants, who wrote in the age of the Ptolemies, according to Ascherson, p. 736.

† *ἀνθρωπόμορφον*, Diosc., 1. 570.

‡ semihominis . . . Mandragorae, De Re Rust., 10. 19 f.

§ This resemblance is readily seen in the picture of the mandragora in Bodaeus's Theophrastus (Amst., 1644), p. 585. See also the illustration in the Standard Dictionary under the word *mandragora*.

‖ Plin., 22. 20.

¶ This, with the passage quoted above from the same section, is a typical version of the middle-age story, and presents all the essentials; it appears elsewhere with slight differences in the details. For example, according to one account the criminal must be a hereditary thief; according to another, a thief who has never had sexual intercourse; another version says that the dog must be snow-white, etc.

> 3. 851 ff: This sprang first when in the glades of Caucasus the flesh-devouring eagle sent dripping to the earth the gory ichor of wretched Prometheus. And a cubit's height above appeared a flower, in color like to the Corycian crocus, supported on twin stalks, and the root in the earth was like to new-cut flesh. . . . The dark earth shook beneath as the Titanian root was cut, and the son of Iapetus groaned himself, for his heart was mad with anguish.*

This is plainly the source of our story. For Prometheus was condemned to his punishment for *theft* (and wrongly condemned, we should say); the flower sprang from his gore as it dripped to the ground.† The idea that the plant was alive under the ground appears to be a reminiscence of the line in Apollonius in which it is said that the root in the earth resembled new-cut flesh; the groans of the plant when taken from the earth go back to the latter part of the above passage. In the story of the Promethean plant it is said to spring from the gore which drips from the Titan's body. Since gore does not drip from the bodies of hanged thieves, a change had to be made here in adapting the story to the mandragora, and so the plant is said to spring from the thief's *urine*.‡ And lastly, the sudden death of the digger plainly goes back to Josephus and Aelian, who say, as we have seen, that the one who pulls *baaras* or *aglaophotis* must surely die unless he takes the precautions indicated.

The images made from the roots which it was alleged were obtained in this way were kept as fetiches or talismans, and were thought to bring the owner good fortune in various ways: they were considered useful in divining secrets,§ in increasing one's material prosperity,§ in gaining the lover the object of his affection,‖ in making sterile women fertile,¶ and for other purposes.* The images were bathed and dressed and carefully tended.†

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* This passage is imitated by Valerius Flaccus, Argon., 7. 355–370.

† With this story compare what Galen says of the crocus, 13. 269: *μειράκιον καλούμενον Κρόκος, ἅμα τῷ Ἑρμῇ δισκεύων, εἶθ' ἑστὸς ἀμελέστερον, ἐμπεσόντος αὐτῷ δίσκου τῇ κεφαλῇ, συνέβη μὲν ἀποθανεῖν αὐτίκα, τοῦ δ' αἵματος εἰς τὴν γῆν ἀναχθέντος, ἐξ αὐτοῦ φῦναι τὸν κρόκον.* On other similar stories see Peter's note on Ovid, Fasti, 5. 226.

‡ This is probably to be regarded as a reminiscence of Josephus' story about *baaras*, which retreated from those who wished to pull it, unless a woman's urine was sprinkled upon it. Bell. Iud., 7. 6. 3.

§ Beyer, p. 742.

‖ Levinus Lemnius, Herb. Bibl. Explic., cap. 2.

¶ Matthiolus, Comm. in Diosc., lib. 4, cap. 76 [61].

This custom, too, is not without its counterpart in antiquity. In the pseudo-Orphic Lithica, written probably about the 4th century after Christ, we find a *stone* treated in essentially the same way, and considered valuable for very similar purposes. Here we read of the stone *ophites:*

> Lith., 363 ff.: Washing the sentient stone in an ever-flowing spring, he cherished it in soft garments like an infant. . . . He tended it in his arms, lifting up the divine stone, like a mother holding in her arms her infant son. And do you too, when you wish to hear a heaven-sent voice, do this, that your mind may behold a marvel; for when you exert yourself, shaking it hard in your hands, straightway you shall arouse the voice of a new-born child, crying for milk on its nurse's bosom. But rouse it ever, being of a stout heart, . . . and dare to ask for a prophecy; for it will declare everything unerringly.

The virtues of the stone thus cared for are several:‡ (1) it has the gift of divination; (2) it prevents attacks by serpents; (3) it cures their bites; (4) it causes sterile women to bear children; (5) it restores sight; (6) it cures headache; (7) it restores hearing; (8) it remedies impotency in wedlock. Have we not here in late Greek literature — evidently an echo of the folk-lore of this or an earlier time — the essential part of the alraun myth of the middle ages? §

We have now observed each of the important features of the later mandragora superstition (except the digging story in its simplest form, as related by Theophrastus — the nucleus about which the rest grew) in an earlier setting, apart from the mandragora, in Greek or Roman folklore: (1) details of the digging story, such as the pulling of the root by a dog, related by Josephus and Aelian in connection with *baaras* and *aglaophotis;* (2) the story about the mysterious origin of the plant from the urine of a thief, and the groans emitted by it when taken from the ground, modelled upon the older story of Prometheus; (3) the story of the reproduction of sexual distinctions, reported by Pliny of *eryngе;* (4) the custom of keeping the root as a fetich, and caring for it as a human being, which finds its pattern in the Lithica. This furnishes us a basis for the following theory: The chief elements of the middle-age mandragora superstition originated either wholly in the Orient, the peculiar home of the magic lore of antiquity, or part of them there,* part of them among the Greeks and Romans;† few of them were at first connected with the mandragora. The elements of Eastern origin passed into the folk-lore of the Greeks and Romans, where they began to unite, and where a story-complex began to form about the mandragora.‡ Passing on to the people of northern Europe,§ this story-complex was augmented in the middle ages by the addition to it of

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* Joan of Arc was accused at her trial of having a mandragora. The words of the charge are given by Quicherat, Procès de Condamnation et de Réhabilitation de Jeanne d'Arc, vol. 1, p. 213 f.

† Figures made from mandragora are kept for the same purposes in eastern countries to-day, and probably have been for centuries. See v. Luschan, p. 728.

‡ (1) 378 ff.; (2) 412 ff., 458; (3) 430–450, 342 ff., 452; (4) 453; (5) 459; (6) 459 f.; (7) 460 ff.; (8) 463 ff. These references are to Hermann's edition (Lipsiae, 1805).

§ Among a number of less important passages bearing on mandragora which I have collected from the works of mediaeval writers, some of the glosses on the word may be mentioned here. The authors of these knew it chiefly as a wonder-plant, and were evidently acquainted with the traditions about its power over the mind. They probably (as no doubt the majority of the middle-age writers) had no personal acquaintance with the plant; so it is several times confused with *hyoscyamus*, the Latin name for which was *apollinaris:* see CGL 3. 543. 41; 3. 536. 10; 3. 550. 5; 3. 550. 1 (*apollea*). 3. 569. 60 gives *malacanina*, with reference to the later digging story. In 3. 558. 65 and 3. 622. 57 we read *circelon*, evidently a corruption of Κιρκαία. We twice find the term *orcibeta* (herb of Orcus?) applied to it: 3. 536. 10 and 3. 550. 5. 3. 592. 42 and 3. 626. 17 call it *herba periculosa*, and 3. 614. 4 reads *mandragora idest periculosa.* The fullest gloss in CGL is 3. 585. 1: *mandragora herba*

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quae odorem habet grandem, qui hominem extra mentem facit, et est eius radix in similitudine corporis humani. In 3. 554. 48 and 3. 619. 3, *bulloquilon mandragora*, we perhaps have an attempt to reproduce βομβόχυλον, a name cited by the interpolator in Diosc., lib. 4, cap. 76.

In a medico-botanical glossary published in Mowat's Anecdota Oxoniensia, vol. 1, p. 109, are two glosses on mandragora. They are plainly taken wholly from Dioscorides, with the exception of the last sentence in the second: *ista, ut dicitur, bibita homines amantes facit.* This glossary is taken from a 14th-century manuscript in the library of Pembroke College, Oxford.

* For example, using a dog to pull the root, an element evidently of Eastern origin, as shown above, p. 492 f.

† The circles made about the plant before digging, which I cannot trace back of Theophrastus. Whether or not the ῥιζοτόμοι were indebted to the East for their digging stories I have been unable to ascertain. It is, of course, possible that the custom of making circles about the root, as well as the story about the herb of Prometheus, and the habit of treating *ophites* as a fetich, go back ultimately to the East.

‡ By the 5th century (the date of the Juliana Anicia manuscript of Dioscorides) this included the pulling of the root by a dog, the report that the root represented the human form, with sex-distinctions, as shown by the miniatures in the manuscript referred to, and doubtless also the encircling of the plant with a sword, and the pronouncing of aphrodisiac formulas, which of course the miniatures cannot indicate.

§ As to the path taken by the mandragora superstitions in their passage north, still other features,* which appear to have been derived independently from the same source. Finally, from northern Europe this group of mandragora stories spread back to the Orient, and the custom of digging and using mandragora there to-day in the mediaeval fashion is the result of this backward movement.†

In the preceding pages I have attempted to give a general idea of the nature of the superstitions which have been connected with the mandra-

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I am not able from my study of the subject to reach any conclusion more definite than that they came to northern Europe from the Greeks and Romans rather than directly from the East. As to whether they passed north from the Greeks *or* the Romans, I am not ready to venture an opinion. Ascherson's article in the Sitzungsber. d. Gesell. f. naturf. Freunde, Berlin, 1890, pp. 59–82, in which he contends that certain beliefs about the mandragora formerly held in Greece passed to Germany (in connection with another plant, *Scopolia carniolica*, Jacq.) by way of Dacia, Galicia, and southern Russia, shows how the particular stories which we are dealing with *might* have gone north from the Greeks. Doubtless there were also other channels. Further investigation may make more definite conclusions possible.

* The Prometheus element and the fetich idea. By "the same source" I mean by way of the Greeks and Romans.

† We know that accounts of the medicinal uses of the plant went from west to east; witness Serapion's chapter on mandragora, based almost wholly on Greek sources (see p. 517 below). What more natural than that the superstitions now at length connected with it should have passed along with these accounts of its uses in medicine? Medicine and magic went hand in hand from the earliest times.

Wetzstein (in a letter to Ascherson, p. 890 f.) and Veth (p 200 f.) attempt to show that the mandragora story (just how much of it I cannot make out from their articles) originated in *Persia*, and spread from there to neighboring countries. Arguments for this view are: (1) that μανδραγόρας may be derived from *merdom gia*, "man-like plant," a Persian name for the mandragora; (2) that another Persian name for it is *segken*, "dog-dug"; and (3) that the plant is dug with the same ceremonies and used for the same purposes in Persia to-day as it was in northern Europe in the middle ages. But the derivation of μανδραγόρας from *merdom gia* (which, if proven, would show that superstitions about the plant were current in Persia in very early times, for which there is now no evidence) is by no means certain; and as regards the name *segken* and the existence of ceremonies and beliefs in Persia to-day similar to those of mediaeval Europe, there is no more reason to believe that they passed from Persia to the West than that the opposite movement took place.

In attempting to show the Eastern origin of portions of the mandragora superstition, previous writers have often, it seems to me, asserted too positively that this plant was believed by Oriental peoples from very ancient times to be an aphrodisiac and a promoter of fecundity, citing the passages in which the word occurs in the Old Testament as proof of this. As I have said below (p. 504), *there is no positive proof* gora, and to show to what extent the later European stories about it were based upon classic antiquity. It has been shown that, while the elements of these superstitions were known to the Greeks and Romans, they were not generally connected with this plant. I wish now to show what they thought about the mandragora itself.

Before discussing its legitimate uses in medicine, certain beliefs should be noticed which are not entirely free from superstition, and yet are apparently based on observation of the actual effects of the plant. These are (1) that it induces madness; (2) that it influences sexual relations, being effective (*a*) in philtres, (*b*) as an aphrodisiac, (*c*) to produce fecundity.

In the treatise *De Hominum Locis* (formerly ascribed to Hippocrates, and in any case to be referred to the 3d or 4th century B. C.[*]) direction is given to administer "less of the drug than causes madness."[†]

Suidas gives the verb μανδραγορίζω without defining it; judging by the analogy of words like ἐλλεβορίζω it seems to mean "to dose with mandragora."

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that the *dûdâ'îm* were actually μῆλα μανδραγορῶν. The supposed discovery of representations of and mention of the mandragora on Oriental and Egyptian monuments, such as Perrot and Guillaume (*Les Monuments de la Ptérie*, Rev. Arch., **23** (1872), pp. 284–289) and Brugsch (*Die Alraune als altägyptische Zauberpflanze*, Zeitschr. für Aegypt. Spr., **29** (1891), pp. 31–33) describe, where the connection might indicate that it had a reputation as a wonder-plant among these peoples, is altogether too much a matter of conjecture to be admitted as evidence.

Some of the recent writers appear to hold the belief (though I do not find it definitely stated by them) that this story-complex developed in the East, and passed from there substantially in its later form, with all its features already connected with the mandragora. In that case, the superstition, it seems to me, would have passed *as a whole* to the Greeks and Romans, in connection with the mandragora, — it would not have been broken up into parts, the circles with a sword staying with the mandragora, the sex-feature going to *eryngo*, digging with the aid of a dog to *baaras* and *aglaophotis*, the use as a fetich to *ophites*, and so on. Veth evidently realized as inconsistent with this view the fact that Josephus tells his digging story about *baaras*, and not about mandragora; but Veth maintained that *baaras* and mandragora were the same plant, but that Josephus did not know the *Greek name*. Veth would gladly believe that Aelian, too, was only telling of mandragora under another name when he wrote of *aglaophotis*, but he confesses himself unable to identify them. He thinks that the uncritical Aelian made a mistake and wrote down the name of the plant wrong! But all difficulty vanishes when we believe that in the time of Josephus and Aelian this feature of the later story had not got as far as the mandragora.

[*] See Kühn's Hippocrates, vol. 1, p. cliii; Neub.-Pag., p. 217.

[†] Hipp., 2. 139: ἔλασσον ἢ ὡς μαίνεσθαι.

Athenaeus gives five fragments of a play of Alexis[*] called ἡ Μανδραγοριζομένη. There are thirty-two verses in all, but unfortunately none of them explains the name of the comedy. The only possible interpretation seems to be to take this name as the passive participle of μανδραγορίζω, understood as above, i. e., "The Mandragora-drugged Lady." Whether the drug was represented as a love-charm or as causing madness it is impossible to determine.

There is a possible reference to madness induced by mandragora in Columella:[†]

> Quamvis semihominis vesano gramine foeta
> Mandragorae pariat flores.

Here some have understood that *gramen* means *plant*,[‡] and that the "mad plant" (*vesanum gramen*) is the mandragora, translating "although, rich in the mad plant, it brings forth the flowers of the half-human mandragora." [§] Others explain *vesanum gramen* as *luxuriously growing grass*, translating "although, rich in luxuriously growing grass, it brings forth the flowers of the half-human mandragora." [‖] It seems impossible to determine which view is the correct one.

In Aretaeus we read:

> De Caus. et Sign. Diut. Morb., 1. 6. init.: Certain edibles, such as mandragora and hyoscyamus, induce madness, but these affections have never been called *mania*.

That mandragora was used in philtres, and that it was believed to have aphrodisiac properties, is not directly affirmed by any ancient writer, but it appears from a number of passages that it had this reputation to a greater or less extent.

Theophrastus and Dioscorides report its use in philtres on hearsay:

> Theophr., 9. 9. 1: They say (φασί) that the root is useful for philtres.
>
> Diosc., 1.570: Some call it *circaea*,[¶] because the root appears (δοκεῖ) to be useful for making philtres.

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[*] Kock, Com. Att., vol. 2, p. 347 ff. (Athen., 3. 123 f; 8. 340 c; 8. 365 d; 12. 552 e f; 14. 621 d e).

[†] De Re Rust., 10. 19 f. I have given the Latin text because of the uncertainty as to the correct translation.

[‡] On *gramen* in this sense compare Verg., Georg., 4. 63, Aen., 12. 415; Ovid, Met., 7. 137.

[§] See Lemaire, Poetae Lat. Min. (Paris, 1826), vol. 7, p. 30.

[‖] See Barth, in Gesner's Script. Rei Rust. (Lips., 1735), vol. 1, p. 692.

[¶] The name refers to Circe. Dierbach, Flor. Myth. (1833), p. 204, thinks that

A reference to the belief in its aphrodisiac properties is doubtless to be recognized in the remark of Theophrastus quoted above, that when mandragora was dug it was customary to repeat aphrodisiac formulas; and in the mention by Hesychius of Μανδραγορῖτις as an epithet of Aphrodite.*

A passage in a letter of Julian the Apostate to the priestess Callixena also appears to refer to this belief:

> Epist. 23: Who can prefer in a woman conjugal love to piety, without being thought to have taken large draughts of mandragora? †

A striking indication of the belief in the plant's aphrodisiac powers is taken from the so-called Physiologus, a work of uncertain date, but probably compiled during the early centuries of the Christian era.‡ In the fourth chapter, which treats of the elephant, we read:

> The female seeks out the so-called mandragora plant, and partakes of it; straightway she is inflamed, and, going to the male, gives him the plant; he partakes of it, and straightway is inflamed, and cohabits with the female.§

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mandragora was the magic herb with which Circe bewitched the companions of Odysseus. Odys., 10. 235 f:

ἀνέμισγε δὲ σίτῳ
φάρμακα λύγρ', ἵνα πάγχυ λαθοίατο πάτριδος αἴης.

It may be, indeed, that we have in the name *circaea* a reminiscence of some lost story in which the plant was brought into connection with this famous incident; and the note of Suidas on mandragora, λήθης ποιητικός, may possibly be another.

* Hesych., s. v. Μανδραγορῖτις.

† This passage *might* be interpreted as referring to the mandragora's power to induce madness ("Who but a madman would place conjugal love before piety?"), or lethargy ("Who but a dullard," etc.). But the context is against this; Julian is contrasting Penelope's persevering fidelity to Odysseus with Callixena's long and faithful service in religion.

According to Gesner's note on Colum., 10. 19, Aetius, 13.45, says that mandragora induces madness. I have not been able to gain access to an edition of Aetius properly arranged for reference.

‡ This book, a favorite in the middle ages, is the source of many well-known wonder stories from the realm of nature, such as the rising of the phoenix from its ashes, the pelican nourishing its brood from its own blood, and others of the sort. On its date and authorship see Christ, Gr. Litt.^{3}, p. 911; Krumbacher, Byz. Litt., p. 874 f.

§ Here follows, as regularly throughout the work, a symbolical interpretation, by which the story is brought into connection with the Bible: the male and female elephant represent Adam and Eve, and the mandragora represents the forbidden

Evidence of belief in the mandragora's aphrodisiac properties, as well as in its power to produce fecundity are found in two passages in the Septuagint version of the Scriptures, where the Hebrew *dûdâ'îm*, meaning literally *love-apples*, is translated by *μανδραγόραι* or *μῆλα μανδραγορῶν*.

In the seventh of the Canticles, 10–13, we read:

> I am my beloved's,
> And his desire is toward me.
>
> Come, my beloved, let us go forth into the field;
> Let us lodge in the villages.
> Let us go up early to the vineyards;
> Let us see whether the vine hath budded and its blossom be open,
>
> And the pomegranates be in flower:
> There will I give thee my love.
> The mandrakes give forth fragrance,
> And at our doors are all manner of precious fruits, new and old,
> Which I have laid up for thee, O my beloved.

Here both the meaning of the Hebrew *dûdâ'îm*, and the context indicate that an aphrodisiac is referred to.

The other passage, Genesis 30. 14–16, is as follows:

> And Reuben went in the days of wheat harvest, and found mandrakes in the field, and brought them unto his mother Leah. Then Rachel said to Leah, Give me, I pray thee, of thy son's mandrakes. And she said to her, is it a small matter that thou hast taken away my husband? and wouldest thou take away my son's mandrakes also? And Rachel said, Therefore he shall lie with thee to-night for thy son's mandrakes. And Jacob came from the field in the evening, and Leah went out to meet him, and said, Thou must come in unto me; for I have surely hired thee with my son's mandrakes.

Why Rachel wanted the *dûdâ'îm*, was a source of much discussion to the fathers who wrote commentaries on Genesis. The theory commonly accepted by these writers, that the *dûdâ'îm* were supposed to promote conception,* is evidently the correct one. In the first place, Rachel's bar-

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fruit. Compare the interpretation of Michael Glycas, Script. Hist. Byz., vol. 26, p. 100 f.

Compare with this story about the elephant the statement of Dioscorides (1. 572): *μαλάσσειν δὲ καὶ ἐλέφαντα λέγεται ἡ ῥίζα συνεψομένη αὐτῷ ἐπὶ ὥρας ς´, καὶ εὔπλαστον αὐτὸν εἰς ὃ ἄν τις βουληθῇ σχῆμα παρασκευάζειν.* Can there be any connection?

* Accepting this theory, and identifying *dûdâ'im* and mandragora, the fathers

renness was a source of keenest grief to her,* which furnishes a motive for her request. Furthermore, the whole chapter in which the *dûdâ'îm* are introduced, up to the twenty-fifth verse, bears upon the sterility of Rachel and the fecundity of Leah, and nothing else. This interpretation is further strengthened by Leah's reluctance to give up the *dûdâ'îm;* for her power over Jacob depended upon the fact that her rival bore him no children.

There seems, then, sufficient reason for supposing that Rachel wanted the *dûdâ'îm* for the purpose of relieving her sterility.† Whether or not *dûdâ'îm* and mandragora are identical is another question, — one which has never been certainly answered, and which probably never will be, inasmuch as there were doubtless other plants which were thought to favor conception, and, as far as the passage in Genesis is concerned, the *dûdâ'îm* may have been any one of these as well as mandragora. But in any case it seems certain that a word meaning *love-apples* would not have been so rendered by the Seventy if mandragora had not been well known as an aphrodisiac at the time the translation was made. ‡

The fullest and best description of the mandragora is found in Dioscorides. § Pliny gives a somewhat shorter one. ‖ Theophrastus ¶ does not attempt a connected description of the plant known to him by this name, but gives a few of its characteristics, thereby showing certainly that it was neither the mandragora of Dioscorides and Pliny nor the plant known as mandragora in Greece and Italy in later times.

We learn from Dioscorides that the mandragora had a variety of names; he himself mentions four, and the interpolator gives several more. A number of these were also applied to other plants, with

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went farther and attempted to show on physiological grounds how the mandragora may have favored conception. See, for example, the opinions cited by Barth. Angl., De Propr. Rerum, lib. 17, cap. 104.

* Gen. 30. 1.

† Frederick Starr, Amer. Antiq., **23** (1901), p. 267, says that the Jews still believe in the power of mandragora to induce fertility, and that American Jews to-day import specimens of the root from the Orient for this purpose.

‡ Mention should be made in passing of a reference of somewhat uncertain significance in the pseudo Orphic Argonautica (917–922), where mandragora and fifteen other plants are represented as surrounding the tree on which the golden fleece is suspended. Most of these others as well as the mandragora were associated with superstitions of one sort or another in antiquity, and they seem to figure here simply as magic plants likely to harm the intruder.

§ Diosc., 1. 570 ff. ‖ Plin., 25. 147–150. ¶ Theophr., 6. 2. 9.

the result that they are often confused with it in the works of ancient writers.[*]

Two species were well-known to the ancients, and both are found to-day in southern Europe and in Asia. Dioscorides mentions a third species, with which he apparently had no personal acquaintance.[†]

The "female" mandragora (*mandragora autumnalis*, Spreng.) is of a dark shade, with small, narrow leaves;[‡] these grow spread out along the ground, and have a strong, unpleasant odor. Among the leaves appears the fruit, which is of a yellow color, of the size of service berries or filbert nuts,[§] and has a pleasant smell. The fruit contains a seed much like that of a pear. The roots are two or three in number, and grow intertwined with one another; they are black[‖] or reddish[¶] on the outside, but white within, and the interior is fleshy and tender; the bark is thick; the average length of the root is about a cubit. Pliny says that this species has a hairy stem; Dioscorides says that it has none.

The "male" mandragora (*mandragora vernalis*, Bert.) is of a light color, which the writers generally call white. The leaves are large, white, smooth, and broader than those of the first species; they resemble the leaves of the beet[‖] or garden sorrel.[¶] The fruit of this species is twice as large as that of the other; its color is yellow, and its odor pleasant, though strong. The root resembles that of the first species, but is larger and whiter. Dioscorides states that it has no stem, and Pliny is silent on this point.[**]

Pliny's description differs from that of Dioscorides in five points:

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[*] See footnote above, p. 497.

[†] Diosc., 1. 573: *ἱστοροῦσι δὲ καὶ ἑτέραν.* As to the existence of other more or less clearly defined varieties in Europe and Asia to-day, see Ascherson, p. 729, note 4.

[‡] Pliny says "narrower than lettuce"; this is a favorite standard of Pliny when speaking of the relative size of leaves. The interpolator in Dioscorides (1. 570) says that mandragora was also called *θριδακίας*; this too seems to refer to some resemblance to the lettuce (*θρίδαξ*).

[§] Service berries and filbert nuts are of about the same size. Bodaeus, Comm. ad Theophr., p. 584.

[‖] Dioscorides.

[¶] Pliny.

[**] Dioscorides adds that the "male" is also called *νώριον* (*τώριον*, cod. N), but this seems to be a corruption of *μόριον*, the name of his "third species." Pliny mentions the name *morion* and also *arsen* (*ἄρσην*) and *hippophlomos* in connection with the "male."

The description given above is that of the ancient writers; I have made no attempt to treat the plant from the standpoint of a modern botanist.

A. Female species. (1) Dioscorides says that the leaves are narrow; Pliny says that they are narrower than the leaves of the lettuce; (2) Dioscorides says that the fruit resembles service berries; Pliny says that it resembles filbert nuts; (3) Dioscorides says that the root is black outside; Pliny says that it is reddish; (4) Dioscorides says that it has no stem; Pliny says that it has a stem. B. Male species. (5) Dioscorides says that the leaves resemble those of the beet; Pliny says that they resemble those of the garden sorrel.

In all this there is no real conflict, except in the matter of the stem. Pliny probably used lettuce, filbert nuts, and sorrel as standards of comparison, while Dioscorides used service berries and the beet, because each thought the ones which he used would be readily familiar to his readers; "black" and "reddish," applied to the root, are only relative terms, such as "white" and "black," applied to the two species. But there is no compromise possible in the matter of the stems. Pliny must be wrong here, for the mandragora as identified by later writers has no stem. In spite of this discrepancy it is evident that Dioscorides and Pliny were writing of the same plant.

The mandragora of Theophrastus is very briefly characterized: (1) It has a stem; (2) its fruit is (*a*) black, (*b*) similar to a grape,[*] and (*c*) has a vinous juice.

The third species of Dioscorides, which he says is also called *μόριον*, grows in thickly shaded places, and about caves. The leaves resemble those of the "male," but are smaller, being about a span broad, and whitish in color; they grow in a cluster about the root, which is tender and white, somewhat more than a span in length, and about as thick as the thumb.

There has been a great deal of discussion about the mandragora of Theophrastus and the third species of Dioscorides. Some have attempted to identify them. This seems to me impossible from the start, for the two authors happen to mention an entirely different sort of facts about each; while Dioscorides tells about *habitat*, *leaves*, and *root*, Theophrastus speaks only of *stem* and *fruit*. There is, then, no proper basis of comparison, and it is useless to conjecture further as to their identity. On the other hand, it is a pretty generally accepted view that the mandragora of Theophrastus is to be identified with belladonna.[†]

Various parts of the plant were employed medicinally. The fresh

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[*] *ῥαγώδης*: apparently the best interpretation of this word.

[†] See Ascherson, p. 734 f., note 2.

leaves mixed with barley meal were applied as a poultice;[*] they were sometimes preserved in brine for future use.[†] The *fruit* had various uses, as did the juice extracted from it, and the *seeds*. Dioscorides distinguishes between the simple juice (ὀπός) obtained by scratching or pounding the bark and that obtained by boiling (χύλισμα, χυλός); Pliny does not make this distinction, so that we do not always know which kind his *sucus* refers to. Juice was obtained both from the *root* and from the fruit,[‡] that from the root being more powerful.[§] The root was pounded and mixed with oil of roses and wine for external application.[||] The *bark* was pounded from the fresh root, put in a press,[¶] and the juice thus obtained allowed to thicken in the sun, after which it was stored in earthen vessels.[*1] The bark was sometimes stripped from the root and hung away on strings, to be used later in making *mandragora wine*.[†1] We have two receipts for the manufacture of this: (1) It was prepared by boiling down the roots to a third in wine and allowing the liquor obtained to thicken;[‡1] (2) The bark was stripped off and hung on strings and let down into a vessel containing sweet wine, which was allowed to stand thus for three months.[§1] An account of a recent attempt to reproduce the mandragora wine of the ancients is given on page 509 below.

The uses of mandragora in medicine were very many. Its external application, in various forms of plasters and poultices, seems to have had a soothing and cooling effect on the parts to which it was applied. Thus it was a remedy for erysipelas[||1] (which the Latin writers frequently call *ignis sacer*), and for many other diseases accompanied by inflammation; it was therefore much used in the treatment of eye troubles.[¶1] Hippocrates mentions its use for inflammation of the anus.[*2] It was applied to abscesses,[†2] indurations,[‡2] and tumors.[§2] We are further told that it was good for ulcers [*] and wounds. [†] According to Pliny, leaves of mandragora with barley meal would remove foreign substances adhering to the surface of the body. [‡] It was used to remove marks from the face or body. [§] A poultice of the scraped root mixed with vinegar is recommended for gout by Theophrastus, [||] while Pliny cites as a remedy for this disease a poultice made of the leaves and barley meal, or of the root and wild cucumber, or the root alone steeped in water. [¶] It was applied to snake-bites. [*1] It was used in preparing pessaries [†1] and suppositories. [†1] Taken internally it was employed in various ailments. Thus it is mentioned as a remedy for different female troubles. [‡1] Hippocrates recommends it for quartan fever. [§1] It was efficacious in curing defluxions of humors. [||1] The juice mixed with some liquid, like honey and water, was thought to draw phlegm, or black bile, [¶1] the presence of which in the body the ancients regarded as the cause of many diseases. [*2]

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[*] Diosc., 1. 572.

[†] Ibid.; Plin., 25. 149.

[‡] Diosc., 1. 571; Plin., 25. 148.

[§] Diosc., 1. 571.

[||] Plin., 25. 147.

[¶] This press is mentioned in the pseudo-Galenian treatise De Simpl. Med. ad Patern. (Junta, Venet., 1609, Sp 91G): succus organo exprimitur, quod Graece mandragorochylon appellamus.

[*1] Diosc., 1. 571.

[†1] Ibid.

[‡1] Ibid.

[§1] Id., 1. 736.

[||1] (Theophr., 9. 9. 1); Diosc., 1. 572; Plin., 26. 121; Theophr. Non., Epit., cap. 246.

[¶1] Plin., 25. 147; Diosc., 1. 572.

[*2] Hipp., 3. 338.

[†2] Diosc., 1. 572; Plin., 26. 145.

[‡2] Diosc., 1. 572.

[§2] Plin., 26. 93 (*panus*); Diosc., 1. 572, Plin., 26. 24 (*struma*); Diosc., 1. 572 (*tuberculum*).

Not only was it thought to be efficacious in the treatment of the body, but it was also employed in mental disorders. Consequently it was recommended for those suffering from convulsions, [†2] or those oppressed with melancholia with suicidal tendency, [†2] and for the treatment of the insane, the idea here being generally to produce sleep, as appears from a passage in Celsus. [‡2]

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[*] (Theophr., 9. 9. 1); Plin., 26. 145.

[†] Plin., 26. 145.

[‡] Plin., 26. 149.

[§] Diosc., 1.572, 2. 151; Plin., 25. 175.

[||] (Theophr., 9. 9. 1). Although the plant known to Theophrastus by the name of mandragora was surely not the same as the mandragora of Dioscorides and Pliny, it does not seem out of place to cite the uses which he mentions, inasmuch as it is by no means certain that he took the description of the plant and his account of its uses from the same source; it might easily be the case that the former is wrong and the latter correct. Moreover his account of its uses agrees very well with those of other writers.

[¶] Plin., 26. 104–105.

[*1] Diosc., 1. 572; Plin., 25. 150.

[†1] Hipp., 2. 711; Diosc., 1. 572.

[‡1] Different uses are mentioned in Plin., 26. 156, Diosc., 1. 572, 573, Hipp., 2. 862.

[§1] Hipp., 2. 251.

[||1] Diosc., 1. 736; Plin., 25. 147.

[¶1] Hipp., 2. 739; Diosc., 1. 571; Plin., 25. 150.

[*2] In the so-called Dynamidia (Ang. Mai's Cl. Auct., vol. 7, p. 457) mandragora is said to relieve persons suffering from nausea. This fact is recorded nowhere else, and Pliny (25. 150) states that it *produces* vomiting.

Dioscorides (1. 574) notes that the root of the "third species" is said to be efficacious as an antidote (see Galen, 14. 1 ff.) when taken with the so-called maddening nightshade.

[†2] Hipp., 2. 139.

[‡2] Cels., 3. 18 (ed. Daremberg, p. 100).

Its soporific power was surely known from very early times, — so well known, in fact, that it became proverbial.

In Xenophon we read:

Symp., 2. 21: Wine, moistening the soul, lulls cares to sleep, just as the mandragora does men's bodies.

Plato, in the Republic, speaks of rendering the captain of a ship incapable by means of it:

Rep., 488 C: Having chained up the noble captain's senses with mandragora or strong drink or otherwise.

Demosthenes compares the torpor of his fellow-citizens to that of persons who have drunk mandragora:

Philip. 4. 6: It is not in these respects alone, men of Athens, that we are behindhand; we cannot even be aroused, but are like to men who have drunk mandragora.

Aristotle includes it in a list of narcotic plants:

De Somn., 3: . . . the narcotics; for they all cause heaviness of the head, both the drinkables and the eatables, — poppy, mandragora, wine, lolium.

Philo the Jew refers to it in a simile in a way that indicates familiarity with this property of the plant in his time:

De Vita Contempl., § 5: Others of the apparently more moderate feasters, when they have drunk unmixed wine as though it were mandragora, fall down and . . . are overcome by deep sleep.

Frontinus tells how the Carthaginian general Maharbal captured a whole host by means of wine drugged with mandragora:

Stratag., 2. 5. 12: Maharbal, having been sent by the Carthaginians against the rebellious Africans, knowing that these people were very fond of wine, mixed a great quantity of it with mandragora, the effect of which is half poisonous and half narcotic. Then, after beginning a light skirmish, he left off by design, and in the middle of the night feigned flight, leaving part of the baggage and all the drugged wine in camp. The barbarians seized the camp in great delight, and eagerly drank the wine. Soon they were stretched on the earth like dead men. Maharbal then returned, and captured or killed them.

Polyaenus relates a very similar story about the Carthaginian general

Himilco;[*] he also attributes Caesar's getting the better of the pirates who captured him near Malea to his sagaciously including wine drugged with mandragora among the presents which he caused to be sent along with his ransom money:

> Stratag., 8. 23. 1: Epicrates, at Caesar's command, provided along with the ransom money supplies for a great feast, and a vessel filled with swords, and wine mixed with mandragora. . . . The pirates, rejoicing at the great amount of money, hastened to indulge in a sumptuous banquet, drank the drugged wine, and were thrown into a stupor. Then Caesar ordered his sleeping victims to be put to death.

Whether or not there was any historical basis for such stories as these, they would not have arisen had not the soporific properties of the plant been a matter of common knowledge.

Plutarch intimates that the mere fact that mandragora grows near vines is sufficient to impart lethargic properties to them:

> Moral., 15 F.: The mandragora, growing near vines and imparting its force to the wine, makes the sleep of those who drink it gentler. [†]

Further references to the narcotic properties of the plant are found in Lucian. In *Timon* we read:

> Timon, 2: You sleep as though under the influence of mandragora, neither hearing those who swear falsely nor regarding wrong-doers.

In the *Verae Historiae*, describing the City of Sleep in the Island of Dreams, he says:

> Ver. Hist., 2. 33: A wood has sprung up all around it, and the trees are lofty poppies and mandragoras.[‡]

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[*] Stratag., 5. 10. 1.

[†] Compare with this Plin., 14. 110: Mira vitium natura saporem alienum in se trahendi, quare et salicem redolent Patavinorum in palustribus vindemiae. Plutarch also alludes to the soporific properties of mandragora in Moral., 652 C.

[‡] A third passage in Lucian referring to mandragora admits of two interpretations: Adv. Ind., 23: *ἀλλ', ὦ κατάπυγον, οἴει τοσοῦτον μανδραγόραν κατακεχύσθαι αὐτοῦ, ὡς ταῦτα μὲν ἀκούειν, ἐκεῖνα δὲ μὴ εἰδέναι, οἷος μέν σου ὁ μεθ' ἡμέραν βίος, οἷοι δέ σοι πότοι, ὁποῖαι δὲ νύκτες καὶ οἵοις καὶ ἡλίκοις ξυγκαθεύδεις*; One might see here a reference to madness produced by mandragora: "do you think him so *mad* that, while he hears this, he does not investigate your record?"; but it is probably better to understand a reference rather to *lethargy;* this is of course in line with the other allusions to mandragora in Lucian.

In the spurious Demosthenis Encomium, 36, there is a reminiscence of Demosth., Philip. 4. 6, quoted above.

Apuleius relates a striking story of a youth put to sleep by mandragora; he makes a physician say:

> Metamorph., 10. 11: I did give him a drug, but it was a soporific, — the famous mandragora, celebrated for the lethargy which it is known to produce, and causing a stupor very like death.

Macrobius alludes to mandragora as a remedy for insomnia:

> Saturn., 7. 6. 7: Physicians apply cooling preparations — the juice of poppy, mandragora, and the like — to persons suffering from insomnia.*

The patristic writers, whose interest centres in the question why Rachel desired Leah's son's "mandrakes," occasionally refer incidentally to the use of mandragora in medicine, and a few mention its use as a soporific.

Cyrillus, in a commentary on Genesis, writes:

> Ad Gen. 30. 14: Mandragora is a soporific, and overcomes with a deep stupor those who partake of it.

In the commentary of Theodoretus on Canticles we have:

> Ad Cantic. 7. 13: Mandragora has the property of producing sleep, as the physicians say. . . . Those who have drunk mandragora have no perception of matters affecting the body.

These are the words of Basilius:

> Hom. 5.: Physicians induce sleep by means of mandragora.

It is evident that its soporific power was regarded by the lexicographers as its most important characteristic.

Hesychius says, under the word *μανδραγόρας*:

> A kind of plant, intoxicating and soporific.

And under the word *ὑπνοτικόν*:

> The mandragora.†

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* A number of references to the medical and botanical writers may be cited here; they are of the same import, and it does not seem necessary to quote them: (Theophr., Hist. Plant., 9. 9. 1, Caus. Plant., 6. 4. 5); Cels., 3. 18 (p. 100 Daremberg), 5. 25. 2; Diosc., 1. 571 f., 736; Plin., 25. 150; Seren. Samon., 998; Coel. Aurel., Acut. Morb., 2. 4; Alex. Trall., 1. 11 (p. 18 Andern.); Theoph. Non., Epit., capp. 25 and 145.

† Hesychius also gives *μανδραγόρας· ὁ Ζεύς*, which no one seems able to explain. Is there possibly some connection with Lucian, Timon, 2, quoted above?

In Photius, under the word *μανδραγόρας*, we read:

A soporific fruit.

Suidas says, under the same word:

A soporific fruit, or one capable of producing forgetfulness. "And he appears to have drunk a great quantity of mandragora."

The scholiast on Plato remarks on the passage from the Republic cited above:

The fruit of this plant is soporific.

And lastly Joannes Siceliota says in a note on the *Ideae* of Hermogenes:*

The mandragora is a soporific and deadly plant.

References to this property of mandragora are very frequent also in the works of mediaeval and early modern writers. They are, however, in most cases reminiscences of the classic authors, and indicate slight or no knowledge of the plant from personal observation.†

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* Walz, Rhet. Graec., vol. 6, p. 253.

† Compare the words of Pierius Valerianus, Hieroglyph., lib. 58: Suspicari possum apud scriptores nostros parum de mandragora liquere antiquissimumque eius usum et utilitatem esse temporibus nostris incognitam.

The following three passages are worth quoting as containing something out of the ordinary about the soporific power of mandragora:

Rhazes (9th–10th cent.), in Serapion's chapter on mandragora (ed. Venet., fol. 98): Dixerunt mihi quidam ex antiquis Babyloniae quod puella quaedam comedit quinque poma mandragorae et cecidit sincopicata et facta est tota rubicunda. Et quidam superveniens effudit super caput aquam nivis totiens donec surrexit.

Petrus Victorius (1499–1585), Var. Lect. (Florent., 1553), lib. 4, cap. 3: Accepi Turcarum regem alere tigrim quae catenis etiam vinctis tuto tractari non possit. Quare cum aliquo convehere velint curatores ipsius sucum mandragorae praebere solitos.

Levinus Lemnius (1505–1568), Herb. Bibl. Explic., cap. 2 (ed. Frankof., 1591, p. 9): Mihi vero quid de mandragora accidit obiter enarrabo. Quum autem aestivis mensibus (nam eo tempore poma mandragorae se proferunt) semel atque iterum in museo nostro amabilem ac speciosum eius stirpis fructum negligenter collocassem, ita somnolentus sum effectus ut aegre sopor excuti possit. Quum autem obnixe obluctarer somnolentiae illamque excusisse conarer, aegre id obtinui, nec rationem tanti veterni inire potui; tandem quum quaquaversum dimovissem oculos, obtulit se a tergo pomum mandragorae, quo amoto atque in alium locum translato factus sum alacrior atque torporem depuli omnemque oscitantiam discussi.

To these may be added a rather amusing receipt for a potion calculated to

It is probable, indeed, that the most general use of mandragora depended on its soporific properties. It was an ingredient of an enormous number of those compounds which the ancients called *anodynes* (*ἀνώδυνα*), meaning by the word remedies which relieve pain by causing sleep.* To gain an idea of how common this use was, one has only to turn, for example, to the 13th volume of Galen, in Kühn's *Medici Graeci*, and to note the number of times mandragora is mentioned in the anodynes there recommended.

Of all the statements that have come down to us regarding the uses of mandragora in ancient medicine, the most interesting are those which concern its use as an anaesthetic before surgical operations. Since anaesthetic agents began to be used in modern surgery, in the first half of the last century, the tradition in regard to their use by the ancients has been the subject of alternate wonder and disbelief. Says the New York Medical Journal of June 23, 1888 :

> It is probably with almost complete incredulity that most of us have read of the anaesthetic virtues of the mandrake as recounted by the writers of antiquity.

Most modern writers on anaesthetics have paid comparatively little attention to the tradition of their use by the ancients, and those who have undertaken to examine this tradition more fully have often made very inaccurate statements about it,† frequently taking their material at

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cause temporary mental aberration, in which the soporific properties of the plant are again referred to :

Giov. Batt. della Porta (1536-1615), Mag. Nat. (Lugd. Bat., 1650), lib. 8, cap. 2 : Morion si drachma propinetur infatuare dicit Dioscorides. Nos id faciliter fecimus vino ; quod his constat : Mandragorae radices accipito, in mustum adhuc fervens et in bullas tumens demittito, operculum indito, aptoque loco serveter binis mensibus. Quum eo indigueris potui dabis. Qui id hauserit, ubi multum profundo fuerit demersus somno, mente capitur, ut per diem non parum deliret ; ac post somnum solvitur dementia, nec damnum affert ; multumque fuerit voluptuosum videre : periclitamini (!).

* See Cels., 5. 25. 1 ; Galen, 10. 816 f. ; Aret., De Cur. Acut. Morb., 2. 5.

† Thus a writer in the Philadelphia Medical News, 66 (1895), p. 313, speaks of "Dioscorides mentioning the method of preparing the sleeping-apple" (!), a thing which Dioscorides nowhere mentions, nor any other ancient writer, so far as I am able to find. The same writer attributes the following to Herodotus : "Herodotus states that in early times anaesthesia was produced by the inhalation of bhang or hasheesh, even before the use of mandragora, and it was used for the express purpose of operative procedure. He also states that the Greeks were familiar with

second hand, and presenting it without due regard to the credibility of the sources. There is surely room for a fuller treatment of the subject than it has yet received.

The statement that physicians used mandragora before surgical operations to prevent sensation, while repeatedly mentioned by writers of later times, does not occur in any work which has come down to us of a date earlier than the first century of our era. In that century we find it in the writings of Dioscorides and the elder Pliny.

Dioscorides refers to the matter three times, as follows:

> 1. 571: Some persons boil down the roots in wine to a third, strain it, and put it away, using one cyathus in the case of persons suffering from insomnia or severe pain, or those about to be cut or cauterized, when they wish to produce anaesthesia.
>
> 1. 572 f.: Wine is also made from the bark of the root without boiling; for this purpose put three minae into one metretes of sweet wine. Three cyathi of this should be given to persons about to be cut or cauterized, as has been said above. They do not feel the pain on account of the ensuing stupor.
>
> 1. 574: This (the "third species" of mandragora) is said to produce stupor when drunk . . . or . . . eaten. The patient falls asleep in the position in which he takes it, having no sensation for three or four hours from the time at which it is administered. Physicians use this also when they are about to cut or cauterize.

These are the words of Pliny:

> 25. 150: Its narcotic power varies with the strength of the patient; an average dose is one cyathus. It is drunk . . . before incisions and punctures to remove sensation; some persons can be put to sleep merely by the odor.

Nothing further regarding this use appears in literature until several centuries after the time of Pliny. The last chapter of a work called *De Viribus Herbarum*,* formerly wrongly ascribed to Apuleius of Ma-

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the use of the drug. Literature on the subject also shows that the Scythians were in the habit of producing intoxication by means of inhalation of the vapor of hemp." The ultimate source of the above is Herod., 4. 75: *ταύτης ὦν οἱ Σκύθαι τῆς καννάβιος τὸ σπέρμα ἐπεὰν λάβωσι, ὑποδύνουσι ὑπὸ τοὺς πίλους καὶ ἔπειτα ἐπιβάλλουσι τὸ σπέρμα ἐπὶ τοὺς διαφανέας λίθους· τὸ δὲ θυμιᾶται ἐπιβαλλόμενον καὶ ἀτμίδα παρέχεται τοσαύτην ὥστε Ἑλληνικὴ οὐδεμία ἄν μιν πυρίη ἀποκρατήσειε. οἱ δὲ Σκύθαι ἀγάμενοι τῇ πυρίῃ ὠρύονται.* A very different story when traced to the fountain-head!

[*] Or De Herbarum Virtutibus, or De Medicaminibus Herbarum; the title is variously given. This passage is from chapter 131 in the edition published with "Celsus et Alii Medici" by Chr. Wechel, Parisii, 1529.

daura, and now generally assigned to the 5th century, contains a description of mandragora, in which we read these words :

> If any one is to have a member amputated, cauterized, or sawed, let him drink an ounce and a half in wine; he will sleep until the member is taken off, without either pain or sensation.

In Isidorus (6th–7th century) we meet with it again:

> Orig., 17. 9: Its bark is given in wine to those who are about to be surgically operated upon, in order that they may fall into a stupor and may not feel the pain.

Not long after Isidorus this use is referred to in the chapter on mandragora in the so-called *Dynamidia* :[*]

> Put three librae of the bark of the root into an amphora of wine, and set it away to season. Give four cyathi to a person about to undergo a surgical operation. It puts one to sleep[†] immediately after drinking it, and one does not feel the cutting.

We read the following in Serapion's (9th century) chapter on mandragora : [‡]

> If it is necessary to cut or cauterize any member, and we wish the patient not to feel the pain, let him be given *more* to drink.
>
> A measure of four obols is given to drink to a person whom it is necessary to cauterize or cut. He will not feel the cauterizing or cutting because of the stupor which ensues.
>
> Surgeons administer it when they wish to cut or burn a member.

Avicenna (980–1037 A. D.) says : [§]

> If any one wishes any of his members cut, let him drink three obols of it in wine, and it will produce stupor.

In the 11th century Bruno Astensis mentions this use:

> Expos. Super Cantic., 7. 13: It is said that those who are to be surgically

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[*] This passage is from A. Mai's Cl. Auct., vol. 7, p. 457. On the probable date of the treatise (6th–8th century) see Teuffel, Röm. Litt.^{4}, 498. 3.

[†] The text gives *saporem facit*, which seems to be a mistake for *soporem f.*

[‡] The passages are taken from the Latin translation of the Tractatus Primus, fol. 98 of the edition published at Venice in 1479.

[§] The passage is from the Latin translation of the Liber Canonis de Medicinis, etc., by Gerardus Cremonensis, published by Junta at Venice in 1555, lib. 2 cap. 366.

operated upon do not feel the cutting or burning if they have drunk the outer bark * of this fruit.

Again, Bartholomaeus Anglicus, compiler of a work called *De Proprietatibus Rerum*, written near the middle of the thirteenth century,† who devotes considerable space to mandragora, mentions this use:

> Lib. 17, cap. 104: Its bark mixed with wine is given to drink to those who are to be surgically operated upon, in order that they may fall into a stupor and may not feel the pain.

Pierius Valerianus, referred to above, who flourished in the latter part of the fifteenth and beginning of the sixteenth century, says:

> Hieroglyph., lib. 58: Physicians have had a great deal to say about the soporific power of mandragora, especially this, that its use is very common and very effective in putting those who are to be cauterized or cut into a stupor; for it is said that after they have taken a draft thus prepared (i. e. with mandragora) their sleep continues to be very heavy for about four hours, so that they feel neither the cautery nor the knife.

Lastly, we may cite Giovanni Battista della Porta (1536–1615 A. D.):

> Mag. Nat., lib. 8, cap. 1: Dioscorides says that persons will sleep right on in the position in which they have drunk mandragora, all sensation being destroyed for three or four hours from the time when the draft was administered, and that physicians use this when they wish to cauterize or cut any one.

This is the testimony of writers regarding this use of mandragora, so far as I have been able to collect it, and I feel reasonably confident that in the investigation of the question I have not overlooked any reference that would have any considerable weight. I have included the passages from the mediaeval writers for the reason that if it can be shown that it is probable that mandragora was used for this purpose in the middle ages, our belief in this use in antiquity — which is attested by so few passages from ancient writers — will be greatly strengthened.

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* It is not clear just what Bruno means by *exteriorem huius pomi corticem.* He uses the word *pomi* evidently because he is commenting on *dudâ'îm, i. e.*, ***μανδραγορῶν μῆλα.*** Probably *exteriorem huius pomi corticem* is a condensed expression for *exteriorem corticem radicis herbae e qua nascitur hoc pomum*, in which case *corticem* has its usual meaning of *bark*; otherwise we should have to translate "the outer rind of this fruit," which could only be a mistake of the writer, referring to the fruit the virtues elsewhere recorded of the bark.

† See the introduction to the extracts translated by Robert Steele, Medieval Lore (London, 1893), p. 1.

From the passages above cited it appears that there has never been a time since the first century of the Christian era when this use of mandragora was wholly unknown to *writers*. But what is the evidence that throughout this time *physicians were actually using the plant as an anaesthetic?* Is there any such evidence, or must we believe that they simply handed on this story of its virtues without ever testing it? We shall best arrive at a conclusion by examining the passages in these several authors separately. For the sake of convenience I shall begin with the one cited last.

1. Giovanni Battista della Porta. He simply quotes Dioscorides, and names him as his authority. Besides, the words I have quoted are taken from the latter's description of his "third species," the identification of which, as we have seen, is very doubtful.

2. Valerianus. In the note on p. 494, above, I have quoted this author as saying that the ancient uses of mandragora were probably little known in his time. He cites Dioscorides and Pliny, and he had perhaps read elsewhere of the use of the plant to produce anaesthesia,* but there is no reason to suppose that he had any personal knowledge of its use for this purpose.

3. Bartholomaeus Anglicus. The work from which our passage is taken does not profess to be more than a compilation from many sources. In his chapter on mandragora the author cites Dioscorides, Pliny, and later writers. The chapter begins with an excerpt from Isidorus, to which the passage I have quoted belongs. There is no independent evidence here.

4. Bruno Astensis. The words of this writer do not indicate that he had any personal knowledge of this use of the plant. It is evident from the wording of the passage quoted that he took it from Isidorus.

5 and 6. Serapion and Avicenna. It will prove convenient to treat these writers in their chronological order.

The works of Serapion, a Syrian, were early translated into Arabic, and came down to posterity as the oldest treatise on medicine in the Arabic language.† The bulk of his chapter on the mandragora is an almost word for word translation from Dioscorides; a few lines of it appear to be taken from Galen, a few resemble the pseudo-Apuleian chapter, and a few are of wholly uncertain source; all the passages on the use of the plant as an anaesthetic are based upon Dioscorides.

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* His remark, on the authority of "physicians" (*medici multa tradiderunt*), that this use of mandragora was very common (*plurimus*) is worthy of note. But he does not cite the names of his authorities, and it hardly seems likely that he had any particular physicians in mind.

† See Neub.-Pag., vol. 1, p. 596.

Two words in one of these passages are worthy of especial notice. The passage in Dioscorides,

> Some persons boil down the roots in wine to a third, strain it, and put it away, using one cyathus in the case of persons suffering from insomnia or severe pain, or those about to be cut or cauterized, when they wish to produce anaesthesia,

is given by Serapion thus:

> Some persons boil down the root of the mandragora with wine until a third part of it disappears,* strain it, and put it away. They take a quantity of one obol and give it for insomnia and to allay pain. And if it is necessary to cut or cauterize any member, and *we wish* the patient not to feel the pain, let him be given *more* to drink.

The two words to which I wish to call attention are *we wish* (*volumus*) and *more* (*plus*). Dioscorides says:

> Some use a cyathus for insomnia and severe pain, and when they wish to produce anaesthesia before cutting or cauterizing;

while Serapion says:

> Some take an obol and use it for insomnia and severe pain, and when *we wish* to produce anaesthesia, let *more* be given.

Serapion, then, if the Latin translator has correctly reproduced his words, used a verb in the first person (*volumus*) where he found a verb of the third person (*βούλονται*) in Dioscorides; moreover, *he himself added* the word *more* (*plus*), for Dioscorides says nothing about any distinction in quantity here. This goes to show that Serapion did not depend entirely upon Dioscorides for this information; and since he was able to add something to the words of Dioscorides, there seems ground for believing that he knew of this use of mandragora from personal experience. This view is confirmed by an examination of Avicenna's chapter on the plant.

The works of Avicenna, the most celebrated Arabian physician and philosopher, were in common use during the middle ages even in the universities of Europe. His chief sources in general are Greek; the sources of his account of the mandragora are not so easy to trace as those of Serapion's, but it is evident that he draws here principally from Dioscorides and Galen; a very considerable part of the chapter shows a marked resemblance to the chapter on mandragora appended to the work of Pseudo-Apuleius.

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* There is a discrepancy here between Dioscorides and Serapion, perhaps due to a mistranslation.

The suggestion made above, that in the word *plus* used by Serapion we have evidence of an addition to Dioscorides, is nicely confirmed by an examination of Avicenna's words. Serapion says that one who has taken mandragora before an operation does not feel the pain *because of the stupor which ensues* (*propter subât quod accidit*); furthermore, that when physicians wish the patients not to feel the pain (i. e., when they wish to produce *subât* [an Arabic word which the translator was not able to give in Latin]) they must administer more than when they merely wish to relieve insomnia (i. e., merely wish to produce sleep [*somnus*]). One immediately infers that there is a distinction between *subât* and *somnus;* and the Arabic bears us out: the word *subât*, primarily meaning *rest*, while it may occasionally be used of ordinary sleep, properly denotes a *deep sleep* or *stupor.** And that these writers had this distinction in mind is conclusively proven by two passages from Avicenna:

> It produces stupor (*subât*), and it produces sleep (*somnum*).
> To produce *stupor* MORE is given, to produce *sleep* LESS.

Furthermore, the repetition of Serapion's *more* (*plus*) here, with the *less* (*minus*) in the following clause removes any suspicion that *plus* got into the Latin version of Serapion merely through the carelessness of the translator.†

In the writings of these two physicians, then, we have a distinct addition to the words of Dioscorides and his successors who wrote about the plant, and good evidence, I think, that they both had a personal knowledge of its use as an anaesthetic.

7. Dynamidia. Little is known about the authorship of this work, which Teuffel includes in a list of Latin writings derived from Greek sources composed on the border-line of antiquity and the middle ages.‡ The chapter on mandragora is drawn mostly from Dioscorides, and the statement about its use as an anaesthetic is manifestly taken from him. There is no reason to think that we have here any independent evidence.

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* For the proper interpretation of this word, and for other helpful suggestions, I am indebted to Professor George F. Moore, of Harvard University.

† Some one might object here that, granting that Serapion wrote *plus*, Avicenna probably copied his account from him. I am confident that any one who will compare the accounts of mandragora given by these writers with each other and with their sources will agree with me that Avicenna is the more independent of the two in his treatment of the subject. It seems therefore very unlikely that he would have blindly taken from his predecessor a fact of such importance in connection with the plant.

‡ See footnote, p. 515.

8. Isidorus. This writer used as the chief source of the 17th book of the *Origines*, in which the passage above quoted occurs, a Latin translation of Dioscorides.[*] The section on mandragora is brief, containing nothing not found in Dioscorides except a few unimportant comments. It is to be observed that, while the sentence on the use of the plant as an anaesthetic seems to be simply an excerpt from Dioscorides, yet this is the *only* medicinal use which the author thinks it worth while to mention.

9. Pseudo-Apuleius. The authorship of this work is uncertain, and the chapter on mandragora is evidently an interpolation.[†] I have had no means of ascertaining when it — the last chapter in the work —

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[*] See Stadler, Archiv für Lat. Lexicog., **10**, p. 403 ff.

[†] It is a recognized fact that this work is interpolated in innumerable places throughout; certain parts of almost any chapter can be picked out as interpolations (see Schanz, Röm. Litt., Teil 3, p. 114 f.). The chapter on mandragora stands apart from all the rest, and I am persuaded that it is a later addition to the already interpolated work.

From information given by Payne, English Medicine in the Anglo-Saxon Times (London, 1904), p. 72 ff., it is evident that Anglo-Saxon versions of the pseudo-Apuleian work present the chapter on mandragora in a form very different from that which appears in Wechel's edition, giving many things that are not found there, and omitting some that are. This goes still further to show that the latter part of the work was tampered with.

Several things have led me to this conclusion: (1) The chapter does not appear in all the printed editions, which indicates that it is lacking in some of the manuscripts; for example, the Basel edition of 1597 (where the De Medicaminibus Herbarum is printed with the works of Apuleius of Madaura) has no chapter on mandragora; in the Paris edition of 1529 (Wechel) it is the last chapter in the book. So with various other editions: see Meyer, vol. 2, p. 316 ff. The information about the manuscripts contained in Koebert's dissertation (see footnote, p. 488, for the title) goes to confirm this (see next footnote, p. 521). (2) The style of this chapter as it appears in Wechel's edition differs in several points from the rest of the work: (*a*) Here we have a connected description of the plant and its virtues, without frequent repetition of the name; elsewhere the account of the virtues of the plant under discussion is divided into several *tituli* (the term used by Ackermann; see Koebert, p. IV, note), at the beginning of which the name of the plant is almost invariably repeated. (*b*) There is no list of synonyms at the beginning of this chapter, as there usually is elsewhere. (*c*) While conditional sentences with *si* (like *si dabis . . . sanabitur*) are rare in the other chapters, this one has no less than six *si* clauses. (3) The interpolated plant descriptions throughout the rest of the work show a marked resemblance to Dioscorides (see Rose, Hermes, **8** [1874], p. 57, and Koebert, p. 39), while the description of the mandragora shows a wide divergence from him; it records the resemblance of the root to the human form, says that its color is ashen (*cinericius*), compares the leaves of the "male" to those of *blitum*, and of the "female" to those of lettuce. None of these things is stated by Dioscorides or his interpolator.

became attached to the rest. The noteworthy fact about it is that it corresponds throughout to Avicenna's chapter on mandragora.* The

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* The resemblance between Avicenna's account of mandragora and this has been referred to above. The following table, based upon a very careful comparison of this chapter with Serapion and Avicenna, will make it clearer:

(1)	Facts recorded about mandragora in this chapter	25
(2)	Facts common to this chapter, Serapion, and Avicenna	9
(3)	In this chapter and Avicenna and not in Serapion	6
(4)	In this chapter and Avicenna and nowhere else	5
(5)	In this chapter and Serapion and not in Avicenna	3
(6)	In this chapter and Serapion and nowhere else	0
(7)	In this chapter and in neither Serapion nor Avicenna	4
(8)	In this chapter and nowhere else	3
(9)	Partly the same in this chapter, Serapion, and Avicenna	2
(10)	Doubtful	1

In this table, (4) and (6) seem to me most significant; it is not necessary to suppose that the author of this chapter used Serapion at all, but it can hardly be accidental that he and Avicenna have five facts in common which are found nowhere else. As regards (8), it is not necessary to attribute any originality to the author of this chapter; of the three facts which he states and which no other writer seems to have recorded ([*a*] cortex est subtilis, [*b*] folia habent virtutem stringendi, [*c*] poma curant erysipelas), [*a*] is perhaps a mistake, and [*b*] and [*c*] may easily have been inferred from virtues recorded by other writers.

Koebert, pp. 1–10, describes five manuscripts of Pseudo-Apuleius: (1) *Monacensis*, 6th or 7th cent., (2) *Vratislaviensis*, 9th or 10th cent., (3) *Cassinensis*, 9th or 10th cent., (4) *Lugdunensis*, 15th cent., (5) *Lugdunensis alter* (*Lat. Voss. E.* 13), 15th cent. (The numbers in parentheses are mine, and the manuscripts, it will be observed, are arranged in chronological order; Koebert numbers the manuscripts differently on pp. 1–3 and 3–10, and I have been unable to discover the principle on which he has arranged either list.) (4) and (5) are fragmentary, and do not contain the chapter on mandragora (chapter 131) (1) has only the beginning of the chapter on mandragora (Koebert, p. 1); (2) has chapter 131, but Koebert says of the manuscript (p. 5): Hic continet 118 folia, quorum 1–113 integra, reliqua valde corrupta sunt. As the mandragora chapter is always the last, it is evidently among the *corrupta*. (3) has chapter 131 (Koebert, p. 2), but Koebert says that he had examined only certain chapters in this manuscript, when the monastery which owns it was closed to *res profanae* for Holy Week, and that he had to leave with the work unfinished. He did not examine chapter 131. There is thus nothing in Koebert's account of the manuscripts which makes it appear improbable that the bulk of the chapter on mandragora was joined to the pseudo-Apuleian work after the time of Avicenna (A. D. 980–1037).

What is most needed in order to prove that the chapter was drawn from Avicenna is information as to the time at which it first appeared in the manuscripts, — information which I am unable to get in this country. I have referred in the preceding note to the lack of a list of synonyms at the beginning of the chapter.

latter appears to be by far the more original of the two, and I believe that investigation will prove that this chapter was added to the pseudo-Apuleian work after the publication of Avicenna's writings, and is based on him.* The emphatic way in which the anaesthetic virtues of mandragora are proclaimed attracts attention; but the language is somewhat grotesque — *if any one is to have a member amputated* (? : *emutilandum*) *or burned or sawed* (!) — and it sounds more like that of an enthusiastic scholar who has stumbled on a very remarkable statement which he is eager to promulgate, than of a competent medical writer. We can hardly base any argument for this use of mandragora in antiquity on this passage.

10 and 11. Pliny and Dioscorides. The lives of these writers were contemporary, though Pliny appears to have finished his work a little later than Dioscorides.† The multitude of striking parallels in their writings shows that they stand in close relation to each other. Dioscorides has been thought by some to have used Pliny, and others have thought that Pliny drew from Dioscorides. The correct view is beyond doubt that of Wellmann, that they based their botanical writings upon common sources, and chiefly upon Sextius Niger, who flourished in the first half of the first century of the Christian era. The work of Sextius, however, appears to have been a compilation, so that we are unable to trace to their original source the parts which Dioscorides and Pliny took from him.‡

A careful comparison of the mandragora passages in Dioscorides and Pliny makes it evident that they are among those which the two writers took from a common source. As has been pointed out, the passages essentially agree; the slight discrepancies have been discussed above.

The interpolator of the synonyms in the rest of the work might easily have obtained them for mandragora by consulting Dioscorides. That they are not given indicates that the chapter did not belong to the work when the synonyms were added to the other chapters.

* Even if it should be proven that Avicenna drew from this chapter, instead of *vice versa*, it would not at all weaken the force of the argument that Avicenna knew of this use of mandragora by personal experience, for there is no mention here of a distinction in the quantity necessary to produce sleep and stupor respectively. But the very character of the pseudo-Apuleian work — it was written and circulated for the layman's use, to enable him to do without the services of the physicians, *qui saeviores ipsis morbis existant* (Koebert, p. III) — makes it highly improbable that a medical writer would have used it.

† Meyer, vol. 2, p. 100.

‡ On the whole question see Wellmann, Hermes, **24** (1889), p. 530 ff.

The statement about the use of mandragora as an anaesthetic as it appears in Pliny is so unemphatic, so purely incidental, and Pliny's generally uncritical habits are so well known, that it furnishes no argument for its truth.

On the other hand, how strong an argument for its truth is the fact that it is found in Dioscorides? The claim to *originality* made in the preface of his work * must, in the light of Wellmann's arguments, be admitted to be considerably exaggerated. Yet he is not to be charged with being grossly uncritical in the admission of material. A careful reading of all his writings reveals a decidedly conservative spirit, tempered in the main by sound judgment. "Child of his time" (Wellmann) he undoubtedly was, and it is somewhat disappointing to find in his work a certain amount of nonsense mixed in here and there alongside of perfectly sound doctrine. Yet where he makes mention of amulets, "sympathetic" † remedies, and the like, his tendency is to report them with the addition of "they say" (ἱστοροῦσι), "it is said" (ἱστορεῖται), "some record" (γράφουσί τινες), as saving clauses. Doubtless many of the bogus remedies are mentioned only because of the feeling common to writers of his time that everything pertaining to a given subject must be brought in; witness his reason for describing a very great number of ways of preparing wine:

> 1. 713: Not because the use of them is extensive or necessary, but in order that our report may not be found wanting in regard to any of them.

Furthermore, it is very probable that he was a practising physician, ‡ and in that case he must have had an especial interest in such a matter as this, with an excellent opportunity for observation. So that his unreserved admission into his work of the statements here recorded should not be without weight in our estimate of the evidence before us.

The use of the plant is stated three times in his chapter on mandragora, as we have seen. The third of these must be disregarded, since all that he says about the "third species" of mandragora is given on hearsay; the plant referred to has not been surely identified, and this use is nowhere else asserted of a plant resembling and yet distinct from the mandragora.* What is said about this use in the other two passages in Dioscorides has every appearance of being based on fact. The two ways of preparing the liquor — with and without boiling — are carefully described, and the proper doses are stated, — one cyathus of the former, three of the latter. It seems conclusive to me that whoever originally wrote the statements which appear here (and I recognize in Dioscorides not the original author of them, but rather the intelligent transmitter) *knew* of this use of the plant.

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* Diosc., 1. 4.

† On the nature of and ancient belief in "sympathetic" remedies (the word in this sense is not defined in the Century or Standard Dictionaries), see Riess, in Pauly-Wissowa's Realencyclopädie, vol. 1, col. 36.

‡ This is the opinion of Sprengel (Praef. ed. Diosc., p. x) and Meyer (vol. 2, p. 100), and is thought probable by Häser (vol. 1, p. 302).

At the beginning of this investigation I had hoped to find somewhere in classical or later literature an account of the use of anaesthetics in the case of some definite individual, such, for example, as the account of the operation on Marius for varicose veins, from which it is at once apparent that the patient took no anaesthetic.† I am now quite certain that no such testimony is to be found. We must base our faith in the tradition upon presumptive evidence.

To recapitulate: Eleven writers state that mandragora was used for this purpose; all of them probably had access to earlier writings in which the statement appeared. In the case of eight of these eleven — Giovanni Battista della Porta, Valerianus, Bartholomaeus Anglicus, Bruno Astensis, the author of the Dynamidia, Isidorus, the author of the chapter in the work of Pseudo-Apuleius, Pliny — the reputation of the writer, or the manner in which this use is stated, is such as to render the testimony of no importance. Regarding the other three, we may draw the following conclusions: The words of Serapion and Avicenna, on account of the important distinction observed in the quantities necessary to produce sleep and stupor respectively, indicate very strongly that the plant was used for this purpose in their time; and the fact that this seems probable for that time gives us much more confidence in the words of the *ancient* writers who record the use. The general tone of the writings of Dioscorides is sober and reliable; for this reason the appearance of the statement in his work gives to it to some extent a guarantee of truthfulness. Furthermore, the manner in which this use is stated in Dioscorides makes it all but certain that the one who first wrote the statement founded his record on fact.

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* R. Kobert is inexact when he says (Ueber den Zustand der Arzneikunde vor 18 Jahrhunderten, Halle, 1887, p. 21), "Jedenfalls führt Dioscorides von mehreren Solanaceen ausdrücklich an, dass sie die Schmerzempfindung so stark herabsetzen, dass man Glieder amputieren und das Glüheisen anwenden kann, ohne dass der Patient erheblich dabei zu leiden hätte." Dioscorides mentions no plant in this connection except the true mandragora and the doubtful "third species."

† Cicero, Tusc. Disp., 2. 35 and 2. 53; Plutarch, Marius, 6. Seneca writes (Epist. Moral., 10. 2 [78]), without giving the name, of a man who *read* during a similar operation: Tibi fortissimus quisque et victor doloris occurrat: ille qui cum varices exsecandas praeberet legere librum perseveravit.

While the writers of antiquity have not very much to say about the employment of other anaesthetic agents, yet there are a few passages bearing on the subject which go to confirm the above conclusions.

Both Dioscorides and Pliny mention local anaesthesia:

Diosc., 18. 17: The stone *memphites* is found in Egypt, near Memphis, being of the size of a pebble, brilliant and variegated. It is said that when it is reduced to powder and applied to parts of the body which are to be cut or cauterized it produces a safe anaesthesia.

Plin., 36. 56: It is called *memphites* from the place where it is found, and it sparkles like a gem. It is reduced to powder, mixed with vinegar, and applied to parts of the body which are about to be cauterized or cut · the body is thus benumbed, and does not feel the pain.

These two passages, which must be taken from a common source, agree perfectly; Dioscorides, however, adds that the anaesthetic is *safe* (ἀκίνδυνον). This is a most excellent indication that the *physician* Dioscorides understood better and was paying closer attention to the subject in hand than Pliny. For if he copied the word ἀκίνδυνον from the common source, while Pliny disregarded it wholly, it is manifest that Dioscorides, in noting this important point, had a more intelligent idea of what he was recording. If, on the other hand, he added the word himself, the same thing is indicated, and it is further plain that he personally knew of the use of anaesthetics, since he thus distinguishes between safe and dangerous ones.* It has often been observed by those who have written of mandragora that it was *dangerous*.† Perhaps we may even think that the author who wrote of this *safe* anaesthetic had mandragora in mind as a dangerous one.

Pliny writes about the crocodile:

28. 110: The ashes of the hide of either species mixed with vinegar and applied to parts of the body which are to be cut, or the smell of the burnt hide, removes all sense of the knife.

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* This argument holds good regarding the original author of this statement, whether Dioscorides or some one earlier. One who writes of safe anaesthetics must have known of dangerous ones; but the fact that some were safe and others dangerous could have been learned only by experiment.

† (Theophr., Caus. Plant., 6. 4. 5); Diosc., 1. 571; Plin., 25. 150.

In the book *De Euporistis*, which has commonly been attributed to Dioscorides,* we have the most direct testimony regarding the practice of using anaesthetics in surgery. Here, in enumerating several compounds used to induce sleep, the author remarks:

> Diosc., 2. 100: We also use soporifics in the case of persons whose suffering is continued, — when one who is about to be cut or cauterized wishes to become insensible to the pain.

The use of the verb in the first person (*we use*) is to be noted.

We are told that a preparation of hemp was used in China in the early part of the Christian era to effect painless surgery. The following passage occurs in a sixteenth-century Chinese work on medicine in the biography of Hoa-tho, an eminent physician of the third century of our era: †

> When he determined that it was necessary to employ acupuncture, he applied it in two or three places; and so with the *moxa*, if that was indicated by the nature of the affection to be treated. But if the disease resided in parts upon which the needle, the *moxa*, or liquid medicaments could not operate — for example, in the bones, or the marrow of the bones, in the stomach or the intestines — *he gave the patient a preparation of hemp*. . . . and after a few moments he became as insensible as if he were drunk or dead. Then, as the case required, he performed operations, incisions or amputations, and removed the cause of the malady; then he brought together and secured the tissues, and applied liniments. After a certain number of days the patient recovered, without having experienced during the operation the slightest pain.

Unfortunately we do not know the date at which the above biography was written, — whether it was composed shortly after the death of Hoa-tho, when the truth of the recorded facts could be known accurately, or whether we have here simply a record of a distant tradition. However this is, it is another indication that antiquity was busy seeking relief from the pain of surgical operations.‡

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* Though his authorship of it is questioned. See Christ, Gr. Litt.^{3}, p. 862.

† A copy of this work is preserved at Paris in the Bibliothèque Nationale. The passage quoted is given by Julien in Comptes Rendus des Séances de l'Académie des Sciences, **28** (1849), p. 197.

It was perhaps confusion of this account of the use of hemp with the one given by Herodotus which led to the extravagant statements cited in the footnote, p. 495 f.

‡ No other passages in ancient writers, so far as I have been able to ascertain, refer certainly to the use of anaesthetics. Two additional passages from Pliny may be cited as bearing indirectly on local anaesthesia:

A very strong argument for the truth of the statements about the use of the plant as an anaesthetic is to be drawn from certain experiments made a few years ago by the English physician and medical writer, Dr. Benjamin Ward Richardson.* Having procured a specimen of mandragora root from Greece, he made a tincture by four weeks' maceration of it, in fine powder, in alcohol diluted with five times its bulk of water, attempting to reproduce as nearly as possible the ancient wine of mandragora (*μανδραγορίτης*). This product he found to exhibit "the most active properties — properties faithfully represented by the ancients in their observations." † I give his conclusions in his own words: ‡

> The wine of mandragora is a general anaesthetic of the most potent quality. The action no doubt depends on the presence of an alkaloid which is like, if not identical with, atropine, and from it an alkaloid could be extracted which might be used medicinally, and which would, I have no doubt, be one of the most active anaesthetics we have yet discovered. From the circumstance that the heart continues to beat after the respiration has ceased § we may infer that as a general anaesthetic the alkaloid might, under

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20. 125 (of *eruca*): Aiunt verbera subituris potum ex vino duritiam quandam contra sensum induere.

25. 24: Tradat M. Varro Servium Clodium, equitem Romanum, magnitudine doloris in podagra coactum veneno crura perunxisse, et postea caruisse sensu omni aeque quam dolore in ea parte corporis.

Mention may also be made here of the soporific compound for use in surgery described by Theodoric, a famous surgeon of the 13th century, in his Cyrurgia, lib. 4, cap. 8. After prescribing the measure of the several ingredients (opium, belladonna, hyoscyamus, mandragora, and others) he says: Haec omnia in unum commisce in vase aeneo ac deinde in istud mitte spongiam . . . et naribus apponatur quousque somnum capiat qui incidendus est, et sic fiat cyrurgia. Qua peracta, ut excitetur aliam spongiam in aceto infusam frequenter ad nares ponas.

Other references to the use of anaesthetics in mediaeval and early modern times could probably be collected.

* These experiments are fully described in the New York Medical Journal, **47** (1888), p. 684 f., and in Dr. Richardson's publication, the Asclepiad, **5** (1888), p. 174 ff.

† Kobert, Ueber den Zustand der Arzneikunde vor 18 Jahrhunderten, Halle, 1887, p. 21 f., says that he was persuaded to believe in the ancient statements about the use of mandragora as an anaesthetic by certain experiments with *hyoscin*, and by the information conveyed to him by the Japanese pharmacologist Takahashi that a solanaceous plant is successfully used in surgery in Japan to-day.

‡ Asclepiad, **5** (1888), p. 183.

§ The experiments were mostly upon small animals, — pigeons, rabbits, and the like; Dr. Richardson also experimented upon human subjects "in doses not sufficient to produce actual narcotism."

necessity, be once more employed, as in the olden time, to deaden the pain of a surgical operation, and that, too, with comparatively little risk to life.

The arguments presented are certainly sufficient to warrant the belief that mandragora was used as an anaesthetic in antiquity. In order to arrive at some idea of *how common* the use of anaesthetics was, I have examined a considerable number of passages (about a hundred and fifty in all) in Greek and Roman literature which bear on surgery, — most of them incidentally. The majority of these are so general that no information can be obtained from them; some of them show plainly that the author knew nothing about anaesthesia, and none of them give evidence that the use of anaesthetics was *common* in ancient surgery. The most important of these passages I submit here:

Hipp., 1. 60: When the operation consists of a single incision it should be made quickly; for, owing to the fact that the patients suffer pain, the time of the operation ought to be as short as possible.

The words which follow these in the text show still further that the author had no thought of anaesthesia when he wrote the passage.

Xenoph., Mem., 1. 2. 54: They allow physicians to cut and cauterize them with sufferings and pains.

Plato, Pol., 293 A: The physicians . . . if . . . they heal us, cutting or cauterizing or inflicting some other painful thing upon us.

Id., Gorg., 479 A: Like a child fearing cutting and cauterizing because it is painful.

Id., Ibid., 480 C: Not to flinch, but to close the eyes and submit manfully, just as one allows a physician to cut and cauterize one, aiming at the good result, not reckoning the pain.

Aristotle, Probl., A. 35: Is the incision easier and freer from pain because iron takes a keener edge?

Cic., Tusc. Disp., 5. 73: Shall Epicurus be allowed to say that the wise man finds no time, even though he be burned, or wrenched, or cut, when he cannot exclaim, "How I despise (these pains)!" ?

Plin., 29. 13: It is said that he (Archagathus, the first Greek physician at Rome) was a skilful surgeon, and that his coming was wonderfully acceptable at first, but that a little later they changed his name to *the Executioner*, from his cruelty in cutting and cauterizing.

Cic., Tusc. Disp., 2. 35: When Gaius Marius had his varicose veins cut he suffered pain.

Id., Ibid., 2. 53: Gaius Marius, a man of rustic breeding, but undoubtedly a *man*, when he was about to have his veins cut, as I have said above, at the outset refused to be bound, yet no one before Marius is said to have been cut without binding. . . . And . . . Marius gave evidence, too, that the bite of the pain was keen.

Propert., Eleg , 1. 1. 27: Bravely will we endure both the knife and the cruel cauteries.

Ovid, Rem. Amor., 229: To restore the body, you will endure the knife aud the cauteries.

Id., Heroid., 20. 183: Others, that they may be well, endure the knife and the cauteries.

Seneca, Ad Marciam, 22. 3: Add . . . the torments of the physicians, who remove bones from living bodies, putting their whole hands into the bowels, and treating the organs with divers pains.

Plut., Moral., 74 D: We must imitate the physicians; for when they perform a surgical operation they do not leave the wounded member in suffering and pain, but soothe it gently and foment it.

Martial, Epig., 11. 84. 5 f.: More gently does Alcon cut a strangulated hernia, and hew broken bones with his rude hand.*

Dio Chrysost., Orat. 14: If ever the physician sees fit to bind the patient, he is straightway bound; and if he sees fit to cut or cauterize him, the knife and cauteries are applied. . . . Often the very servants of the sick man bind their master, and bring the fire to cauterize him, and furnish other assistance.

Aristides, Orat. 24: Finally, the physicians began at my chest and carved (*κατέτεμνον*) me all the way down to my bladder. And when the cupping-glasses took hold, my breath left me completely, and a pain benumbing and very hard to bear went through me, and everything was wet with blood, and I was filled with anguish.

Tertull., Scorpiac., cap. 5: Being cut and cauterized . . . is not an evil, for the reason that it brings useful pains. . . . The patient who howls and groans and bellows in the hands of his physician will presently load those same hands with pay, and call them the best of operators, and no longer say that they are cruel.

Themist., Orat. 22: Just as physicians often leave the knife alone, and by applying certain drugs heal the malady without pain.

From the last clause it follows that cure with the knife *would* be attended by pain.

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* That is, Alcon, bungler that he is, performs a surgical operation with less pain to the patient than that caused his customers by the barber Antiochus.

None of the passages which I have quoted implies in any way that the writer was familiar with the use of anaesthetics. I have found two passages upon which some argument for a knowledge of their use on the part of the writer might be founded:

> Plut., Moral., 102 D: "May we not be ill at all," says the Academician Crantor, "but if so, let us have our senses, even if one of our members be cut or taken away. For that state of insensibility does not come to one without great cost, since in that case one is likely to become brutish not only in body, but also in mind."

At first sight it seems that "that state of insensibility" may be the anaesthesia which we are looking for; but when the passage is examined in relation to the context, the word seems rather to refer to *apathy* or *indifference* to pain (ἀπάθειαν, further up in the passage of which I have quoted the latter part) than to anaesthesia produced by drugs.

The other passage is from Lucian:

> Apol., 2: Now is the time for me to be quiet and to endure cutting and cauterizing, if need be, for my welfare; and for you to sprinkle on the drugs, holding in readiness the knife and the flaming cautery.

Here it is *possible* that we have a reference to local anaesthesia. If the drugs were to be applied *after* the cutting and burning, it would be clear that we had to do with something applied to the lacerated parts to aid the healing process, a custom which appears to have been very old.* But it is to be noted that Lucian instructs his friend *first* to sprinkle on the drugs, and *then* to hold the knife and cautery in readiness.

After carefully weighing all the evidence, I think that we may safely conclude (1) that the use of mandragora as an anaesthetic before surgical operations was known to, and to some extent employed by physicians at least as early as the first century of the Christian era; (2) that mandragora was the principal, and almost the only anaesthetic of antiquity; (3) that the use of anaesthetics never became very general either in ancient or mediaeval times, this being doubtless due to the danger attendant upon their use. It was this danger which led the medical writers to urge caution in the use of all *anodynes.*†

WORCESTER, MASSACHUSETTS.

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* Compare Hom., Il., 11. 830, 844 ff.; and Plut., Moral., 74 D, quoted above.

† See Celsus, 5. 25. 1; Galen, 10. 816.

APPENDIX.

Text of Greek and Latin passages translated above.

(P. 489.) Theophr., 9. 8. 8: *περιγράφειν δὲ καὶ τὸν μανδραγοραν εἰς τρὶς ξίφει, τέμνειν δὲ πρὸς ἑσπέραν βλέποντα· τὸν δ᾽ ἕτερον κύκλῳ περιορχεῖσθαι καὶ λέγειν ὡς πλεῖστα περὶ ἀφροδισίων.*

(P. 490.) Plin., 25. 148: Effosuri cavent contrarium ventum et tribus circulis ante gladio circumscribunt, postea fodiunt ad occasum spectantes.

(P. 493.) Schmidel, § 53: Die Veneris ante Solis exortum auribus gossypio oppletis, ceraque aut pice occlusis exire iubent, crucis figuram terno tripodis circumductu plantae inscribere, tum circumfodere totam, sic ut terrae non nisi extrema fibra obhaereat; ita solicitatam caudae canis alligere funiculo, statimque aufugere, frusto panis cani obverso; id illum appetentem nitendo radicem evellere, moxque audito eius eiaculatu concidere mortuum.

(P. 495.) Schmidel, § 53: Ex urina quam homo innocenter ad suspendium furti crimine damnatus inter extrema vitae effuderit, herbam nasci aiunt sub patibulo, foliis latis, flore croceo, radice hominem examussim simulante, pilis quoque et ipsis sexuum notis apparentibus. Nonnulli adiciunt, vitalem fuisse subterraneum hunc foetum. . . . Nostri Alrunam vocant. Eam eruere rem esse plenissimam periculi: ab eruta enim radice gemitum edi tam terribilem, ut audienti fossori moriendum sit e vestigio.

(P. 496.) Apollon. Rhod., Argon., 3. 851 ff.:

851. *πρωτοφυὲς τόγ᾽ ἀνέσχε. καταστάξαντος ἔραζε*
αἰετοῦ ὠμηστέω κνημοῖς ἔνι Καυκασίοισιν
αἱματόεντ᾽ ἰχῶρα Προμηθῆος μογεροῖο.
τοῦ δ᾽ ἤτοι ἄνθος μὲν ὅσον πήχυιον ὕπερθεν
χροιῇ Κωρυκίῳ ἴκελον κρόκῳ ἐξεφαάνθη,
καυλοῖσιν διδύμοισιν ἐπήορον· ἡ δ᾽ ἐνὶ γαίῃ
σαρκὶ νεοτμήτῳ ἐναλιγκίη ἔπλετο ῥίζα.

861. *μυκηθμῷ δ᾽ ὑπένερθεν ἐρεμνὴ σείετο γαῖα*
ῥίζης τεμνομένης Τιτηνίδος· ἔστενε δ᾽ αὐτὸς
Ἰαπετοῖο πάις ὀδύνῃ περὶ θυμὸν ἀλύων.

(P. 497.) Pseudo-Orpheus, Lith., 363 ff.:

363. *ἀενάῳ δ᾽ ἐνὶ πέτρον ἐχέφρονα πίδακι λούων.*
φάρεσιν ἐν μαλακοῖσιν. ἅτε βρέφος. ἀλδήσσασκε.

368. χέρσιν ἑαῖς ἀτίταλλε, θεουδέα πέτρον ἀείρας,
μητέρι νήπιον υἱὸν ἐοικὼς ἀγκὰς ἐχούσῃ.
καὶ σὺ δ', ἐπὴν ἐθέλῃς ὄπα δαιμονίην ἐπακοῦσαι,
ὧδ' ἔρδειν, ἵνα θαῦμα μετὰ φρεσὶ σῇσι δαείης·
ὁππότε γάρ μιν πάγχυ κάμῃς ἐνὶ χείρεσι πάλλων,
ἐξαπίνης ὄρσει νεογιλοῦ παιδὸς ἀϋτήν,
μαίης ἐν κόλπῳ κεκληγότος ἀμφὶ γάλακτι.
χρὴ δέ σε τετληῶτι νόῳ ἀκταινέμεν αἰεί,

378. . . . τόλμα δὲ θεοπροπίην ἐρεείνειν·
πάντα γὰρ ἐξερέει νημερτέα.

(P. 501.) Aretaeus, De Caus. et Sign. Diut. Morb., 1. 6. init : ἐκμαίνει δὲ καὶ τῶν ἐδεστῶν μετεξέτερα, ἢ μανδραγόρη, ἢ ὑοσκύαμος, ἀλλ' οὔτι πω μανίη τάδε κικλήσκεται.

(P. 501.) Theophr., 9. 9. 1: τὴν δὲ ῥίζαν χρήσιμον (sc. εἶναί φασι) . . . πρὸς φίλτρα.

(P. 501.) Diosc., 1. 570: οἱ δὲ Κιρκαίαν καλοῦσιν, ἐπειδὴ δοκεῖ ἡ ῥίζα φίλτρων εἶναι ποιητική.

(P. 502.) Julian, Epist. 23: εἶτα μετὰ τοῦ φιλάνδρου τὸ φιλόθεον τίς ἐν γυναικὶ δεύτερον τίθησι, καὶ οὐ φανεῖται πολὺν πάνυ τὸν μανδραγόραν ἐκπεπωκώς ;

(P. 502.) Physiologus, cap. 4: ἡ θῆλυς δὲ ζητεῖ βοτάνην τὴν λεγομένην μανδραγόραν καὶ μεταλαμβάνει ἐξ αὐτῆς καὶ εὐθέως πυρυῦται καὶ πορεύεται πρὸς τὸν ἄρρενα καὶ παραπεζεύουσα κομίζει αὐτῷ τὴν βοτάνην καὶ μεταλαμβάνει ὁ ἄρρην ἐξ αὐτῆς καὶ εὐθέως πυροῦται καὶ συγγίγνεται τῇ θήλει.

(P. 503.) Cantic. 7. 10–13: ἐγὼ τῷ ἀδελφιδῷ μου, καὶ ἐπ' ἐμὲ ἡ ἐπιστροφὴ αὐτοῦ. ἐλθὲ ἀδελφιδέ μου, ἐξέλθωμεν εἰς ἀγρόν, αὐλισθῶμεν ἐν κώμαις· ὀρθρίσωμεν εἰς ἀμπελῶνας, ἴδωμεν εἰ ἤνθησεν ἡ ἄμπελος, ἤνθησεν ὁ κυπρισμός, ἤνθησαν αἱ ῥοαί· ἐκεῖ δώσω τοὺς μαστούς μου σοί. οἱ μανδραγόραι ἔδωκαν ὀσμήν, καὶ ἐπὶ θύραις ἡμῶν πάντα ἀκρόδρυα νέα πρὸς παλαιά, ἀδελφιδέ μου, ἐτήρησά σοι.

(P. 503.) Genesis, 30. 14–16: ἐπορεύθη δὲ Ῥουβὴν ἐν ἡμέρᾳ θερισμοῦ πυρῶν καὶ εὗρε μῆλα μανδραγορῶν ἐν τῷ ἀγρῷ, καὶ ἤνεγκεν αὐτὰ πρὸς Λείαν τὴν μητέρα αὐτοῦ· εἶπε δὲ Ῥαχὴλ τῇ Λείᾳ τῇ ἀδελφῇ αὐτῆς Δός μοι τῶν μανδραγορῶν τοῦ υἱοῦ σου εἶπε δὲ Λεία Οὐχ ἱκανόν σοι ὅτι ἔλαβες τὸν ἄνδρα μου; μὴ καὶ τοὺς μανδραγόρας τοῦ υἱοῦ μου λήψῃ; εἶπε δὲ Ῥαχὴλ Οὐχ οὕτως· κοιμηθήτω μετὰ σοῦ τὴν νύκτα ταύτην ἀντὶ τῶν μανδραγορῶν τοῦ υἱοῦ σου. εἰσῆλθε δὲ Ἰακώβ ἐξ ἀγροῦ ἑσπέρας, καὶ ἐξῆλθε Λεία εἰς συνάντησιν αὐτῷ καὶ εἶπε Πρὸς ἐμὲ εἰσελεύσῃ σήμερον· μεμίσθωμαι γάρ σε ἀντὶ τῶν μανδραγορῶν τοῦ υἱοῦ μου.

(P. 509.) Xenoph., Symp., 2. 24: ὁ οἶνος ἄρδων τὰς ψυχὰς τὰς μὲν λύπας ὥσπερ ὁ μανδραγόρας τοὺς ἀνθρώπους κοιμίζει.

(P. 509.) Plato, Rep., 488 C: τὸν δὲ γενναῖον ναύκληρον μανδραγόρᾳ ἢ μέθῃ ἤ τινι ἄλλῳ συμποδίσαντας.

(P. 509.) Demosth., Philip. 4. 6: ἡμεῖς δ' οὐ μόνον τούτοις ὑπολειπόμεθ', ὦ ἄνδρες Ἀθηναῖοι, ἀλλ' οὐδ' ἀνεγερθῆναι δυνάμεθα, ἀλλὰ μανδραγόραν πεπωκόσιν . . . ἐοίκαμεν ἀνθρώποις.

(P. 509.) Aristotle, De Somn., 3 (ed. Acad. Bor., vol. 1, p. 456 b 30): . . . *τὰ ὑπνωτικά· πάντα γὰρ καρηβαρίαν ποιεῖ, καὶ τὰ ποτὰ καὶ τὰ βρωτά, μήκων, μανδραγόρας, οἶνος, αἶραι.*

(P. 509.) Philo Iud., De Vita Contempl., 5 (ed. Richt., vol. 5, p. 313): *Ἕτεροι δὲ τῶν μετριοτέρων δοκούντων εἶναι συμποτῶν ὥσπερ μανδραγόραν τὸν ἄκρατον πιόντες ὑποβεβλήκασι καὶ . . . ὕπνῳ βαθεῖ πιέζονται.*

(P. 509.) Frontin., Stratag , 2. 5. 12: Maharbal missus a Carthaginiensibus adversus Afros rebellantes, cum sciret gentem avidam esse vini, magnum eius modum mandragora permiscuit, cuius inter venenum et soporem media vis est. Tum, proelio levi commisso, ex industria cessit. Nocte deinde intempesta, relictis intra castra quibusdam sarcinis et omni vino infecto, fugam simulavit; cumque barbari, occupatis castris, in gaudium effusi avide medicatum merum hausissent, et in modum defunctorum strati iacerent, reversus, aut cepit eos aut trucidavit.

(P. 510.) Polyaen., Stratag., 8. 23. 1: *Ἐπικράτης ἐντολὴν ἔχων παρὰ Καίσαρος μετὰ τῶν χρημάτων παρασκευὴν εὐωχίας μεγάλης κομίζει καὶ ὑδρίαν πλήρη ξιφῶν καὶ οἶνον μανδραγόρᾳ κεκερασμένον. . . . οἱ δὲ ὑπερχαίροντες τῷ πλήθει τῶν χρημάτων ὥρμησαν ἀνέδην εὐωχεῖσθαι καὶ τὸν οἶνον πεφαρμακωμένον πιόντες ἀπεκοιμήθησαν. Καῖσαρ καθεύδοντας ἀναιρεθῆναι κελεύσας, κτλ.*

(P. 510.) Plut., Moral., 15 F: . . . *ὁ μανδραγόρας ταῖς ἀμπέλοις παραφυόμενος καὶ διαδιδοὺς τὴν δύναμιν εἰς τὸν οἶνον μαλακωτέραν ποιεῖ τὴν καταφορὰν τοῖς πίνουσιν.*

(P. 510.) Lucian, Timon, 2: . . . *καθάπερ ὑπὸ μανδραγόρᾳ καθεύδεις, ὃς οὔτε τῶν ἐπιορκούντων ἀκούεις οὔτε τοὺς ἀδικοῦντας ἐπισκοπεῖς.*

(P. 510.) Id., Ver. Hist., 2. 33: *κύκλῳ μὲν περὶ πᾶσαν αὐτὴν ὕλη ἀνέστηκε, τὰ δένδρα δ᾽ ἐστὶ μήκωνες ὑψηλαὶ καὶ μανδραγόραι.*

(P. 511.) Apul., Metamorph., 10. 11: Dedi venenum, sed somniferum, mandragoram, illud gravedinis compertae famosum et morti simillimi soporis efficax.

(P. 511.) Macrob., Saturn., 7. 6. 7: Insomnem medici frigidis oblinunt, modo papaveris suco, modo mandragora vel similibus.

(P. 511.) Cyrill., ad Gen. 30. 14: (sc. *μανδραγόρας ἐστὶ*) *ὑπνηλὸν χμῆμα, καὶ βαθεῖ καταμεθύσκον κάρῳ τοὺς μετεσχηκότας.*

(P. 511.) Theodoret., ad Cantic. 7. 13: *ὕπνον οἶδεν ἐμποιεῖν ὁ μανδραγόρας, ὡς ἰατρῶν παῖδές φασιν . . . . οἱ μανδραγόραν πεπωκότες οὐκ αἰσθάνονται τῶν τοῦ σώματος κινημάτων.*

(P. 511.) Basil., Hom., 5: *διὰ γὰρ τοῦ μανδραγόρου ὕπνον ἰατροὶ κατεπάγουσιν.*

(P. 511.) Hesychius, s. v. *μανδραγόρας*: *εἶδος βοτάνης, οἰνωτικὸν καὶ ὑπνωτικόν.*

(P. 511.) Id., s. v. *ὑπνωτικόν*: *ὁ μανδραγόρας.*

(P. 512.) Photius, s. v. *μανδραγόρας*: *ὑπνωτικὸς καρπός.*

(P. 512.) Suidas, s. v. *μανδραγόρας*: *ὑπνωτικὸς καρπός, ἦ λήθης ποιητικός. "καὶ φανεῖται πολὺν τὸν μανδραγόραν ἐκπεπωκώς."*

(P. 512.) Schol. ad Plat. Rep. 488 C: *ὑπνωτικὸς ὁ καρπὸς τοῦδε τοῦ φυτοῦ.*

(P. 512.) Ioan. Sicel., ad Ideas Hermog. (Walz, Rhet. Graec., vol. 6, p. 253): ἔστι δὲ ἡ μανδραγόρα βοτάνη καρωτικὴ καὶ ἀναιρετική.

(P. 511.) Diosc., 1. 571: ἔνιοι δὲ καθέψουσιν οἴνῳ τὰς ῥίζας ἄχρι τρίτου καὶ διυλίσαντες ἀποτίθενται, χρώμενοι ἐπὶ τῶν ἀγρύπνων καὶ περιοδυνώντων κυαθῷ ἑνί, καὶ ἐφ' ὧν βούλονται ἀναισθησίαν τεμνομένων ἢ καιομένων ποιῆσαι.

(P. 514.) Id., 1. 572 f.: σκευάζεται δὲ καὶ δίχα ἑψήσεως οἶνος ἐκ τοῦ φλοιοῦ τῆς ῥίζης· δεῖ δὲ ἐμβάλλειν μνᾶς γ' εἰς μετρητὴν οἴνου γλυκέος, διδόναι τε ἐξ αὐτοῦ κυάθους γ' τοῖς μέλλουσι τέμνεσθαι ἢ καίεσθαι, ὡς προείρηται· οὐ γὰρ ἀντιλαμβάνονται τοῦ ἀλγήματος διὰ τὸ καταφέρεσθαι.

(P. 514.) Id., 1. 574: ἥν φασι πινομένην . . . ἢ . . . ἐσθιομένην ἀποκαροῦν· καθεύδει γὰρ ὁ ἄνθρωπος ἐν ᾧπερ ἂν φάγοι σχήματι, αἰσθανόμενος οὐδενὸς ἐπὶ ὥρας γ' ἢ δ' ἀφ' οὗπερ ἂν προσενέγκηται· χρῶνται δὲ καὶ ταύτῃ οἱ ἰατροί, ὅταν τέμνειν ἢ καίειν μέλλωσι.

(P. 514.) Plin., 25. 150: Vis somnifica pro viribus bibentium; media potio cyathi unius. Bibitur . . . ante sectiones punctionesque ne sentiantur; ob haec satis est aliquis somnum odore quaesisse.

(P. 515.) Pseudo-Apul., De Virib. Herb., cap. 131: Si aliqui aliquod membrum fuerit emutilandum, comburendum, vel serrandum, bibat cum vino unciam mediam, et tantum dormiet quousque abscindatur membrum aliquo sine dolore et sensu.

(P. 515.) Isidor., Orig., 17. 9: Cuius cortex vino mixtus ad bibendum iis datur quorum corpus propter curam secandum est, ut soporati dolorem non sentiant.

(P. 515.) Dynamidia (ap. Ang. Mai, Class. Auct., vol. 7, p. 457): Corticis radicum eius libras III. vino mixtas in amphoram mittito et reponito ut maturescat: exinde dato potui [cyathos][*] IIII. cui aliquid secandum est; mox ut biberit, soporem[†] facit et incisionem non sentit.

(P. 515.) Serapion, Tract. Prim. (ed. Venet., 1479, fol. 98): Qui fuerit necessarius incidere aut cauterizare aliquod membrum et volumus quod dolor non sentiatur, detur in potu ex eo plus.

(P. 515.) Id., Ibid.: Datur in potu ex eo quantitas quattuor obolorum[‡] ei quem necesse est cauterizare aut incidere, et non sentiet cauterizationes sive incisionem propter subât quod accidit.

(P. 515.) Id., Ibid.: Medici cyrurgiae dant eam quando volunt incidere seu coquere aliquod membrum.

(P. 515.) Avicenna, Can. Med., lib. 2, cap. 366: Qui autem vult ut aliquid ex membris suis secetur, bibat tres obolos[§] ex ea in vino, et faciet subât.

(P. 515.) Brun. Astens., Expos. Super Cantic., 7. 13: Fertur eos qui ob curam secandi sunt, si exteriorem huius pomi corticem biberint, non sentire sectionem vel adustionem.

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[*] *Cyathos* addidi; deest in libro impresso.

[†] *Saporem*, Mai.

[‡] *Obolorum*, Prof. G. F. Moore, ex corrupto *onolos*, quod in libro impresso legitur.

[§] *Obolos*, Id., ex corrupto *anulusat*.

(P. 516.) Bart. Angl., De Propr. Rerum, lib. 17, cap. 104: Cuius cortex vino mixtus porrigitur ad bibendum his quorum corpus est secandum, ut dolorem non sentiant soporati.

(P. 516.) Pier. Valer., Hieroglyph., lib. 58: De soporifera mandragorae vi medici multa tradidere, idque praecipue, quod plurimus et efficacissimus est eius usus in soporandis iis qui vel inurendi vel secandi sunt: altissimum enim per quatuor ferme horas somnum, potione ita medicata exhausta, perseverare, ut neque ignem neque ferrum sentiant.

(P. 516.) Giov. Batt. della Porta, Mag. Natur., lib. 8, cap. 1: Dioscorides eos homines protinus dormituros eo habitu dicit, quo mandragoram hauserunt, ereptis omnibus sensibus in ternas quaternasque a quo potionati sunt horas, et hac uti medicos quum urere aut secare quem volunt.

(P. 518.) Diosc., 1. 571: (The first passage from Dioscorides quoted on p. 534.)

(P. 518.) Serap., loc. cit.: Sunt quidam coquentes radicem mandragorae cum vino donec minuatur tertia pars eius, et colant ac reponunt et accipiunt ex eo quantitatem unius oboli * et administrant ad superfluas vigilias et ad sedandum dolores. (The rest of the text given above in the first passage from Serapion quoted on p. 534.)

(P. 519.) Avicenna, loc. cit.: Facit subât et facit somnum. . . . Administratur ex eo ad hoc ut faciat subât plus, et ad hoc ut dormire faciat minus.

(P. 523.) Diosc., 1. 713: *οὐχ ὅτι πολλή τίς ἐστιν ἡ ἀναγκαία ἡ χρῆσις αὐτῶν, ἀλλ' ἵνα κατὰ μηδὲν αὐτῶν ἐλλείπειν δοκῶμεν.*

(P. 525.) Diosc., 1. 817: *Λίθος μεμφίτης εὑρίσκεται ἐν Αἰγύπτῳ κατὰ Μέμφιν, ἔχων ψηφίδων μέγεθος, λιπαρὸς καὶ ποικίλος· οὗτος ἱστορεῖται καταχρισθεὶς λεῖος ἐπὶ τῶν μελλόντων τέμνεσθαι ἢ καίεσθαι τόπων, ἀναισθησίαν ἀκίνδυνον ἐπιφέρειν.*

(P. 525.) Plin., 36. 56: Vocatur et memphites a loco, gemmantis naturae; huius usus conteri et iis quae urenda sint aut secanda ex aceto inlini; obstupescit ita corpus nec sentit cruciatum.

(P. 525.) Plin., 28. 110: Corii utriusque cinis ex aceto inlitus iis partibus quas secari opus sit, aut nidor cremati, sensum omnem scalpelli aufert.

(P. 526.) Diosc., 2. 100: *χρώμεθα δὲ τοῖς ὑπνωτικοῖς καὶ ἐφ' ὧν ἐστι χρόνια τὰ πάθη, ὅτε τεμνόμενός τις ἢ καιόμενος ἀναντίληπτος ἐθελήσειε τῶν ἀλγηδόνων γίγνεσθαι.*

(P. 528.) Hipp., 1. 60: *ἐν οἷς μὲν γάρ ἐστι διὰ μιῆς τομῆς ἡ χειρουργία χρὴ ποιέεσθαι ταχεῖαν τὴν διαίρεσιν. ἐπεὶ γὰρ συμβαίνει τοὺς τεμνομένους πονέειν μὲν τὸ λυπέον, ὡς ἐλάχιστον χρόνον δεῖ παρεῖναι.*

(P. 528.) Xenoph., Mem., 1. 2. 54: *τοῖς ἰατροῖς παρέχουσι μετὰ πόνων τε καὶ ἀλγηδόνων καὶ ἀποτέμνειν καὶ ἀποκαίειν.*

(P. 528.) Plato, Pol., 293 A: *τοὺς ἰατροὺς . . . ἐὰν . . . ἡμᾶς ἰῶνται. τέμνοντες ἢ κάοντες ἤ τινα ἄλλην ἀλγηδόνα προσάπτοντες.*

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* *Oboli*, Prof. G. F. Moore, ex corrupto *conos*.

(P. 528.) Id., Gorg., 479 A: φοβούμενος ὡσπερανεὶ παῖς τὸ κάεσθαι καὶ τὸ τέμνεσθαι, ὅτι ἀλγεινόν.

(P. 528.) Id., Ibid., 480 C: μὴ ἀποδειλιᾶν, ἀλλὰ παρέχειν μύσαντα καὶ ἀνδρείως, ὥσπερ τέμνειν καὶ κάειν ἰατρῷ, τὸ ἀγαθὸν καὶ καλὸν διώκοντα, μὴ ὑπολογιζόμενον τὸ ἀλγεινόν.

(P. 528.) Aristotle, Probl., A. 35 (ed. Acad. Bor., vol. 2, p. 863 a 27): εἴπερ ἀκμὴν μᾶλλον ὁ σίδηρος λαμβάνει, ῥᾴων καὶ ἀπαθεστέρα ἡ διαίρεσις;

(P. 528.) Cic., Tusc. Disp., 5. 73: An Epicuro . . . dicere licebit . . . nullum sapienti esse tempus, etiamsi uratur, torqueatur, secetur, quin posset exclamare: "Quam pro nihilo puto!"?

(P. 528.) Plin., 29. 13: Vulnerarium eum fuisse tradunt, mireque gratum adventum eius initio, mox a saevitia secandi urendique transisse nomen in carnificem.

(P. 528.) Cic., Tusc. Disp., 2. 35: Cum varices secabantur C. Mario, dolebat.

(P. 529.) Id., Ibid., 2. 53: At vero C. Marius, rusticanus vir, sed plane vir, cum secaretur, ut supra dixi, principio vetuit se alligari, nec quisquam ante Marium solutus dicitur esse sectus. . . . Et . . . fuisse acrem morsum doloris idem Marius ostendit.

(P. 529.) Propert., Eleg., 1. 1. 27:

Fortiter et ferrum saevos patiemur et ignes.

(P. 529.) Ovid, Rem. Amor., 229:

Ut corpus redimas ferrum patieris et ignes.

(P. 529.) Id., Heroid., 20. 183:

Ut valeant aliae, ferrum patiuntur et ignes.

(P. 529.) Seneca, Ad Marciam, 22. 3: Adice . . . lacerationes medicorum ossa vivis legentium et totas in viscera manus demittentium, et non per simplicem dolorem pudenda curantium.

(P. 529.) Plut., Moral., 74 D: δεῖ μιμεῖσθαι τοὺς ἰατρούς· οὔτε γὰρ ἐκεῖνοι τέμνοντες ἐν τῷ πονεῖν καὶ ἀλγεῖν καταλείπουσι τὸ πεπονθός, ἀλλ' ἐνέβρεξαν προσηνῶς καὶ κατῃόνησαν, κτλ.

(P. 529.) Martial, Epig., 11. 84. 5 f.:

Mitior implicitas Alcon secat enterocelas
Fractaque fabrili dedolat ossa manu.

(P. 529.) Dio Chrysost., Orat. 14 (ed. Reisk., vol. 1, p. 438): κἂν δόξῃ ποτὲ τῷ ἰατρῷ τὸν κάμνοντα δῆσαι, παραχρῆμα ἐδέθη, κἂν τέμνειν ἢ καῦσαι, καυθήσεται καὶ τμηθήσεται . . . πολλάκις οἱ τοῦ νοσοῦντος οἰκέται αὐτοὶ καταδοῦσι τὸν δεσπότην, καὶ τὸ πῦρ κομίζουσιν, ὅπως καυθήσεται, καὶ τἄλλα ὑπηρετοῦσι.

(P. 529.) Aristides, Orat. 24 (ed. Dind., vol. 1, p. 482): καὶ τέλος οἱ ἰατροὶ κατέτεμνον ἐκ τοῦ στήθους ἀρξάμενοι πάντα ἑξῆς ἄχρι πρὸς τὴν κύστιν κάτω· καὶ ὡς ἀνθήψαντο αἱ σικύαι, παντάπασι τὸ πνεῦμα ἀπελήφθη καὶ διῆλθεν ὀδύνη ναρκώδης καὶ ἄπορος φέρειν καὶ πάντα αἵματι ἐπέφυρτο καὶ γίνομαι ὑπεραλγεινός.

(P. 529.) Tertull., Scorpiac., cap. 5: Non tamen secari et inuri . . . idcirco malum, quia dolores utiles affert. . . . Ululans denique ille et

gemens et mugiens inter manus medici, postmodum easdem mercede cumulabit, et artifices optimas praedicabit, et saevas iam negabit.

(P. 529.) Themist., Orat. 22 (ed. Dind., p. 335) : ὥσπερ οἱ ἰατροὶ τὸ μὲν σιδήριον πολλάκις ἐῶσι, προστιθέμενοι δὲ φάρμακα ἅττα οὐ ξὺν ἀλγηδόνι τὸ πάθος ἰῶνται.

(P. 530.) Plut., Moral., 102 D : “μὴ γὰρ νοσοῖμεν” φησὶν ὁ ἀκαδημάικὸς Κράντωρ, “νοσήσασι δὲ παρείη τις αἴσθησις, εἴτ᾽ οὖν τέμνοιτό τι τῶν ἡμετέρων εἴτ᾽ ἀποσπῷτο. τὸ γὰρ ἀνώδυνον τοῦτ᾽ οὐκ ἄνευ μεγάλων ἐγγίγνεται μισθῶν τῷ ἀνθρώπῳ· τεθηριῶσθαι γὰρ εἰκὸς ἐκεῖ μὲν σῶμα τοιοῦτον ἐνταῦθα δὲ ψυχήν.”

(P. 530.) Lucian, Apol., 2 : ὥρα . . . ἐμὲ μὲν σιωπᾶν καὶ ἀνέχεσθαι τεμνόμενον καὶ καιόμενον, εἰ δέοι, ἐπὶ σωτηρίᾳ, σὲ δὲ ἐπιπάττειν τῶν φαρμάκων καὶ τὴν σμίλην ἅμα πρόχειρον ἔχοντα καὶ τὸ καυτήριον διάπυρον.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

A MANOMETER DEVICE FOR AIR THERMOMETERS.

By Harold Edwards.

A MANOMETER DEVICE FOR AIR THERMOMETERS.

BY HAROLD EDWARDS.

Presented by John Trowbridge, June 8, 1904. Received October 10, 1904.

FIVE years since an investigation was begun involving the use of an air thermometer. Before the investigation was completed a publication of the Reichanstalt more than covered the field contemplated. The investigation was, therefore, discontinued. In the meantime an air thermometer had been constructed containing some novel features, one of which in particular is perhaps worth recording, — the method of measuring the height of the mercury column. This is a very old problem in connection with both the air thermometer and the normal barometer, and it is recognized by every one who has worked with either of these instruments as still far from a satisfactory solution. The best methods used have been those suggested by Pernet, Marek, and Thiesen. These depend on the observations of the direct and reflected images of points or lines through the cylindrical walls of heavy glass tubing, and involve errors due to the displacement and distortion of the images by the non-parellelism of the surface, and by the heterogeneity of the glass. Further, the resolving power of the telescope and the brilliancy of the images observed in these methods are diminished greatly when one brings the mercury surface very near to a fixed glass surface, as is necessary at the lower meniscus of a constant-volume gas thermometer. There are besides these many small sources of error familiar to any one who has faced the problem. When the main experiment was abandoned, the work on the air thermometer had been carried far enough to show practically the merits of the device which is here described.

The general arrangement of the apparatus is shown in Figure 1. A, A are two glass plates containing on their inner surfaces the scales on which the mercury height is to be measured. B, B is the mercury column ; C an air reservoir at the top of the barometer column, exhausted by an air pump in the usual way. D is the tube leading through the wall to the bulb of the air thermometer in the adjacent room. E, E are the devices for determining the position of the surface of the mercury.

It is to the latter that attention is particularly called in the present note.

In Figure 2 is shown the details of the device E. Each consists of a plate of optically plain parallel glass. Opposite edges of this plate are polished and ruled with a diamond midway between the upper and lower surfaces. Each of these side lines, as they may be called, faces one of the large glass scales. The position of the scales is so adjusted that the distance from plate to scale is as small as possible. On the under side of the plate E and near its centre is a diamond line, perpendicular to the sides turned toward the scales. On the top of the plate E rest two prisms whose position and purpose will be explained later.

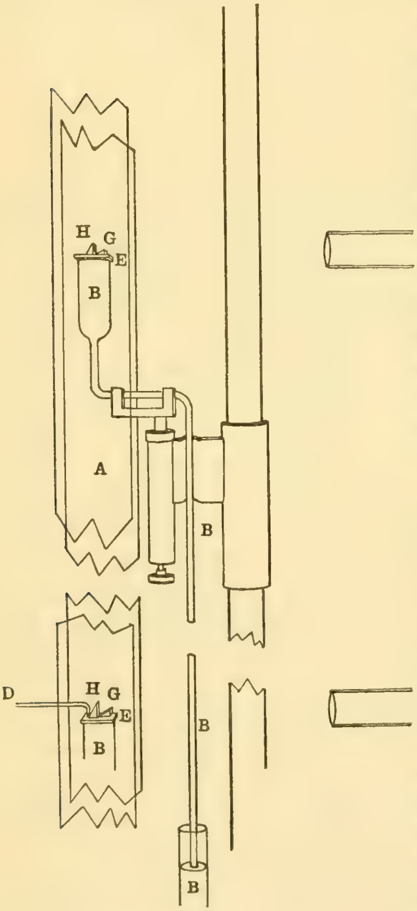

FIGURE 1.

The position of the side lines with reference to the glass scales is read by means of cathetometer telescopes not shown in the drawing.

The distance between each side line and the lower surface of the plate can be determined with great accuracy before the plate is mounted on the apparatus, by observing under a microscope the apparent displacement of the line when the opposite plain surfaces are brought against a fixed stop. Half this displacement is equal to the distance from the line to the central plane of the plate. Knowing the thickness of the plate, the distance from the side lines to the under surface of the plate is determined.

This distance determined, we have left for determination the interval from the lower surface of the plate to the surface of the mercury. This is accomplished by means of the two glass prisms G and H cemented with Canada balsam on the top of the plate.

In the case of the prism G a beam of light directly from the line is reflected at the surface of the prism and gives an image in the telescope D. Seen in the same prism there will be another image somewhat above the first and coming by reflection from the mercury surface. Measuring the vertical distance between the images by means of a cathetometer telescope we can calculate accurately and easily, if the plate and the telescope are horizontal, the distance between the glass and the mercury surfaces. The prism H gives a pair of images precisely similar, except that the one coming by reflection from the mercury is now below the direct image. Let b and b_1 be the vertical displacement of the telescope for each pair of images, d the distance from the line to the mercury surface, x and x_1 the values of the ratios $\frac{b}{d}$ and $\frac{b_1}{d}$, i. e., the effective magnifying powers of the prisms and plate. The unaccented letters refer to prism G, the accented letters to prism H.

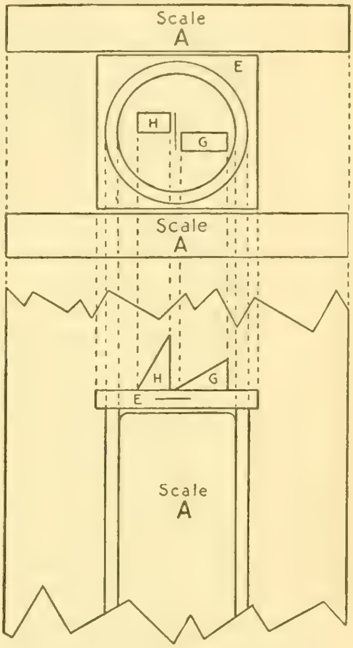

FIGURE 2.

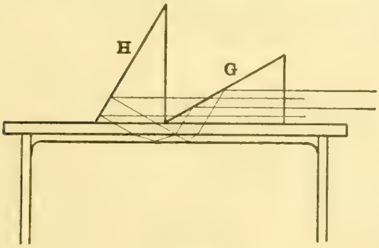

FIGURE 3.

Assuming for the present that the plate and telescope are level, we have, according to Figure 3, α being the angle of the prism and μ the index of refraction of the glass, and the analysis being so simple and obvious as to scarcely need entering here, —

$$x = x' = \frac{b}{d} = \frac{\mu \sin 4\,\alpha}{\sqrt{1 - \mu^2 \cos^2 2\,\alpha}}.$$

In selecting the best values for μ and α, the index of refraction of the glass and the angle of the prism, one has to bear in mind that the angle θ, the angle of incidence of the reflected light on the under surface of the glass plate, must not be so great as to cut down seriously the intensity of the reflected beam. If θ has a given value, which is equivalent to $\mu \cos 2\alpha$ having a given value, then x increases with μ, for if μ is increased the perpendicular distance between the two rays is increased; but if we increase α we must increase μ if $\mu \cos 2\alpha$ is to remain a constant, so that μ should be as large as possible. The glass having been selected α should be chosen as large as is consistent with the proper value of θ. The angle α was taken 30°, making $\theta = 58°$ and x about 2.75, μ being 1.6936.

The definition of the reflected image will be affected by the dispersion of the light in entering the under surface of the plate. The magnitude of this effect is obviously calculable from the above equation. In the particular glass employed, both the plates and the prisms being of heavy flint glass, the indices of the refraction were $\mu_D = 1.6936$ and $\mu_E = 1.7032$. This gave for the values of x corresponding to the lines D and E the values 2.757 and 2.807. The variation in the value of x thus produced was about 1.8 per cent, corresponding to a difference in the value of d of about .001 mm., d being .05 mm. Thus the dispersion does not seriously injure the definition of the reflected image, and the latter, though somewhat weaker than the direct image, is of excellent definition.

The object of using two prisms G and H and thus an image reflected both in front of and behind the fiducial line is to correct for any error in horizontality of the plate E, and the axis of the observing telescope. In order to examine the effect of this let β be the angle between the axis of the telescope and the horizontal, and let γ be the angle between the horizontal and the intersection of the surface of the plate with a vertical plane parallel to the plane of the figure. The values of x and x_1 are given by the formulae:

$$x = \frac{2 \sin\left[2\alpha + (\gamma - \beta)\right] \tan\left[\sin^{-1}\left\{\mu \cos 2\alpha + (\gamma - \beta)\right\} + \beta\right]}{\cos\beta + \sin\beta \tan\left[\sin^{-1}\left\{\mu \cos 2\alpha + (\gamma - \beta)\right\} + \beta\right]}$$

$$x_1 = \frac{2 \sin\left[2\alpha - (\gamma - \beta)\right] \tan\left[\sin^{-1}\left\{\mu \cos 2\alpha - (\gamma - \beta)\right\} - \beta\right]}{\cos\beta - \sin\beta \tan\left[\sin^{-1}\left\{\mu \cos 2\alpha - (\gamma - \beta)\right\} - \beta\right]}$$

It is immediately evident, that small variations from the horizontal, changing x in one direction, will change x_1 to about the same extent in the opposite direction, so that the mean magnifying power of the two prisms is unchanged by small values of β and γ. This can be illustrated by the following table of values of x and x_1, and the errors introduced when the mean is taken.

			x	x_1	Mean.	Error.
$\alpha = 30°$	$\mu_D = 1.6936$	$\beta = \gamma = 0$	2.7574	2.7574	2.7574	.000%
" = "	" = "	$\beta = 10', \gamma = 0$	2.8073	2.7088	2.7580	.025%
" = "	" = "	$\beta = 0, \quad \gamma = 10'$	2.7136	2.8027	2.7581	.025%

If $d = .2$ mm. and $\beta = \gamma = 10'$ the error in using the mean of x and x_1 is 0.0001 mm.

If the plate is out of the horizontal by having been rotated around a horizontal axis parallel to the scales, the reading of one edge line will be increased and the other decreased to the same extent, so that the error is eliminated in the mean.

The light coming by reflection from the rear prism H has a somewhat longer path than the light coming from the front prism; therefore to have all the images in focus at once, a block of glass of requisite length, in this case 2.1 cm., is inserted in the path of the light from the rear prism. It is not shown in the drawing.

The advantages of the above procedure may be summed up as follows: The images observed cannot be displaced and distorted, as the path of the light lies entirely in air and optically worked glass. All errors of levelling are eliminated. When used in a constant-volume gas thermometer the quantity of gas between the lower meniscus and the thermometer bulb is reduced very much. The breadth of the beam of light coming to the telescope is unrestricted, so that the resolving power is not cut down. The meniscus does not have to be brought to a definite point, but only near it, the remaining distance then being read in the cathetometer telescope. This is especially convenient in barometer work where two surfaces are to be simultaneously read, and the setting of one alters the other.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

NOTES ON RESISTANCE MEASUREMENTS IN PLATINUM THERMOMETRY.

By Harold Edwards.

NOTES ON RESISTANCE MEASUREMENTS IN PLATINUM THERMOMETRY.

By Harold Edwards.

Presented by John Trowbridge, June 8, 1904. Received October 10, 1904.

Several years ago an investigation was begun in the Jefferson Physical Laboratory, the object of which was to compare the scale of the constant-volume gas thermometer with that of the platinum-resistance thermometer, more especially at temperatures above 100° C. The intention was to extend the work of M. Chappuis, of the Bureau International des Poids et Mesures, and also to obtain results on the constant-volume thermometer for comparison with those of Messrs. Callendar and Griffiths on the constant-pressure instrument.

Before this was completed the publication of other investigations, particularly those of the Physikalsch Technische Reichsanstalt, made such a research hardly necessary. During the preliminary work, however, problems came up leading to experimental methods which in themselves may be of some interest.

In the standardization of the platinum thermometer the greatest possible refinements are called for in the resistance measurements. Errors peculiar to this instrument arise in correcting for the lead resistance, in avoiding possible cooling effects on the platinum on account of the presence of the comparatively large leads, and also in expressing in terms of some selected coil or unit the value of the platinum resistance.

The elimination of lead resistance is usually accomplished by subtracting the values obtained by direct measurement, or by placing in the stem of the thermometer dummy leads which are connected in the opposite side of a Wheatstone's bridge, the ratio coils of which are equal. The first method necessitates duplicate measurements of great absolute accuracy; the second involves the assumption that the two sets of leads change their resistance equally under all possible conditions, an assumption that can never be proved in any particular case. Moreover, alterations in the leads result in a new thermometer, and prevent to a considerable extent the verification of old measurements.

When either of these methods is used it is an advantage to have the leads of low resistance; this means large wire and the possibility of a great cooling effect of these large leads on the fine platinum wire.

It is the first object of this paper to show a method of bridge connection whereby the resistance of the leads may be entirely and directly eliminated.

A difficulty peculiar to resistance thermometers results from the condition that in general we have to measure a resistance varying over a very great range. The regular form of resistance box presents conditions which make it insufficient for this purpose, involving, as it does, the calibration of resistances which are high multiples and sub-multiples of a chosen standard.

The potentiometer methods of measurement suggested a modification of the Carey-Foster form of the Wheatstone bridge, which was finally adopted. A resistance may be measured by connecting it as part of one side of a wire bridge and tapping the galvanometer lead successively to its two ends. Its value is given by the resistance between the balancing points of the other galvanometer terminal, multiplied by the ratio of the resistances of the two sides of the bridge. This may be modified in the following way, so that only a difference has to be measured: — Let T (Figure 1) be the resistance to be measured, with D and F the leads and external resistance in the bridge construction, S the known resistance, M and N the ratio coils, with a low resistance slide wire between them. The resistance of the two portions of this wire we shall denote by w and w'. The wire between the points 3 and 4 can be neglected for the present. The shaded portions of the figure are thus heavy copper connectors, the resistance of a single one being equal to j.

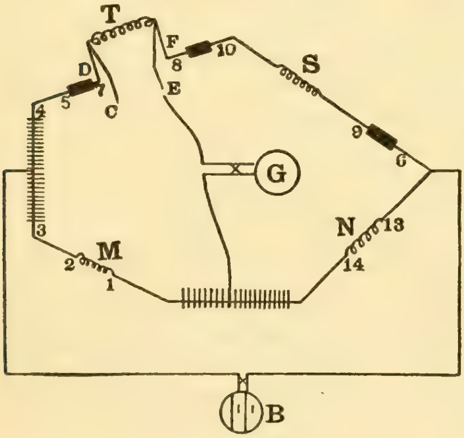

Figure 1.

By shifting the galvanometer lead from E to C a large change in the positions of the other galvanometer terminal on the wire is called for.

However, by having the resistance S very nearly equal to T, and by interchanging it and the two connectors between 8 and 10 and between 9 and 6 with the connector between 5 and 7, we nearly neutralize the required change in the balancing point on the bridge wire. The resulting connections are given in Figure 2. (The numbers refer particularly to Figure 3, and will be explained later.)

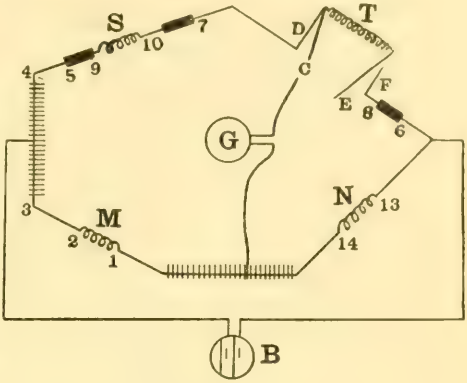

Figure 2.

It is then obvious that as a potentiometer method, where each element of the wire measures an element of resistance in the opposite side of the bridge, the resistance (d) of the bridge wire between the balancing points multiplied by the ratio of the resistance of the two sides of the bridge gives the difference between $T + j$ and S.

As a Carey-Foster method the results may be worked out as follows:

$$\frac{T + j + D}{M + w} = \frac{S + F + 2j}{N + w'} \text{ (Figure 1).}$$

$$\frac{S + 2j + D}{M + w - d} = \frac{T + F + j}{N + w' + d} \text{ (Figure 2).}$$

$$(T + j + D)\,N + (T + j + D)\,w' = (S + F + 2j)\,M + (S + F + 2j)\,w$$

$$(S + 2j + D)\,N + (S + 2j + D)\,w' + (S + 2j + D)\,d$$
$$= (T + F + j)\,M + (T + F + j)\,w - (T + F + j)\,d.$$

$$(S + j - T)\,N + M + w + w' = -\,d\,(S + T + D + F + 3j).$$

$$T - (S + j) = d\left(\frac{S + T + D + F + 3j}{N + M + w + w'}\right).$$

The lead resistances D and F only appear in the ratio by which the difference d is multiplied. If, by the ordinary method of directly measuring the lead resistance and subtracting, a given percentage error is introduced, with the above arrangement, one-half the percentage error is introduced as regards the knowledge of the *difference* between S and T. Let us assume that we are comparing two twenty-ohm resistances differing by one two-hundredth of their value (or .1 ohm) with leads of half an ohm each to one of the coils. If we should disregard the lead resistance entirely, an error of 1 ohm or 5 per cent would result with the ordinary method; here the error would be approximately

$$.1\left(\frac{41}{40}-\frac{40}{40}\right)=\frac{1}{400}\text{ ohms, or .012 per cent.}$$

The error has been reduced to one four-hundredth of the other value.

The knowledge of the lead resistance is thus of less importance, in so far as the difference is less than one side of the entire bridge.

The actual value of the lead resistance is determined most satisfactorily by getting the balancing points on the bridge wire with the galvanometer connected first to the end of the platinum wire by means of leads C and E, and second to the ends of D and F, where they connect to the heavy bridge terminals.

The necessary accuracy in the knowledge of the lead resistance being greatly reduced, we are able to have the leads of much higher resistance, and thus decrease greatly the possibility of heat being conducted to or from the platinum by the comparatively heavy low resistance leads.

We have now the problem of arranging a resistance S, whose value in terms of some particular coil can be varied from a fractional part to four or five times the resistance of the platinum at 0° C., and in steps small enough to permit the use of a low resistance bridge wire. A box of series resistances introduces calibration problems that are sure to give trouble, especially when some of the resistances bear a ratio to the unit of 100 : 1 or 1000 : 1.

A simple and more direct method, but yet more tedious as regards computation, is to arrange a set of five, ten, and twenty-ohm coils (whose values in terms of a ten-ohm unit can be determined with great convenience and accuracy) in series and parallel combinations the resistances of which increase in steps of .1 or .05 ohms from two or three, to thirty or forty ohms. The resistance of any complicated combination can be computed with a little care, and the errors are especially small, as they depend upon the comparison of coils in the ratio of 2 : 1 instead of 100 : 1.

As finally built the bridge consists (Figure 3) of copper bars and forgings 19 × 14 mm. in cross section, supported on a board of first-

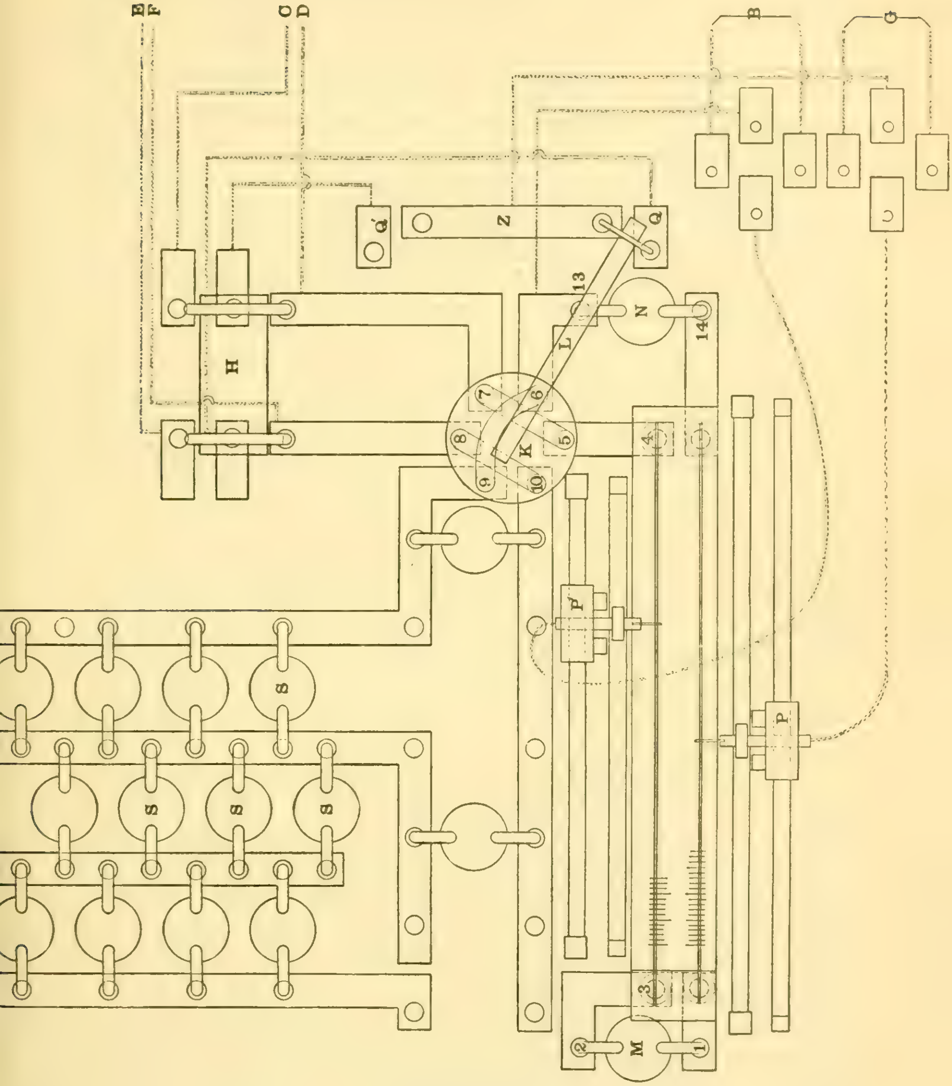

FIGURE 3.

quality cherry, 66 cm. long and 61 cm. wide, with hard-rubber bushings 7 mm. thick under the bars to give satisfactory clearance from the board. The coils were hung from mercury cups in the copper, so as to be immersed in a tank of oil, which was thoroughly stirred and mixed by means of a simple circulating pump driven by a small electric motor. In selecting this oil the aim was to obtain as pure a hydrocarbon as possible, which would give no trouble from evaporation. A pure hydrocarbon should introduce no difficulties on account of oxidation, and should maintain a high degree of insulation indefinitely as a result of its chemical inertness. What in the trade is known as liquid vaseline was adopted — an almost colorless oil, without odor and of approximately the consistency of a thin syrup. After five years no traces of deterioration have been found.

The general arrangement of the bridge is shown in Figure 3, where M and N are the ratio coils, and P and P′ the contact makers for the two manganin bridge wires. The connections are shown diagrammatically on Figures 1 and 2, where the numbers and letters correspond with those given in Figure 3, and serve to distinguish the most important of the mercury cups and coils.

The interchange of connections as required by Figures 1 and 2 is brought about by the "commutator" K, the position shown corresponding to Figure 1, that for Figure 2 being obtained by rotating K in a counter-clockwise direction through an angle of 60°, the connector from 5 to 7 then going from 6 to 8, while the one between 10 and 8 goes from 5 to 9, etc. These heavy connectors are shown by the shaded blocks in Figures 1 and 2.

In the lower right-hand corner of Figure 3 are two reversing switches for the battery and galvanometer circuits, the leads from which are marked by the brackets B and G. From the galvanometer switch a lead runs to the bar Z, from which, by means of a copper connector carried by the rod L (of hard rubber and supported from K), connection is made to Q or Q′, according to the position of K. In this way when the commutator K is shifted, the galvanometer lead is changed from F to D or E to C automatically.

The throw-over switch H allows the galvanometer lead coming from Q or Q′ to connect either to the ends of the platinum wire by means of C and E, or to the ends of the leads D and F. Readings made in this latter case give data as stated above to determine the resistance of the main leads to platinum thermometer T. (Figure 3 shows H thrown to give the connections for determining lead resistance.)

The subsidiary wiring for the galvanometer and battery circuits around the bridge consists of ordinary "flexible cord" drawn into soft rubber tubing.

In the upper left-hand corner of the plan the coils S of accurately known resistance, but approximately 5, 10, and 20 ohms in value, are indicated. These are connected to give roughly the resistance of T, and the desired difference is obtained from the wire readings.

The second wire, shown between mercury cups 3 and 4, is used for calibrating the standard wire, and for bringing the readings of P to convenient portions of the scale.

The entire bridge is kept under cover and operated from the outside by cords and simple levers, so far as the actual readings on the bridge wire are concerned.

JEFFERSON PHYSICAL LABORATORY,
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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. — No. 5.

THE SHOAL-WATER DEPOSITS OF THE BERMUDA BANKS.

By Henry B. Bigelow.

With Four Maps.

THE SHOAL-WATER DEPOSITS OF THE BERMUDA BANKS.*

By Henry B. Bigelow.

Presented by E. L. Mark, December 14, 1904. Received October 19, 1904.

The sea-bottom deposits described in the following pages were collected during the summers of 1903 and 1904 at the Bermuda Biological Station for Research, to whose Director, Professor E. L. Mark, the writer is indebted for many courtesies and much practical assistance. The collection comprises seventy-six samples, the most of them preserved in alcohol, dredged at sixty localities in the lagoons and sounds, and from the beaches, of the Bermuda Plateau, as well as the material taken in twelve dredge hauls made during a three-days' expedition to the Challenger Bank. No attempt was made to secure samples in the deeper waters on the sea faces of the banks.

The general topographic features of the Bermudas have been thoroughly studied by Rein (**'81**), Heilprin (**'89**), and Agassiz (**'95**), and I shall commence with a brief résumé of their accounts. The Bermuda Bank (Map 1, p. 560) is oval in outline, its longest axis, which lies northeast and southwest, being about thirty miles, and its breadth about fifteen; an area of some two hundred and seventy-five square miles. For the purposes of the present description I shall consider the thirty-fathom line as its limit, for at about that depth the abrupt slope to deeper oceanic waters commences. The land mass, which consists of a number of small islands, lies in a somewhat crescentic form on the southern side of the bank, where it forms an almost continuous barrier, but its entire area is not more than fifteen thousand acres (less than 24 square miles). Commencing at either extremity of the land mass and running thence around the northern side of the bank, at about the twelve-fathom line, some three miles inside the thirty-fathom line, is the so-called "boundary reef," or "ledge flat." This is perhaps three-fourths of a mile in breadth, awash in many places at low tide, and penetrated by several narrow "cuts," of which the most important are "Hogfish," on the southwest;

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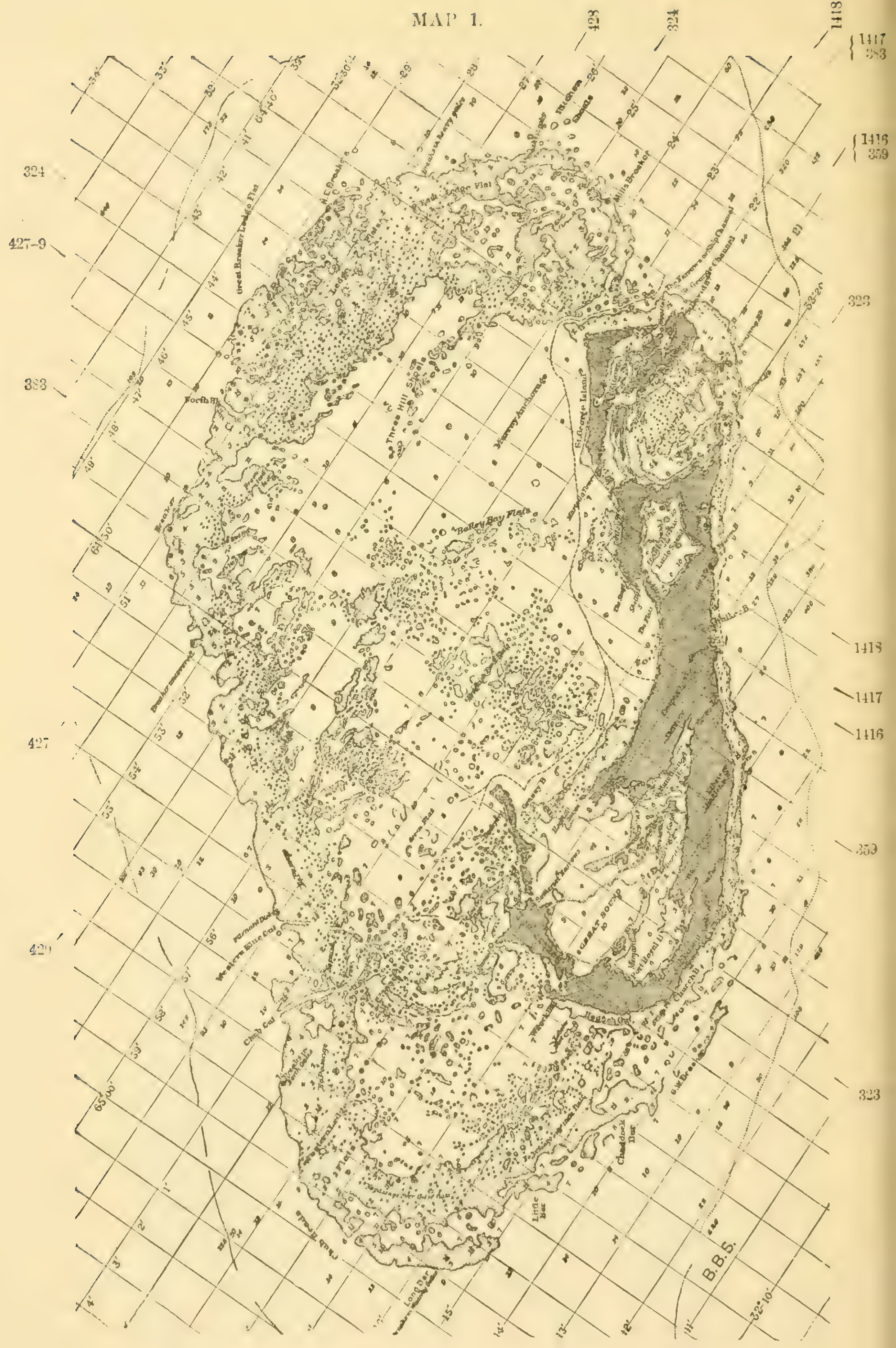
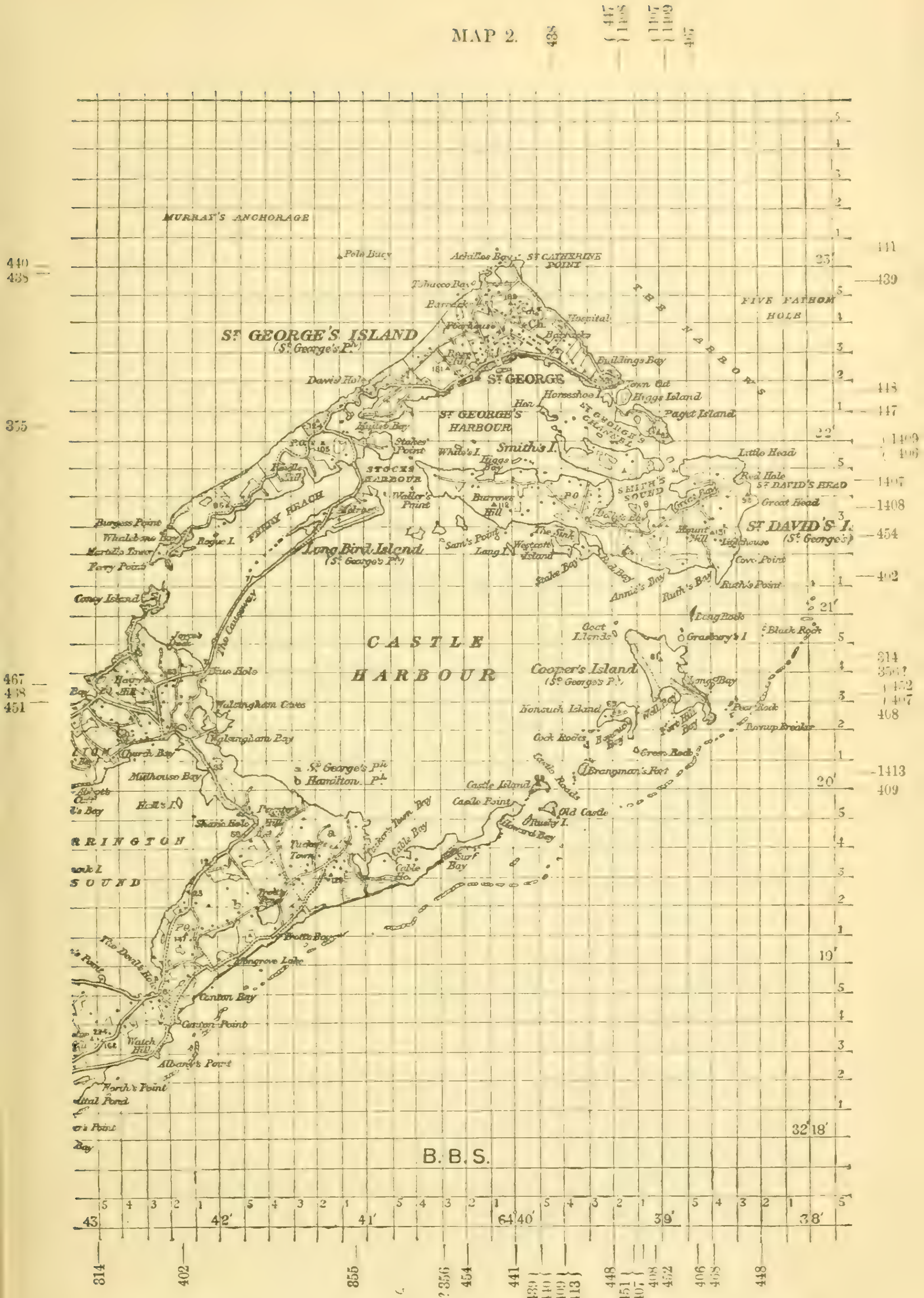
* Contributions from the Bermuda Biological Station for Research, No. 5.

MAP 1.



Map of Bermuda Plateau, based on British Admiralty Chart. Latitude and longitude ruled to minutes. One-hundred-fathom line in dots; Ship Channel in dashes; soundings in fathoms.

MAP 2.



Map of northeast third of Bermuda Islands, based on Ordinance Survey Map of 1902. Latitude and longitude ruled to 10 seconds. Scale about 0.9 inch to one statute mile.

"Chub" and "Western Blue," on the west; "Mills Breaker" and the "Ship Channel," on the east. On the southeast, running about parallel with the land and at a distance of one to three miles, is a line of detached ledges. The great "lagoon," enclosed by the boundary reef, is of rather uniform depth, from five to ten fathoms, but is broken in several localities by great numbers of shoals and ledges, while the Islands themselves enclose several large sounds, which are in more or less free communication with the sea. The rise and fall of the spring tides at Ireland Island (Map 4), as given on the admiralty charts, is four feet, — only about eight inches more than the vertical height of the tidal wave. The inflow of water into the great lagoon takes place over the whole boundary reef, so that it forms no strong currents, except through the cuts. On the bank as a whole, with the exception of one or two inlets, — for instance the Flatts Inlet into Harrington Sound (Map 3), — the tidal currents are too slight to cause any important transference of materials. Bermuda lies nearly one hundred miles south of the southern margin of the Gulf Stream, and there is no general sweep of water across the bank, while the land crescent forms an effective barrier against any surface currents which might follow the prevailing southerly winds.

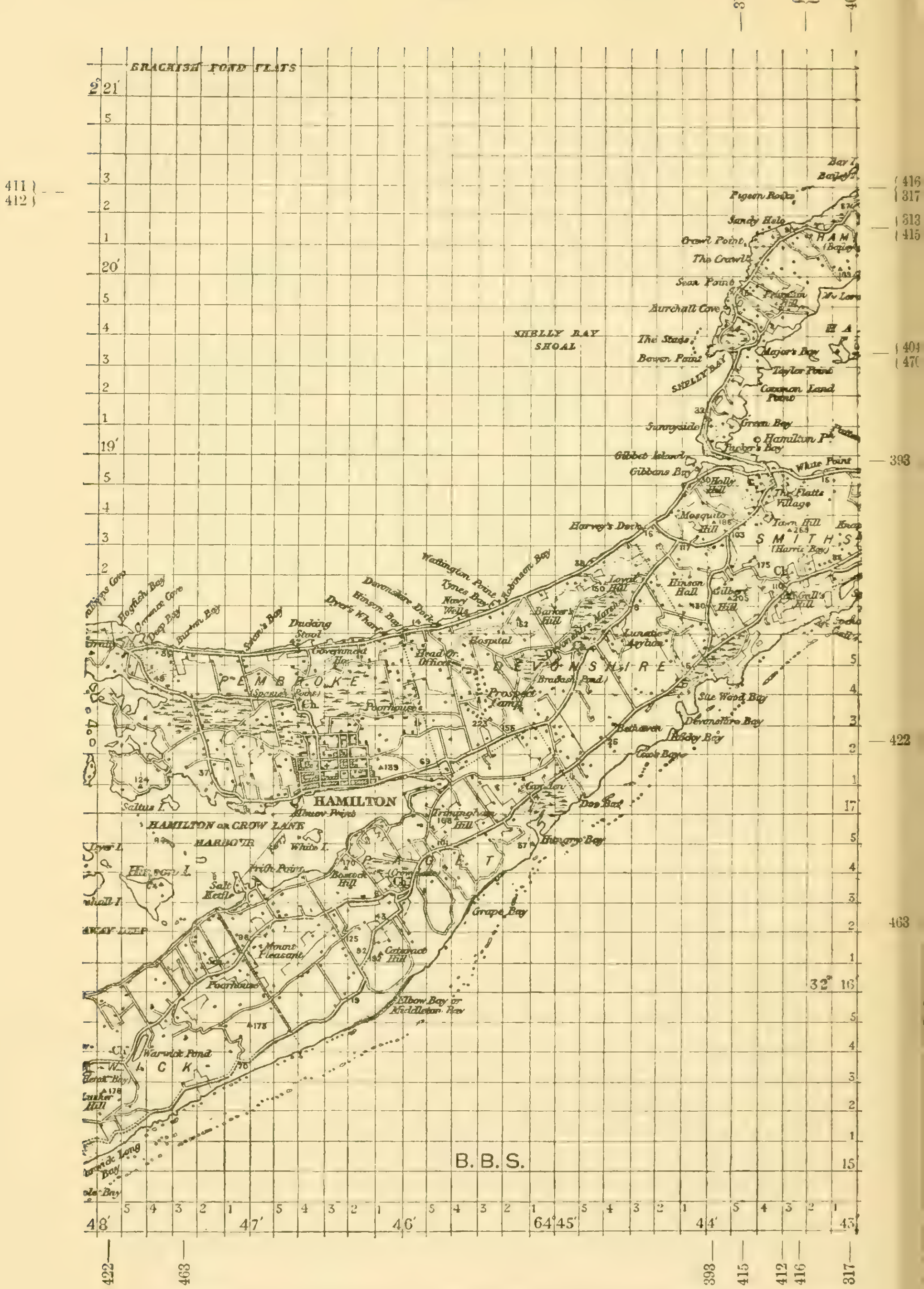
A good account of the probable geologic history of Bermuda — a gradual upbuilding of a volcanic cone by the accumulation of organic débris, followed by a subsequent subsidence — has been given by Agassiz ('95). No outcrop of volcanic rock has ever been found, but it is generally agreed that the core of the bank is of that nature, although buried under a thickness of limestone which is estimated by Agassiz as at least four hundred and fifty feet. The limestone of which the islands consist is of aeolian origin, being formed from wind-blown sand by a consolidation due to the action of water percolating through it, a process which is still going on in the dunes at Tucker's Town (Map 2). This aeolian rock is underlaid in places by the much harder beach rock, which is now being formed at Great Turtle Bay (Map 4), on the south shore. Whether there was ever any considerable elevation of the bank above the level of the sea, or whether the dunes and sandhills were formed near that level, as maintained by Agassiz, is of no great moment in this connection, but it is certain that at one time most of the surface of the bank was dry land and that it has subsequently subsided until only the narrow chain of islands remains above water. An outcrop of this ancient land remains in the "North Rocks." Agassiz has described the formation of the lagoons as resulting from the breaking down of the walls between natural depressions owing to denudation combined with the destructive action of the sea, and he has shown beyond question that the reefs and ledges are composed of aeolian limestones, being the remnants of bluffs or cliffs, undercut and demolished by the sea, a process now taking place actively on the south shore.

The most important point to be borne in mind in any study of the physiography of the Bermudas is that the ring-like outline of the reefs bears no relation whatever to an atoll; and that Bermuda is not in any true sense a coral island, although coral grows there in some profusion. There is no coral reef on the plateau; and we have no reason to suppose that Bermuda ever showed any more of the characters of an atoll than it does to-day. Coral fragments are rather rare in the aeolian rock, though other organic remains are recognizable in abundance. Mr. Agassiz says (**'95**, p. 228) that he never saw a piece of coral in Bermuda above high-water mark where its presence could not be explained by storm winds or waves. The living corals have added very little to the vertical height of the reefs, forming merely a thin covering, which, with the encrusting masses of serpula tubes, protects the limestone of the ledges against the attacks of the surf.

Although the general features of the Bermudas, both geographic and geologic, are so well known, but little attention has been paid to the bottom deposits. Almost the whole information which we possess on this subject is contained in the brief accounts of the Challenger expedition and of Mr. Agassiz's explorations.

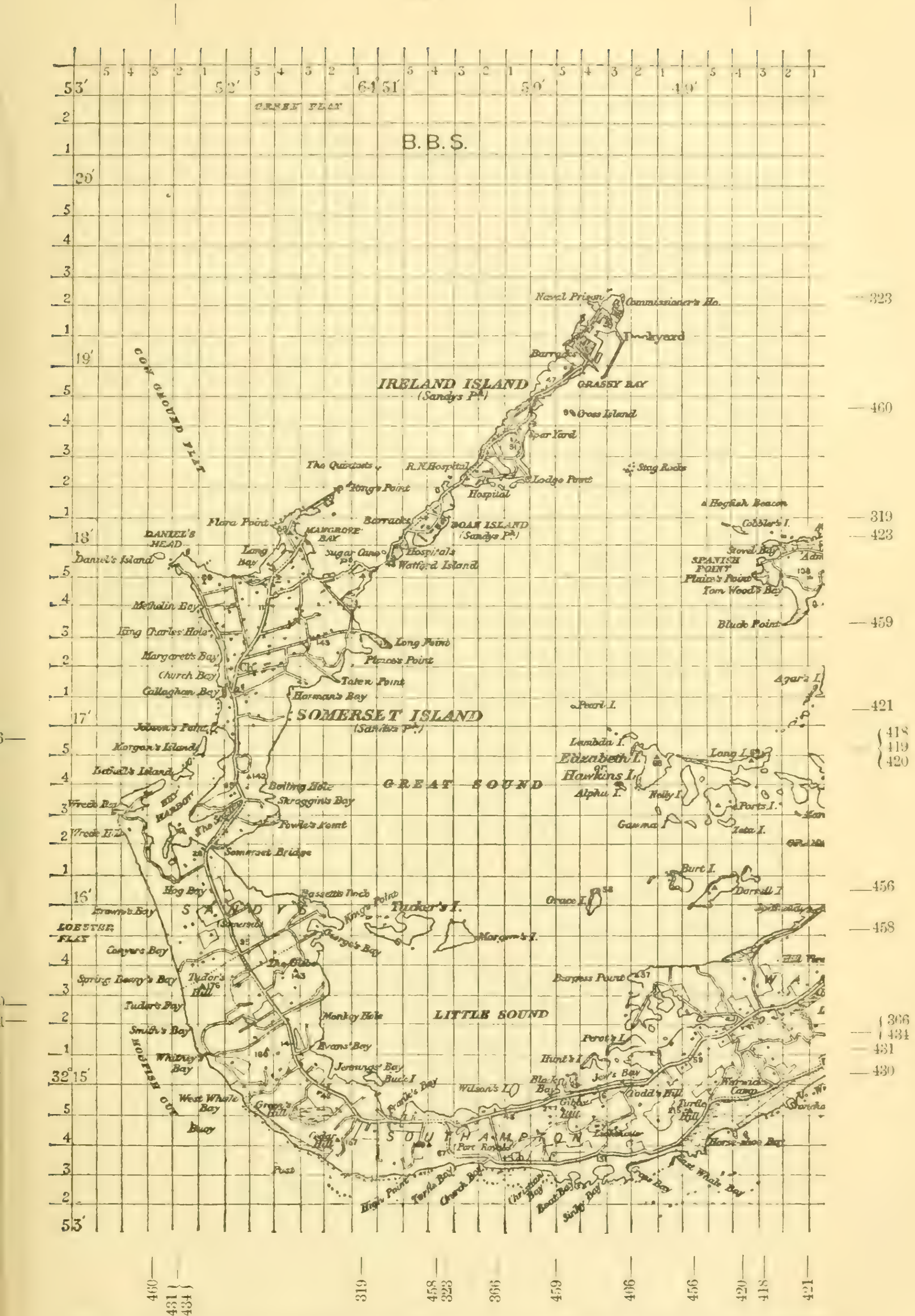
Murray and Reynard (**'91**, p. 48) analyze four samples of coral sand and mud from as many localities inside the reef, and find that calcic carbonate composes the following proportion of the deposits. No. 1, from the northern half of Murray Anchorage, in nine and a half fathoms, 95.43 per cent; No 2, near the Ship Channel, in five fathoms, 91.09 per cent; No. 3, on the Cow Ground Flat, 90.18 per cent; and No. 4, in Murray Anchorage, in six fathoms, 86.77 per cent. In three of the samples calcareous algae and their broken-down parts composed over 50 per cent of the mass, and Foraminifera, 10 to 30 per cent, the balance consisting of serpuline, gasteropod, and lamellibranch fragments, corals and millepores, ostracods, and plates and spines of echinoderms; 1 to 2 per cent was of mineral origin, fine particles of quartz and mica; and 2 to 10 per cent was fine flocculent amorphous dust. Siliceous remains are in small amount. These authors especially emphasize the importance of algae as sand builders. Wyville Thomson (**'77**) describes the bottoms in from four to ten fathoms as coral muds and sands, composed of the triturated fragments of algae, corals, and polyzoans, together

MAP 3.



Map of middle third of Bermuda Islands, based on Ordinance Survey Map of 1902. Latitude and longitude ruled to 10 seconds. Scale about 0.9 inch to one statute mile.

MAP 4.



Map of southwest third of Bermuda Islands, based on Ordinance Survey Map of 1902. Latitude and longitude ruled to 10 seconds. Scale about 0.9 in. to one statute mile.

with Foraminifera, Serpulae, and gasteropods, from 85 to 96 per cent of the mass being calcic carbonate, and the remainder consisting of the siliceous spicules of sponges, casts of Foraminifera and diatoms. Agassiz ('95) has added to this description. He found some of the flats largely formed of coraline, coral, and aeolian sand, and characterizes the deposits westward of Wreck Hill and east of Ireland Island as a fine marl, similar to that which he found in the Bahamas, off Andros Island (Agassiz, '94, p. 253). Heilprin ('89) has called attention to the importance of millepores as sand builders, and emphasized the importance of aeolian sand. But beyond these no detailed study of the deposits has been made.

For the purposes of the present paper the whole of the Bermuda Bank falls naturally into four main topographic divisions: first, the land area, of which only the beaches can be considered here; second, the more or less enclosed lagoons; third, the shoals and flats inside the boundary reefs; and last, the area between these reefs and the thirty-fathom line. Of this last area little examination was made, and our information is only of a very general character.

Commencing, then, with the beaches, a good example is afforded at Newton's Bay, on the south shore, directly south of Harrington Sound. This beach is of rather abrupt slope, the angle being about ten degrees, and is composed of rather coarse shell sand, which has a pinkish color. It consists chiefly of broken fragments of gasteropod and lamellibranch shells, with many living gasteropods and a few bivalves; the three most important genera being Rissoina, Vermetus, and Tellina. Fragments of large and small worm tubes are very numerous, and the pinkish color is due to the great number of bits of a shining red Serpula. Incrusting Bryozoa, nullipores, and corallines are important, and there are many fragments of millepores, but few of any corals. Tests of large Foraminifera occur, chiefly Orbitolites and Orbiculina, with a few smaller species. In this sand there seems to be no fine material whatever; probably it is all washed away by the surf, which is often violent. At Tucker's Town (Map 2), a few miles to the eastward, the surf is causing a rapid destruction of the cliffs, thereby adding to the sand a large amount of finely ground material produced by the mechanical destruction of the aeolian rock, which, being light and therefore easily wind blown, forms the bulk of the modern dunes. The force of the winds here is well illustrated by the large shells found blown many yards inland. Many of the beaches contain enough serpuline fragments to give the sand a pink color, as at Newton's Bay, and none are entirely free from such fragments. A striking feature of all the beaches is the complete absence of coral sand, though fragments of millepores play a considerable rôle as beach builders.

The detached ledges which run parallel to the south shore enclose a more or less clearly marked lagoon, with a depth of about six fathoms, where all conditions are as favorable for marine organisms as possible. Here mollusks, echinoderms, tube-building worms, gorgonians, corals, millepores, corallines, and nullipores flourish, and it is from this area that the beaches of the south shore receive most of their material. The amount of detritus added by the destruction of the cliffs varies in different places, but the total addition must be enormous, because the surf is often very violent. Most of the beaches, however, are swept clean of any of this fine calcareous silt, most of which is probably carried out and deposited in deeper water. Such parts of it, however, as are added to the beaches, as at Tucker's Town, are often blown far inland, to form large dunes. The gorgonians, which are luxuriant just off the shore, can of course add nothing but their spicules to the deposits, and the corals, being mostly porites, meandrinas, and other massive forms, are not easily ground up. The offshore reefs probably furnish most of the tubes of the red Serpulae and other worms and a few mollusk shells; but they are so well protected by incrusting Serpulae and algae that the surf cannot exercise any very vigorous destructive action on them.

Besides the beaches of the south shore, there are others on the north shore — as at Shelly Bay — and in Castle Harbor, — as at Tucker's Town and at Cooper's Island, — all of which consist of shell sand of the type already described, with the addition of a large proportion, probably 25 per cent, of very fine calcareous dust derived from the destruction of the neighboring cliffs and ledges. Here there is little or no surf action, and the slope of the beaches is not nearly so steep, being only about five degrees.

In the second class of localities fall also the landlocked lagoons or sounds. These are all more or less nearly enclosed bodies of water, of considerable size and moderate depth, standing in communication with the sea through several passages. The most important are Castle Harbor, St. George's Harbor, Harrington Sound, Great Sound, and Little Sound. It is generally agreed that they originated as depressions, or sinks, whose walls became broken through by denudation and the mechanical action of the sea. Several such sinks, now in process of formation, are occupied by marshes or marshy ponds, an example being Devonshire Marsh. Some, perhaps many, of the channels which connect the sounds with the open sea originated as partially subterranean channels, such as to-day connect Harrington Sound with the surrounding waters.

Castle Harbor, which is an excellent example of one of these large submerged sinks, communicates with the sea through several channels, so that it is comparatively open. The main portion of the harbor is about three miles in diameter, is nearly round, and has a greatest depth of about seven fathoms; but the bottom is very uneven. On the south it is separated from the ocean by a chain of small islands, between which there are five narrow channels. On the northwest the enclosing wall is more nearly complete, being broken in only one place, — by the body of water known as Ferry Reach, a partially artificial basin nearly cut off from the harbor by a causeway and an island, but communicating on the west with the open water of the great lagoon, and on the northeast with the passage known as Stocks Harbor. Through this passage Castle Harbor connects directly with St. George's Harbor and through it with the outer lagoon. Thus Castle Harbor, communicating with the sea on three sides, combines the advantages of a protected location and a free circulation of water, making it a very favorable locality for many marine organisms. The bottom is of clear white sand, interrupted by numerous ledges covered with corals, millepores, and coralline algae.

A sample of this sand from Station No. 356, near Castle Island, shows the following composition. It is white, slightly flecked with pink and yellow, slightly coherent, sticky; with 20 per cent of fine flocculent amorphous calcareous ooze. Foraminifera are a striking feature, forming 40 per cent or more of the mass; particularly abundant are the large sand-dwelling genera Orbitolites, Orbiculina, and Ammodiscus. A partial list comprises Orbitolites duplex, Orbiculina adunca, Ammodiscus tenuis, Cornuspira foliacea, Pulvinulina menardii, and species of Polystomella, Trochammina, Textularia, Cyclammina, Cassidulina, Biloculina, and a very few Globigerinae. The great majority of them are living, though there are in addition many dead tests and casts. Most important after the Foraminifera are the coralline algae Udotea and Halimeda, which Agassiz (**'88**^{a}, p. 82) has also described as occurring in great masses on the Florida Reef. In this particular sample they form only about 10 per cent, but in certain parts of Castle Harbor they grow in such great abundance that their remains form 30 per cent to 40 per cent of the sand. Living mollusks and their shells, mostly fresh and little worn, form 10 to 15 per cent, the most important genera being the gasteropods Bulla, Codakia, Nassa, Cardium, Gouldia, Rissoina, Olivella, Modulus, Bittium, Cadulus, and Vermetus, and the bivalves Crasinella, Abra, Macoma, Semele, Arca, Lasaea, and Tellina. The remainder consists of serpuline fragments, of millepore, bits of coral, echinoid plates and spines, starfish pedecillariae, holothurian plates, sponge spicules, and a few diatoms. The fine flocculent dust or ooze, mentioned above as forming 20 per cent of the sample and giving it a sticky consistency, is composed of exceedingly minute amorphous calcareous particles. It is identical with the ooze, which, in larger deposits, has already been described by Agassiz ('94, p. 52, and '95, p. 253) as "marl," and is undoubtedly produced by the slow triturating action of the sea on aeolian rock. The fine dust thus formed is held so easily in suspension in the water that it is carried and deposited, often at a great distance from its point of origin, in all localities where absence of wave action or currents allows. It is present, in greater or less degree, in every bottom sample from the Bermuda Bank.

As a whole, this bottom contains a great number of living organisms, and the empty shells and shell fragments are fresh and but little waterworn — good evidence that they are of very recent deposition. But at Station 1412, at about the centre of Castle Harbor, in four fathoms of water, the proportion of fine calcareous silt is very much greater, living organisms are less numerous, and the whole sample has a much more water-worn appearance; while the honeycombed condition of most of the dead shells shows that solution by the sea water is here of some importance. This sample was taken from a small basin nearly surrounded by shoals and ledges, where so much of the fine detritus formed by the mechanical action of the water on these aeolian rocks, together with the various organic remains, is being constantly deposited on the floor of the basin that it effectually chokes most of the living organisms. In such regions as Castle Harbor, and, in fact, over most of the Bermuda Plateau, where the sea water always holds a vast amount of calcareous silt in suspension, fixed and sand-dwelling organisms, such as mollusks, tube-building worms, bryozoans, and hydroids, reach an extensive development only on such bottoms as have an active circulation of water that prevents the rapid deposit of silt.

Castle Roads is a good example of the deeper channels connecting Castle Harbor with the open sea on the south. This passage is about four hundred yards wide, with a greatest depth of about six fathoms, and during both incoming and outgoing tides a strong but not violent current runs through it.

The bottom at Stations 409 and 1413, in five fathoms, is a rather coarse shell sand, containing many large fragments; most of the organisms are macroscopic. It is white, spotted with pink; very clean, with less than 1 per cent of fine ooze. Foraminifera of the same species as those already enumerated from Castle Harbor form about 25 per cent of the deposit; bivalves and gasteropods, many of them living, about 25 per cent; the remainder consist chiefly of calcareous algae, and fragments of millepore, much water worn, with a few Serpula and Polyzoa. Except for the abundance of living organisms, the resemblance of this sand to the beach at Newton's Bay is striking.

Nonsuch Scaur, between Nonsuch and Cooper's Islands, a shoal channel with only two fathoms of water, has a bottom at Stations 407 and 408 of very distinct character, and is a good example of the shallow passages. The bottom is white sand with a faint yellowish tinge, and many pink fragments. It is fine, the average diameter of the fragments being about 1 mm., granular, with about 3 per cent of fine ooze. Foraminifera, — many of them living, the most abundant being Orbitolites and Polystomella, — corallines, and millepore fragments in about equal proportions form 60 per cent of the mass, the remainder being fragments of bivalve and gasteropod shells, Serpula, spines and plates of echinoids, Polyzoa, calcareous spicules, diatoms, and about 5 per cent of rather large, rounded, amorphous calcareous fragments, evidently the coarser detritus from the cliffs on the neighboring islands. Except for the Foraminifera, there are very few living organisms; and this is apparently an area where deposition of sand from the nearby ledges is going on rapidly; its exposed position, however, and the vigorous wave action thoroughly scour the bottom.

In Ferry Reach, the miniature artificial sound on the north of Castle Harbor, the bottom is white sand. A sample near its mouth, Station 402, in three fathoms, is white, mottled with yellowish, rather fine, granular, uniform, with about 10 per cent fine ooze. Foraminifera and their casts form about 10 per cent; joints of the calcareous algae Halimeda and Udotea, and their broken-down fragments, form about 30 per cent; the remainder, consisting of millepores, bivalves, gasteropods, Serpula, coral fragments, ostracods, echinoid fragments, holothurian plates, sponge and alcyonarian spicules, diatoms, and a few radiolarians.

At the inner, southern end of the Reach the bottom is grayish white, coherent, with about 30 per cent ooze, the remaining 70 per cent consisting of the same organisms as above, in about the same relative abundance; whereas at the northern entrance there is less than 2 per cent of fine ooze, and the bottom in every way closely resembles that at Nonsuch Scaur.

In the small basin known as Stocks Harbor there are few ledges and very little living coral. The bottom consists of grayish sand, composed of a small amount of broken shells and a larger amount of fine silt, most of which is either white marl or of terrigenous origin.

St. George's Harbor is an enclosed sound of much smaller area than Castle Harbor, and is much more nearly landlocked, its only direct communication with the sea being through three narrow channels on the northeast, two of which are shallow. On its northwest border there is a considerable bight, Mullet Bay by name, with a shoal bar at its mouth; and at the southern end Smith's Island cuts off Smith's Sound and Dolly's Bay, which together form another practically enclosed bight. The greatest depth of St. George's Harbor is seven fathoms, and the chief tidal current is through St. George's Channel. The bottom samples at seven fathoms consist of a rather coarse dark gray sand of very much water-worn shell fragments, together with the plates and spines of Toxopneustes, a few large Foraminifera, and the calcareous alga Halimeda. There are very few living organisms of any kind, and there is a considerable proportion, about 30 per cent, of fine calcareous silt, which is of limestone origin, but, unlike the "marl" already mentioned, of a bluish gray color. On the north side of the Harbor, between Horseshoe and St. George's islands, in two fathoms (Station 448), the bottom is fine grayish blue sand, coherent, with 75 per cent fine ooze. Foraminifera, especially Polystomella, form 10 per cent, and nullipore, gasteropod, bivalve, millepore, and echinoid fragments, 10 per cent. The fine ooze consists almost wholly of impalpable, flocculent, amorphous, calcareous dust, stained gray; probably in large part of terrigenous origin and very similar to the bottom at Mullet Bay. Although this passage communicates with the sea, it is closed by a very shoal bar; it is a region of rapid deposition and with few living organisms.

The cut between Horseshoe and Paget islands (Station 447) is deeper. A sample from three fathoms at the inner end of the cut is dark blue sticky mud, slightly coherent; 50 per cent fine ooze; Foraminifera, — especially Polystomella biloculina, Orbitolites, Hastigerina, and Textularia, — 15 per cent; the remainder being calcareous algae, millepores, and mollusks in about equal abundance, with a few Serpulae and Polyzoa. The fine ooze is of the same composition as in the last sample and there are few living organisms except the Foraminifera. The coarse portion of the sample is evidently water-worn reef detritus swept in through the channel by the tides.

In St. George's Channel, the chief passage between St. George's Harbor and the outer lagoon, the bottom is bluish gray mud, of the same composition as in the Harbor, the proportion of coarse sand increasing rapidly toward the mouth; a sample from the middle of the opening (Station 1409, four fathoms), just at the mouth, has only a very faint bluish tinge. It is rather coarse, clean, granular, with 1 per cent fine silt. Foraminifera make up 10 per cent, and millepores and algae, in nearly equal proportion, about 40 per cent; the remainder consists of bivalve and gasteropod fragments, Serpula, Polyzoa, echinoid fragments and small water-worn fragments of limestone from the cliffs. There are many living organisms in this bottom, especially Foraminifera and gasteropods, but the greater portion of the deposit consists of the coarser reef detritus from the neighboring ledges.

Mullet Bay, on the northwest side of St. George's Harbor, is a small nearly enclosed body of water cut off by a very shoal bar and with no tidal currents. The depth over the bar is about seven feet, that inside the bay nearly three fathoms. The bottom (Station 355) consists of a sticky and very fine blue mud, which is chiefly if not entirely calcareous. The fragments of which it is composed are so minute as to be wholly indistinguishable to the naked eye. Large numbers of Toxopneustes live in this mud, and it contains great quantities of their plates and spines, but there are few if any other conspicuous living organisms. The character of the mud shows conclusively that it is chiefly terrigenous silt and detritus washed down from the surrounding hills, and it corresponds perfectly in its occurrence and origin to the siliceous "blue mud" of the continental slope, its calcareous nature being of course explained by the fact that the land mass consists entirely of limestone rocks.

Dolly's Bay (Station 1408), another partially landlocked bight of a similar character, has a bottom of the same type consisting almost entirely of sticky blue mud, with a few echinoid plates and spines. In Smith's Sound (Station 1407), which connects Dolly's Bay with St. George's Channel, the bottom is of similar constitution, but much coarser, with a considerable number of both broken and living shells and an appreciable proportion of the spicules of calcareous sponges, gorgonians, and holothurians.

A glance at the maps shows the close correlation between the bottom deposits and the topographic features in the Castle Harbor and St. George's Harbor regions. In Castle Harbor we find a large body of water, of moderate and fairly uniform depth, in rather free communication on two sides with the sea, and with a free circulation of water, though without strong tidal currents. Under these favorable conditions there is a considerable development of stony corals, gorgonians, and millepores; mollusks, worms, and echinoderms also flourish in great numbers. These living organisms, particularly mollusks, millepores, and Foraminifera, are adding constantly to the bottom deposits, except in some parts of the basin, where silt from the limestone ledges is deposited in such large quantities as to practically choke them. In the deep channels on the south, such as Castle Roads, the tidal currents, setting now in one direction and now in the other, make conditions for sedentary animal life still more favorable, by the great access of fresh sea water and the continual scouring which they effect. In St. George's Harbor the conditions are somewhat different. This is much more nearly enclosed, and the channels which connect it with Castle Harbor and with the outer lagoon are very much shoaler than its main basin, the result being that the circulation of water is not rapid, and a considerable quantity of terrigenous detritus settles to the bottom, giving the sand its characteristic gray color. This detritus, added to the calcareous silt from the outer lagoons, is in sufficient quantity to prevent any very active development of animal life on the bottom. In Dolly's Bay and Mullet Bay the same process has gone so much further that the whole bottom is covered to a considerable depth with the blue mud, which is chiefly of terrestrial origin. In St. George's Channel the tidal current is strong enough to scour the bottom of most of this fine material, leaving little but the large fragments.

The much larger basin at the western end of the islands, known as Great Sound, agrees, in many of its topographic features, with Castle Harbor. Like the latter, it probably originated as an enormous sink, which, through the combined action of subsidence and of the denudation of its surrounding hills, has become open to the sea. In its present form it is open on the north by a passage about a mile and a half broad, and on the west by the narrow cuts which separate Ireland Island, Somerset Island, and the main island. The southern part of this large basin is practically cut off by several small islands, forming the smaller basin known as Little Sound, while Hamilton Harbor forms a narrow arm on the east. The greatest depth of Great Sound, about eleven fathoms, is near its southern border; passing thence northward, the water shoals steadily until at the narrowest point, between Ireland Island and Spanish Point, many of the ledges are nearly awash at low tide, thus forming a natural bar. Through this a narrow ship channel has been blasted.

The character of the bottom in the different regions of Great Sound varies considerably. Near the entrance (Station 459) there are numerous luxuriant patches of gorgonians, sponges, millepores, and corals, which alternate with spaces of white shell sand, composed chiefly of calcareous algae and molluscan remains, with about 10 per cent of fine calcareous silt. Further south, east of Boaz Island and Somerset Island, in a depth of about four fathoms, the appearance of the bottom is altered by the complete absence of coral patches or ledges, although there is still a small amount of living, and a good deal of dead, coral. The bottom samples here consist of white sand, in which fine and coarse material are in about equal amount. The former is about 60 per cent calcareous silt or marl, the remainder being calcareous spicules, together with a great variety of small organic particles. The coarse material consists chiefly of algae, millepores, molluscan fragments, and living mollusks, with worm tubes and echinoid plates. There are very few Foraminifera and few coral remains, although there are a considerable number of millepore fragments. As we go still further southward into the sound, we find the bottom near its southern edge in ten fathoms, composed of sticky white sand of similar constitution but containing a much larger proportion of fine ooze. Apparently this deep basin is a region of deposition for the silt collected by the water on the outer flats.

On the south and east sides of Great Sound, between the different islands, there are several shoal bars where the sand is sufficiently distinctive to be worth special notice. On the bar between Tucker's and Morgan's islands, which forms the southern boundary of Great Sound and the northern boundary of Little Sound (Station 458), in three fathoms, the bottom is whitish, thickly mottled with bright yellow, rather coarse, granular, clean sand, with 6 per cent fine ooze. The yellow color is due to a staining of many of the fragments. Large Foraminifera, of the same genera as those found in Castle Harbor, together with their casts, form about 20 per cent; calcareous algae, bivalves, and gasteropods, 60 per cent; the remainder being composed of Serpula, echinoid spines and plates, millepore, and coral. There are many living Foraminifera and mollusks in this bottom.

The same type of sand, of the same yellow color and with many living mollusks, forms the bars between Godet Island and the main island (Stations 463 and 464), and between Moses and Darrell islands (Stations 456 and 457) on the east of Little Sound; it also forms the bottom off the north and west sides of Elizabeth Island (Stations 455 and 456). But in the channel leading into Hamilton Harbor (Station 421), the sand, though yellowish and almost free from ooze, is much finer, contains very few living organisms, and consists chiefly of water-worn fragments of algae, mollusks, and millepores. It is probably an area for deposition of material from the surrounding shores and shoal bars; but all the finer detritus is scoured out by the tide.

Little Sound, situated directly south of Great Sound and forming a basin about two miles long, with a greatest depth of ten or eleven fathoms, is in many respects similar to Mullet Bay. On the south, east, and west it is bounded by the shores of Long or Bermuda Island; its eastern edge is formed by a broad and very shallow flat, with three small islands; and on the north it is separated from Great Sound by a shoal and narrow bar, over which the greatest depth of water is three fathoms. There is no channel cutting this bar, so that the sound is a practically enclosed basin. The bottom at Station 366, in nine fathoms, is composed of a fine, sticky blue mud, made up of extremely minute particles, most of which are of terrigenous origin. But I was able to distinguish also the tests of a few small Foraminifera, as well as a large number of calcareous sponge and gorgonian spicules, and fragments of Halimeda. The only large fragments were a few broken and water-worn lamellibranch shells, and the bottom contains very few living organisms. It is evident from the nature of the bottom that its origin is very similar to that of Mullet Bay, and that much of the mass consists of terrigenous detritus washed down from the hills which form its southern shore. The topography of Little Sound is such that it has acted as a catch-basin for this material and also for the fine calcareous silt from the outer lagoons and reefs, carried in suspension by the sea water. The absence of living organisms is the result of a choking process caused by the rapid deposition of silt all over the bottom; and in such a locality the remains of resident animals are of very slight importance in forming the bottom deposits.

Hamilton Harbor shows somewhat similar conditions, with a bottom of the same type. There is also another basin of this type which, though very much smaller, has a depth of nine fathoms; it is known as "Granaway Deep," and lies directly south of Marshall Island.

Harrington Sound is more completely landlocked than any of the basins hitherto described. At its southwest corner it connects with the open lagoon through the "Flatts Inlet," a channel barely sixty feet wide, and by several smaller subterranean passages. I cannot make any estimate of the amount of water which passes in and out through these channels, but in certain parts of the sound the tidal currents are of appreciable strength. Strangely enough, in spite of its landlocked condition, this sound is one of the richest localities for invertebrate life; from which we must draw the conclusion that its circulation of sea water is probably much greater than the narrowness of the known passages connecting it with the outer lagoon would indicate. The greatest depth is said by Heilprin ('**89**) to be sixteen fathoms; most of the basin is from six to ten fathoms deep. The bottom (at Station 404, one fathom), just off the beach at Trunk Island, is grayish white, mottled with red, brown, or yellow; it is composed of coarse sand, the largest fragments 15 mm. in diameter, and about 1 per cent fine ooze. This bottom differs markedly from any hitherto mentioned. There are very few Foraminifera, not more than 2 per cent; echinoid plates and spines, coralline algae, alcyonarian spicules, mollusks, corals, and polyzoans together form not more than 20 per cent; the remaining 75 per cent consists almost wholly of limestone fragments, ranging in diameter from 1 to 15 mm., of rounded and irregular shapes, and variously stained red and brown. They are the coarsest detritus from the neighboring cliffs.

In four fathoms of water, between Rabbit Island and the shore (Station 470), the sand is of the same color and composed of the same elements, but much finer, and with 5 per cent fine ooze. There are no large fragments, and the particles are of fairly uniform size, 1–5 mm. in diameter. The sand decreases in coarseness from shoal water near the shores to greater depths, with an increase of fine ooze; a sample from eight fathoms consists of very fine sand with 30 per cent ooze, and a few large fragments of dead coral. Living organisms seem to have added very little to the sands of Harrington Sound. The undercut and water-worn condition of the cliffs, and the huge limestone masses recently detached from them, bear witness to the effects of surf action on soft friable limestone, even in so limited an area as the Sound. Enormous quantities of detritus, both coarse and fine, must be constantly added to the beaches, the former remaining in shallow water, the latter being swept out and gradually deposited in greater depths beyond the reach of wave action. The bottom in this sound is more extensively derived from broken-down limestone than that of any other extended area which I have examined, and I believe that it is being added to with greater rapidity.

The most important and widespread of all the formations of sand that are now taking place occur on the northern side of the main islands, between them and the nearly continuous boundary reef. Both Heilprin and Agassiz have shown that this area consists of a series of sunken lagoons, bounded by more or less continuous ridges, the "Flats." At only one spot, the "North Rocks," do any of the flats rise above sea-level, but the depth in many places is so slight that they are nearly awash at low tide. The numerous ledges and shoal patches, although covered by a luxuriant growth of corals and gorgonians, are not of coral origin, but are ledges of the ordinary aeolian rock, over which the corals and other organisms are growing, and they furnish a great deal of material to the bottom sands. The depths of the lagoons are fairly uniform and moderate, being nowhere over twelve fathoms. The more important lagoons from which I have examined samples are Mangrove Bay, the Cow Ground Flats, Green Bay, Murray Anchorage, the Ship Channel, and Brackish Pond Flats.

Mangrove Bay, though open to the sea, is nearly surrounded by Somerset Island and, on the north, by a shoal flat, making it another enclosed catch basin, with a greatest depth of six fathoms. The bottom at Station 319, in five fathoms, is fine gray sand, containing many large fragments of millepore and oculina, evidently washed from the neighboring flat. The bulk of the sand consists of broken shells, sea-urchin plates and spines, and fragments of worm tubes, with about 20 per cent of very fine calcareous silt. The presence of great numbers of the calcareous spicules of sponges and gorgonians, and of the tests of small Foraminifera, show that it is of reef origin ; a small proportion of it, however, is apparently terrigenous silt, or "blue mud," and there are few living organisms. Most of this bottom is evidently derived from the surrounding shoals, being composed of various animal remains, together with a large amount of the fine silt resulting from the erosion of the limestone ledges. The rest of the material is of terrigenous origin, and resident organisms have had little or nothing to do in the formation of the deposit.

Murray Anchorage is the largest of the submerged lagoons, covering an area of about ten square miles. It extends from the shores of St. George's Island on the southeast to the Three Hill Shoals on the north, a distance of three or four miles. On the southwest it is bounded by Bailey's Bay Flats, and on the northeast by the boundary reefs to the north of St. George's Island. The bottom is level, with very few shoals or ledges, and the greatest depth, in the Ship Channel on the southern edge of the lagoon, is ten fathoms. The deposits over the bottom of the lagoon vary considerably in character in different localities, although they are everywhere white sand. North of Murray Anchorage, between Three Hill Shoals and the boundary reef at Station 324, in ten fathoms, this white sand consists of exceedingly minute particles, and has the appearance of a white, chalky ooze. Fragments of algae, millepores, and gasteropod and lamellibranch shells, with a few echinoid fragments and pieces of worm tubes, are distinguishable ; also a few Foraminifera. Over 70 per cent of the mass, however, consists of minute calcareous particles, forming a true marl. There are very few living organisms of any kind on this bottom, and their remains have added but slightly to it. The fine silt consists of the detritus from the destructive action of the water on the surrounding limestone reefs and ledges. In the southern and southeastern part of the lagoon, through which the Ship Channel runs, the character of the bottom is very different. We find here also, at Station 1416, in seven fathoms, a white sand, but it is much coarser, containing large quantities of corallines, molluscan shell fragments, and many living bivalves and gasteropods, with bits of worm tubes, plates of echinoids, millepore fragments, and a few large Foraminifera, chiefly Orbiculina and Orbitolites. There is about 10 per cent of fine calcareous silt, containing some spicules of sponges and gorgonians, which is similar in nature to the white marl of the northern half of the lagoon. This shell sand extends westward along the Ship Channel, at a depth of about eight fathoms, into the so-called "Brackish Pond Flats," directly north of the Flatts Inlet. In this inlet itself (Station 393), the very strong tidal flow has effectually scoured out all the finer materials, leaving only the coarser fragments. From the Ship Channel southward toward Crawl Point and Bailey's Bay, the water gradually shoals to about four fathoms; about a mile off shore, in seven fathoms (Station 00 ?), we again find the white marl bottom already described for the region north of Murray Anchorage. There are here more large shell fragments and more living Orbitolites and Orbiculina than in the more northern area, but organisms in general are very few. The proportion of calcareous sponge and gorgonian spicules is considerable; the rest of the deposit, about 60 per cent, consists of very fine water-worn silt.

Still nearer the shore, around the ledges known as the "Pigeon Rocks," about half a mile from Crawl Point, at Stations 411, 412, and 416, in about three fathoms, shell sand again occurs, the bottom being very clean, with less than 5 per cent ooze, though varying much in coarseness, some of it being very fine indeed. The transition from marl, at Station 00 ?, to this deposit of sand is very sudden, and is no doubt due to the shoaling water and the surf action around these detached ledges. About one mile to the eastward, at the mouth of Bailey's Bay (Station 317) in two to three fathoms the bottom is again coherent, having 30 to 40 per cent ooze and being light gray. The coarse material consists chiefly of molluscan fragments and Foraminifera, with a few living bivalves, gasteropods, and fragments of echinoids and corallines. This station and the Bay itself are sheltered localities, where fine silt is readily deposited.

Close in along the shores of the main or Bermuda Island and St.

George's Island, in one to three fathoms, the bottom is again shell sand. Thus, just off St. Catherine's Point (Stations 438, 439, 440, and 441), in from one to two fathoms, the bottom is very clean, the sand being very coarse, white with a faint yellowish tinge, and with less than 1 per cent of ooze. Foraminifera form less than 10 per cent, and the great bulk, probably 50 per cent, of the sample consists of coralline and millepore fragments of large size, the remainder being mollusk shells, echinoid plates, Polyzoa, corals, and limestone fragments. The fragments are all much water-worn and honeycombed, and there are very few living organisms in the bottom. In this neighborhood there are many detached ledges, and in the shoal water along shore, where the scouring action of the waves is considerable, the bottom consists almost wholly of the coarse detritus from them, and from the shore cliffs.

In all other parts of the submerged lagoons, except those already described, the bottom deposits consist of shell sands and marl, occurring together in varying proportions. But in such of the lagoons as are broken by great numbers of small reefs, the proportions in which these two materials occur often differ widely in closely adjacent localities. In such basins the depths over the floor are usually very uniform, and the shoals and ledges rise very abruptly to near the surface. Their walls are often nearly perpendicular, very much honeycombed, and often undercut by the action of the water. Examples are the Brackish Pond Flats, the flats north of Ireland Island, and the Cow Ground Flats, two miles west of that island. The Brackish Pond Flats occupy the centre of the Bermuda Plateau, and consist of a perfect maze of shoals, deep channels, and basins. At different stations the bottom deposits are very different. Thus at Station No. 359, three miles northeast of the dockyard, in five fathoms of water, it is chiefly shell sand, composed of large living Foraminifera in great abundance, particularly Orbiculina, Orbitolites, and Pulvinulinae, molluscan fragments, living lamellibranchs and gasteropods, millepore fragments, a few coral fragments, worm tubes, corallines, and calcareous spicules, with about 25 per cent of very fine calcareous silt. At another station (No. 383 ?), in a small basin one mile directly north of the last, in seven fathoms of water, the bottom consists almost entirely of white marl, but contains also a considerable amount of broken shells and calcareous spicules, although very few living organisms. On the Cow Ground Flats the topographic conditions are very similar. At a station one mile directly west of the dockyards, in a cut between the ledges eight fathoms deep the bottom was white and coherent, with 80 per cent fine ooze or marl, the remainder being chiefly corallines and millepores from the nearby ledges, with a few molluscan and other fragments.

Hogfish Cut is the only one of the several channels penetrating the boundary reef from which I have examined bottom samples. The collection contains five of these, from as many localities between the cut and the shore (Stations No. 430, 431, 434, 460, 461), which are almost indistinguishable. They are very clean, fine shell sand, whitish, with a faint pinkish tinge, with less than 1 per cent ooze. Although so clean, the sand itself is as fine as the bottom at Nonsuch Scaur. Foraminifera and their casts, especially Polystomella, form about 15 per cent; the remainder consists of corallines, mollusks, millepores, corals, ostracods, echinoderm remains, and Serpula tubes. There are very few living organisms of any kind in this sand, which resembles sand from Nonsuch Scaur so closely as to be almost indistinguishable from it.

From the area of the bank outside the boundary reef, between it and the thirty-fathom line, I have, unfortunately, no samples, and my only information is derived from the reports of fishermen and from what little could be seen through the water glasses. The vigorous growth of corals, gorgonians, millepores, etc., on the ocean faces of the reefs extends down to only about six or seven fathoms. Here begins the "broken ground" described by Agassiz (:**95**), as composed of various fragments and detritus from the reef itself, together with great numbers of nullipores. At about fifteen fathoms large sand patches appear, and in twenty fathoms the bottom is chiefly sand (admiralty chart).

General Considerations.

The study of these samples of sand from the shoal lagoons of the Bermuda Plateau is chiefly of interest for the light which it may throw on the method of growth of a limestone island in a latitude where reef-building corals are of but slight importance, and on the organisms chiefly concerned in this growth. The Bermudas, from their superficial resemblence to an atoll, and from the fact that their reefs are largely covered with corals, were considered as typical coral islands until the time of Agassiz's visit in 1895. It is from this point of view that Heilprin ('**89**) describes them, and Rein ('**81**) uses them as an example of an atoll in combating Darwin's theory of subsidence. Agassiz ('**95**) has, however, shown beyond possibility of doubt that this resemblance to a coral formation is purely superficial, that their ring-like form is due, not to coral growth, but to entirely different causes, and that coral has entered but slightly into their formation; a conclusion which is supported by the composition of the modern sands.

The oceanic character of Bermuda — its great distance from the neighboring continent — prevents, as might be expected, its receiving any important access of material from the continental slope, and consequently its submarine deposits consist almost entirely of materials of clearly local origin. The great scarcity of remains of pelagic organisms, Globigerinae, Radiolaria, Pteropoda, and the like, is a striking feature of all the bottom samples examined; and this is especially interesting in connection with the results of our pelagic towing, from which we were forced to conclude that the pelagic fauna of Bermuda, on the north side of the islands at least, is very poor, both in species and individuals. On the southern exposure of the bank the conditions might prove to be different. But by far the greater area of the bank lies north of the land crescent, which forms an effectual barrier against any surface currents which might be caused by the prevailing southerly winds. Castle Harbor is the only sound to which water from the south has free access, and it is interesting to note that several tropical pelagic organisms, such as Velella, Charybdea, etc., have been taken here, but never on the northern side of the islands, while some species of fish are also known to occur only on the south shore. Shells of Spirula, however, are found in considerable numbers on all of the beaches. The only objects found which might possibly be of volcanic origin were a few small particles of pumice.

The great bulk, at least 90 per cent, of all the bottom deposits, whatever their nature, is composed of calcareous material. Siliceous materials are exceedingly small in amount, and consist chiefly of the spicules of various organisms, there being no siliceous particles of terrigenous origin.

One of the most striking features of all the beaches and submerged sands is the complete absence of any true coral sand, and the great rarity of coral fragments of any kind. This is, of course, strictly in agreement with the fact that the Bermudas are in no sense a coral island, and that coral has had practically nothing to do with their formation and growth. Corals, of course, flourish on the reefs, but form merely a thin incrustation over the underlying aeolian ledges. That they do not enter more largely into the sands is due chiefly to the character of the forms occurring here. In all coral islands and coral reefs it is the madrepores which are chiefly responsible for the formation of sand, these being delicate and easily broken up. But here, as observed by Agassiz ('95, p. 235), the corals are mostly gorgonians, which are of course of no importance in this connection, and porites, astraeans, meandrinas, and oculinas; massive, stony forms, too solid to be ground up except by the very heaviest surf, which occurs only on the south shore. I should add that in speaking in general of corals, I have not meant to include the millepores, which have been of great importance in sand-building, as noted by Heilprin (**'89**).

The bottom deposits of Bermuda fall into three main types: first, the blue muds; second, the white marls; and third, the shell sands. Of course these may be combined in any one locality, but in their features they are tolerably distinct. The blue muds occur only in small, more or less landlocked basins which are cut off by bars or islands from any active circulation of sea water. They seem to be chiefly of terrigenous origin, being the fine detritus washed down by the rains from the surrounding calcareous hills, together with some vegetable remains. They are relatively bare of animal life, except for a few worms and for the sea urchins, which, as in Mullett Bay, may live in them in extraordinary numbers; and they agree perfectly in their mode of origin with the blue muds of the continental slope. The areas where they occur are so limited that they are of little practical importance and need not be further considered.

The white marls consist of white calcareous sand so fine as to form a chalky ooze, containing very few living organisms or recognizable remains. They occur chiefly in the deep basins of shallow waters in sheltered localities where there are no strong currents, and in the neighborhood of cliffs, ledges, and reefs. They are formed, as stated by Agassiz (**'95**), by the "slow trituration of aeolian rock."

A good example of the origin of these marls is afforded by the Cow Ground Flats, just west of Ireland Island, where there are innumerable banks and ledges, enclosing many small channels and basins, which have a depth of about seven fathoms. For some reason the mechanical destruction by the action of the water on these aeolian rocks is very rapid here, and the silt thus formed, which is of almost impalpable fineness, is being deposited in large quantities in the protected basins between the ledges. During strong winds the waters over the whole lagoon are often milky with the great quantity of this silt which they hold in suspension (Agassiz, '**95**, p. 212). In certain restricted localities, as mentioned above, the white marl forms almost the entire mass of the bottom deposit, and it is mixed in greater or less degree with the various deposits of shell sand.

The shell sands are composed of rather coarse fragments, with great numbers of living organisms, with, in every case, a considerable admixture of marl, and of various sorts of silt, calcareous and siliceous spicules, and the like. The organisms chiefly active in the formation of the shell sands are corallines, mollusks, tube-building worms, millepores, and Foraminifera, varying in their relative importance in different localities. Algae probably form the greatest mass of the sand, the most important genera being those described from the Castle Harbor sands. Large sand-dwelling Foraminifera form nearly half of the entire bottom deposits in certain restricted localities, the most abundant forms being Orbitolites duplex, Orbiculina adunca, Bulimina, Cornuspira foliacea, Pulvinulina menardii, Textularia concava, T. luculenta, Ammodiscus tenuis, and a species of Trochammina.

The chief deposits of shell sand occur in rather shoal water, or in the channels or other places where there are tidal currents, or at least strong currents of water. They may mark areas either of the growth of organisms or of deposition; the former being localities, such as parts of Castle Harbor and Castle Roads, where sedentary organisms, chiefly corallines, Foraminifera, and mollusks, flourish in the sand. It is in such places that the larger Foraminifera reach their greatest abundance. But these localities of growth are comparatively rare and restricted, by far the greater part of the shell sands marking areas of deposition of the coarse detritus from the reefs and ledges and from the littoral zone. Such sands are coarsest along shore, for example off St. Catherine's Point, and on the shoalest bars, as in Great Sound, where the action of the surf and of currents reaches its maximum; they decrease steadily and regularly in coarseness as the depth increases, until, as in Hogfish Cut, they may be very fine indeed, though still washed clean of fine ooze by the tides. In these sands, built of the organic fragments from the neighboring ledges, corallines and millepores are of the greatest importance, while mollusks and Foraminifera constitute but a small portion of the deposits. The reefs, as in true coral islands, contribute far more to the sands of the lagoon than do the sand-dwelling organisms, despite the local abundance of the latter. Heilprin ('89), it is true, thought that "all the sand being formed at present is derived from the destruction of existing land masses," that is, either marl or coarser limestone detritus; but I believe that the organic mains, which are constantly settling down from the reefs to the bottom, are in much greater bulk. It is, however, exceedingly difficult to estimate the relative importance of the marl and the shell-sand deposits over the plateau, since they always occur together, though in varying proportions. No doubt the formation of marl over many parts of the plateau is rapid; so rapid that it results in the choking of all organisms on the bottom, except where tides or waves sweep it away. A good example of such distribution of deposits is seen in Murray Anchorage, where, in the sheltered northern half, the bottom is almost wholly marl, while in the Ship Channel near by there is a strong tidal current and a bottom of shell sand. A striking example of the silting in of sedentary organisms was found by Herdman (:03) in his study of the pearl oyster beds of Ceylon, where, during a single year, a bed of several millions of young oysters was destroyed and entirely hidden by shifting sands.

Heilprin ('89) emphatically denies the supposition that solution can have any significant effect in keeping the lagoons and channels open; it is certain that the marl and shell sand are building up and levelling the surface of the plateau much faster than the water can possibly dissolve it, by which, however, I do not mean that the surface of the bank is as a whole rising. Agassiz ('95) has already shown the great importance of the solvent action of sea water on limestones in honeycombing and undermining cliffs and ledges, and in certain localities the same action is clearly shown in the worn and honeycombed condition of the shells and other fragments in the sand.

List of Stations at which Bottom Samples were collected.[*]

No.				Map No.
		1903.		
313.	64° 43′ 47″ W.;	32° 20′ 18″ N.	Off Crawl Point, 5 fathoms.	3
314.	64° 42′ 50″ W.;	32° 20′ 45″ N.	N. of [Bailey's] Bay Id., 8 fms.	2
317.	64° 43′ 03″ W.;	32° 20′ 30″ N.	Mouth of Bailey's Bay, 3 fms.	3
319.	64° 51′ 07″ W.;	32° 18′ 10″ N.	Mangrove Bay.	4
323.	64° 50′ 32″ W.;	32° 19′ 24″ N.	West of Ireland Id., 7 fms.	4
324.	64° 43′ 50″ W.;	32° 25′ 15″ N.	Three Hill Shoals, 8½ fms.	1

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[*] The position of each of the various stations is indicated on the accompanying maps by two marginal lines, one for latitude and one for longitude. After the maps were prepared it was discovered that there was a discrepancy of about 20″ in latitude between the British Admiralty Chart, on which Map 1 is based, and the recent Ordinance Survey Map, on which Maps 2–4 are based. A correction of *plus* 20″ is to be applied to the *latitude* of any place indicated on Maps 2–4 to bring it to the corresponding place on Map 1; or a correction of *minus* 20″ to places on Map 1 to make them fall at the right place on Maps 2–4.

The location of any given station was determined on one or other of these maps, and the latitude and longitude here recorded is that of the given place *on that map*. Where the haul extended over some distance, the middle point of the haul is the one noted on the map. — E. L. M.

355.	64° 41′ 05″ W.;	32° 22′ 05″ N.	Mullet Bay.	2
356.	64° 40′ 30″ W.;	32° 20′ 40″ N. (approximately).	Centre of Castle Harbor, 6 fms.	2
359.	64° 46′ 10″ W.;	32° 21′ 10″ N.	Brackish Pond Flats, 5 fms.	1
366.	64° 50′ 12″ W.;	32° 15′ 17″ N.	Little Sound, 9 fms.	4
1407.	64° 38′ 58″ W.;	32° 21′ 45″ N.	Smith's Sound, 4 fms.	2
1408.	64° 39′ 15″ W ;	32° 21′ 36″ N.	Dolly's Bay, 3 fms.	2
1409.	64° 38′ 57″ W.;	32° 21′ 56″ N.	St. George's Channel, 4 fms.	2
1413.	64° 39′ 40″ W.;	32° 20′ 05″ N.	Castle Roads, 5 fms.	2
1416.	64° 44′ 00″ W.;	32° 21′ 10″ N.	Ship Channel, near Crawl Flats, 6½ fms.	1
1417.	64° 43′ 18″ W.;	32° 22′ 21″ N.	South Side of Murray Anchorage, 8 fms.	1
1418.	64° 42′ 18″ W.;	32° 22′ 38″ N.	South Side of Murray Anchorage, 8 fms.	1

1904.

383.	64° 47′ 45″ W.;	32° 22′ 20″ N.	Brackish Pond Flats.	1
393.	64° 43′ 59″ W.;	32° 18′ 59″ N.	Flatts Inlet.	3
402.	64° 42′ 15″ W.;	32° 21′ 13″ N.	W. Entrance Ferry Reach.	2
404.	64° 43′ 02″ W.;	32° 19′ 35″ N.	Harrington Sound.	3
406.	64° 38′ 47″ W.;	32° 21′ 57″ N.	Entrance St. Geo. Channel.	2
407.	64° 39′ 12″ W.;	32° 20′ 33″ N.	Nonsuch Scaur.	2
408.	64° 39′ 08″ W.;	32° 20′ 28″ N.	Nonsuch Scaur.	2
409.	64° 39′ 40″ W.;	32° 20′ 00″ N.	Castle Roads.	2
411.	64° 43′ 20″ W.;	32° 20′ 28″ N.	Near Pigeon Rocks.	3
412.	64° 43′ 30″ W.;	32° 20′ 28″ N.	Near Pigeon Rocks.	3
415.	64° 43′ 45″ W.;	32° 20′ 17″ N.	Off Crawl Point.	3
416.	64° 43′ 22″ W.;	32° 20′ 30″ N.	Near Pigeon Rocks.	3
418.	64° 48′ 27″ W.;	32° 16′ 53″ N.	North Shore of Long Island.	4
419.	64° 48′ 32″ W.;	32° 16′ 52″ N.	North Shore of Long Island.	4
420.	64° 48′ 35″ W.;	32° 16′ 52″ N.	North Shore of Long Island.	4
421.	64° 48′ 10″ W.;	32° 17′ 05″ N.	Two Rock Passage.	4
422.	64° 47′ 55″ W.;	32° 17′ 25″ N.	Mouth of Fairyland Creek.	3
423.	64° 48′ 32″ W.;	32° 18′ 03″ N.	Cobbler's Cut.	4
427.	64° 45′ 10″ W.;	32° 27′ 30″ N.	North Ledge Flats.	1
428.	64° 45′ 10″ W.;	32° 26′ 30″ N.	Northwest of Three Hill Shoals.	1
429.	64° 45′ 10″ W.;	32° 25′ 20″ N.	West of Three Hill Shoals.	1
430.	64° 52′ 20″ W.;	32° 15′ 03″ N.	Hogfish Cut.	4
431.	64° 52′ 18″ W.;	32° 15′ 12″ N.	Hogfish Cut.	4

434.	64° 52′ 18″ W.;	32° 15′ 16″ N.	Hogfish Cut.	4
438.	64° 39′ 43″ W.;	32° 22′ 58″ N.	Off St. Catherine's Point.	2
439.	64° 39′ 50″ W.;	32° 22′ 55″ N.	Off St. Catherine's Point.	2
440.	64° 39′ 50″ W.;	32° 23′ 00″ N.	Off St. Catherine's Point.	2
441.	64° 40′ 00″ W.;	32° 23′ 05″ N.	Off St. Catherine's Point.	2
447.	64° 39′ 17″ W.;	32° 22′ 10″ N.	Higg's Cut.	2
448.	64° 39′ 20″ W.;	32° 22′ 16″ N.	Town Cut.	2
451.	64° 39′ 12″ W.;	32° 20′ 30″ N.	Nonsuch Scaur.	2
452.	64° 39′ 03″ W.;	52° 20′ 32″ N.	Nonsuch Scaur.	2
454.	64° 40′ 20″ W.;	32° 21′ 28″ N.	North Side of Castle Harbor.	2
456.	64° 48′ 55″ W.;	32° 16′ 05″ N.	Between Burt Island and Darrell Island.	4
458.	64° 50′ 36″ W.;	32° 15′ 51″ N.	Between Tucker's Island and Morgan's Island.	4
459.	64° 49′ 50″ W.;	32° 17′ 36″ N.	North of Great Sound.	4
460.	64° 52′ 28″ W.;	32° 15′ 25″ N.	Near Hogfish Cut.	4
461.	64° 52′ 20″ W.;	32° 15′ 20″ N.	Near Hogfish Cut.	4
463.	64° 47′ 25″ W.;	32° 16′ 23″ N.	Near Godet's Passage.	3
466.	64° 49′ 20″ W.;	32° 16′ 55″ N.	Between Hawkin's Island and Lambda Island.	4
467.	64° 38′ 48″ W.;	32° 20′ 37″ N.	Long Bay, Cooper's Island.	2
468.	64° 38′ 40″ W.;	32° 20′ 34″ N.	Long Bay, Cooper's Island.	2
470.	64° 43′ 20″ W.;	32° 19′ 35″ N.	Harrington Sound.	3

The Challenger Bank.

The Challenger Bank lies southwest of the Bermuda Bank, from which it is separated by a channel nine miles broad, and one thousand fathoms deep at its deepest part. Thus it forms a fairly independent elevation of the sea floor. If we take the one-hundred-fathom line as its margin, it is seven miles long (north and south) by six miles broad, covering an area of about forty square miles, and is roughly oval in outline. In its shoalest spot it rises to within twenty-four fathoms of the surface, but over most of its area the depth is between thirty and forty fathoms. Between the Bermuda and Challenger banks a single sounding of one thousand fathoms has been made (Agassiz, '**95**, Pl. II, fig. 1), and between the latter and the Argus Bank, which lies eight miles to the south, one of five hundred and thirty fathoms. One and one half miles northwest of the one-hundred-fathom line of the Challenger Bank, the depth is given on the admiralty chart as twelve hundred and fifty fathoms, a slope of about forty degrees. But on the eastern side the slope to a depth of seven hundred fathoms is more gradual, being about twenty-five degrees. The bank has thus the character of a very steep, isolated peak, some six thousand feet high. Although the sounding of five hundred and thirty fathoms shows that there is a ridge connecting the Challenger and Argus banks, on each side of which the water is much deeper, this in no way interferes with the isolated character of the bank as far as its fauna and bottom deposits are concerned.

Dredging on this bank was carried on from an ocean-going tug from July 31 to August 2, 1903, hauls being made at a number of stations.* Bottom samples were obtained from depths between thirty and fifty fathoms.

The only dredging ever done before on this bank was carried on by the Challenger, in April, 1873, when a single haul was made while the ship was anchored to allow soundings to be taken. Speaking of the results of this haul, Sir Wyville Thomson (**'77**, I, p. 360), says, "The bank, which seems to be about five miles across, consists mainly of rounded pebbles, of the substance of the Bermudas 'Serpuline reef.' There is an abundant growth all over the pebbles of the pretty little branching corals Madracis asperula and M. hillana." Murray and Renard (**'91**, p. 50–51), in their report on the deep-sea deposits collected by the Challenger, say that the bank is covered with corals, Serpula, and calcareous pebbles. Since that time the area of the bank has been developed by careful soundings, and the bottom is given on the admiralty charts as coral sand; but, so far as I am aware, no other dredging has ever been done on the bank.

Our dredge hauls brought up the same organisms as those taken by the Challenger; that is, Madracis, gorgonians, starfishes, mollusks, etc., together with large numbers of the calcareous pebbles, with which the dredge was ordinarily filled to the mouth. These pebbles ranged from two to six inches in diameter, the latter being the largest that could pass through the mouth of the dredge; but of course much larger ones may occur. Neither sand nor mud was brought up by us; in fact, no deposits whatever except these calcareous masses. These are roughly spherical and of a dark red or brownish color, being entirely incrusted with the nullipore Lithothamnion ungeri, and with Serpula tubes, small corals, bryozoans, and other organisms. Every one of these masses, although entirely coated with organisms, showed on one side or region nullipores that were dead; this side evidently had recently been undermost. They thus agree exactly in external appearance with the "pebbles" of the Challenger. But while both Thomson and Murray believed them to be composed of the same material as the Bermuda serpuline reefs, that is, of aeolian limestone, careful examination has shown them to be of an entirely different origin. When sawed into slices, the sections show that the pebble is composed of successive, more or less regularly concentric layers of Lithothamnion, together with a few worm tubes, bryozoans, and other incrusting organisms. The whole mass is more or less porous, often honeycombed by the borings of mollusks (Lithophagus), and the looser regions alternate with more compact layers. The mode of growth has evidently been that of successive depositions of lime by nullipores around some small central core, which may be very minute. This is not, of course, in any sense of the word a pebble, but is a true nullipore "head," in no way to be confounded with either aeolian pebbles or the mechanical concretions of lime or siliceous materials so often dredged in deep water.

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* For this opportunity the Biological Station was indebted to Captain William E. Myer of St. George's, who generously gave the use of his tug "Gladisfen" and her crew for this purpose.

The spherical form of these heads is of considerable interest. Such spherical nullipore masses occur very commonly in shoal water, either as concretions about some core, or as independent stalked growths, which eventually become detached and free to roll about. Such forms have been taken in great numbers off Eastport, Me., in a few feet of water, and also in many other localities where they are well within the sphere of wave action, and probably owe their spherical form to the fact that they are frequently rolled over and over. Such spherical masses have also been previously dredged in deep waters, where their mode of growth has been something of a problem. That some such process of wave action is at work on the Challenger Bank seems probable from the spherical form and worn appearance of the concretions, the latter condition being made evident by the broken nullipore branches. Without being occasionally overturned they would not attain a spherical form, but would probably take on forms similar to those of porites, astraeans, or other fixed massive corals. To account for the amount of disturbance of the bottom necessary to explain such spherical concretions, we must turn to a consideration of wave action; neither tides nor currents being sufficiently strong in this locality to afford an explanation.

Statements as to the depth to which the action of storm waves may extend are conflicting and unsatisfactory in the extreme. While it is often claimed that one hundred feet marks the limit to which wave action is important in the transference of material, I am assured by Professor

W. M. Davis that the oscillations can probably be detected to a depth of several thousand feet. Moreover, Alexander Agassiz ('**88**^{a}, p. 273) writes that off the New England coast disturbing forces due to the action of waves, tides, and currents may extend perhaps to a depth of nearly three hundred fathoms. But in this case currents are probably chiefly concerned. More definite and satisfactory are the opinions of Admiral Wharton ('**97**), who has suggested that the depth to which the action of waves extends may be indicated by the change of slope generally taking place off shore below a depth of eighty to one hundred fathoms; and, further, that the existence of banks in the open sea at a depth of from thirty to forty fathoms may show the limit of depth to which oceanic waves may cut down a land mass on which they act. Agassiz (:**03**^{a}, p. xviii.) also thinks there is good evidence of such submarine denudation. At first glance the Challenger Bank seems to present a good example of such a worn-down bank; dredging has shown, however, that its surface is not undergoing a process of denudation, but rather one of growth.

If we can accept Admiral Wharton's conclusions, — and they rest upon a much sounder basis than the earlier views which limited wave action to lesser depths, — we shall find in wave action, and especially in the oscillations of storm waves, a sufficient explanation for the present problem. I am convinced that the frequent movements of the nullipore concretions, which undoubtedly occur, are brought about in this way; and I consider the occurrence of such spherical concretions on the bank good evidence that at a depth of from thirty to fifty fathoms the action of storm waves is often of considerable force.

The dredgings from the Challenger Bank add to the evidence already accumulated to prove the great importance of nullipores as reef builders. The active rôle which they play in the economy of coral reefs, especially on the sea faces, and in localities where the corals are dead or dying, has been emphasized by Agassiz ('**88**^{a}, p. 82), who, writing of the Florida reefs, says, "Immense masses of nullipores and corallines grow on the shallowest flats at the tops of the branches of madrepores which have died from exposure to the air." J. Stanley Gardiner ('**98**, pp. 501, 502), in his studies on the island of Funafuti, also comes to the conclusion that they take a very important part in the building up of a coral reef. They are not, however, limited to shallow waters, but occur at considerable depths, being absolutely limited only by the absence of sunlight. Thus Agassiz ('**88**) found them smoothing over the modern limestones on the Pourtales Plateau, in depths of from ninety to three hundred fathoms, and in the borings at Funafuti nullipores seem to be the most important of all rock-building organisms, even at the extreme depth of 1120 feet, a depth of course well within their ordinary bathymetric distribution, as shown by the Pourtales Plateau. Apart from the considerable amount of material which they may add to any shoal, the great importance of the incrusting nullipores lies in their solidifying and cementing action. Thus, on the sea face of a reef, they are most effective in protecting it against the force of the surf, as, for example, in the case of the Bermuda serpuline reefs, a function which they share chiefly with the tube-building annelids. In the upbuilding of submarine banks they carry on a very similar process. The events now going forward on the Challenger Bank are probably as follows. The nullipores gradually form incrusting masses about various objects, or grow up independently on stalks which later become broken; in these ways the spherical concretions begin. Lying free on the bottom, subject to the action of the waves, they are occasionally turned during their growth so that they retain a spheroidal form, until at last they can no longer readily be moved; whereupon, lying motionless side by side, they become cemented together by the incrusting organisms, and thus come to form a solid modern limestone, which is produced much more rapidly than that which is formed from the loose shell sands. This process taking place over the Challenger Bank, where there is no direct evidence of either elevation or subsidence, has raised it to within some thirty to fifty fathoms of the surface of the sea; a depth where a few corals already flourish. If we imagine this process as continuing until the bank rises to within about twenty fathoms of the surface, we should then have excellent conditions for the formation of a coral reef. Of course in such upbuilding the nullipores constitute only a part, though a most important one, of the whole growth; the tests of Foraminifera, and the remains of other lime-secreting organisms, add largely to the accumulation of material.

The reports of the borings in Funafuti, a true coral island of the Ellice group, recently published (Bonney and others, :04) by the Royal Society of London, are of special interest in this connection. After two unsuccessful attempts, the drill was finally driven to a depth of 1114 feet, and the cores have since been carefully studied. Dr. G. J. Hinde has given (pp. 186–361) an elaborate account of the organisms; and the evidence goes to show that whether in the form of solid rock cores, or as incoherent granular particles, the material is entirely of organic character, traceable to the calcareous remains of animals and algae, of which Halimeda and Lithothamnion occur in greatest abundance. Professor Judd remarks that "from top to bottom the same organisms occur, sometimes plants, sometimes Foraminifera, and sometimes corals predominating"; and Mr. Finckh (p. 133), studying the biology of these reef-forming organisms, judges their relative importance to be in the following order: (1) Lithothamnion, (2) Halimeda, (3) the Foraminifera, (4) Corals.

In dredging on the sea slopes of the Funafuti atoll Lithothamnion was found abundant down to two hundred fathoms, and it plays a very important rôle in the modern reefs. One observation of particular interest was on the growth of a cluster of Halimeda (p. 146), which in six weeks attained a height and thickness of three inches.

The most interesting feature of the report, as far as concerns our study of the Challenger Bank, is the very great importance attaching to the coralline algae, both in the modern reefs and in the older rocks, as shown by the core from the borings. Putting aside all question as to changes of level of the sea floor, there can be no doubt that the corallines, with slight aid from other organisms, are capable of building limestones of great thickness; and I believe that they will be found to enter into the composition of most limestone banks, particularly those which, like Bermuda and the Challenger Bank, lie outside the belt of vigorous coral growth. To complete the survey of what I may call the "Bermuda range," it is most desirable that an examination should be made of the Argus Bank, which rises to within eight fathoms of the surface. No doubt, thanks to the shoal water, it will be found much more thickly covered with corals and gorgonians than the Challenger Bank; but in all probability its foundations and structure are the same, and I think we may consider the Challenger and Argus banks as two stages in the growth of such an exposed peak as the Bermudas.

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THE ACHROMATIZATION OF APPROXIMATELY MONOCHROMATIC INTERFERENCE FRINGES BY A HIGHLY DISPERSIVE MEDIUM, AND THE CONSEQUENT INCREASE IN THE ALLOWABLE PATH-DIFFERENCE.

By R. W. Wood.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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By R. W. Wood.

Presented by C. R. Cross, October 12, 1904. Received September 6, 1904.

The results recorded in the present paper were, for the most part, obtained during the progress of an investigation of the dispersion of sodium vapor. As I have mentioned in the previous paper, the path-difference under which it is possible to obtain interference-fringes with helium (D_3) light can be more than doubled by the introduction of a small amount of sodium vapor into the path of one of the interfering beams. This development of fringes far out in the system by the dispersive action of the vapor is accompanied by their complete disappearance at the centre of the system, where the difference of path is zero.

In order to understand this action of the vapor we must first consider briefly the conditions under which fringes may be visible.

Suppose that we have a system of circular fringes formed with white light, and consider a point just outside of the visible ring system, where the illumination appears uniform. Our fringe system is built up of an infinite number of colored systems which are in coincidence at the centre, but which get more and more out of step as we advance out into the system, owing to the fact that the "scale" on which the Newton rings are formed decreases with decreasing wave-length. Let us now consider in what manner fringes may be made to appear at a point where the overlapping is so great as to destroy all trace of the fringes; in other words, how may achromatization be more or less completely secured.

It appears to me that there are only two conceivable ways in which the result can be obtained. If we could, by the introduction of a dispersing medium, increase the diameters of the blue rings without generally affecting the diameters of the red ones, it is obvious that we should greatly increase the number of visible fringes without, however, altering their distinctness at the centre of the system.

A slight inclination of either of the back mirrors of the interferometer increases or diminishes the scale on which the fringes are formed, and since a similar change in the direction of the reflected rays can be effected by the introduction of an acute prism, it is easy to see that, owing to the dispersion of the latter, the change in the scale will be different for the different wave-lengths, more or less perfect achromatization resulting.

The introduction of a medium into the path of one of the interfering beams causes a shift of the fringe system as a whole, and if the medium is dispersing the shifts will be different for the different colors. The red, green, and blue fringes, which are out of step at a given point, may thus be brought into coincidence by the inequality of their respective displacements. In this case, however, since the systems are shifted as a whole, the fringes will be thrown out of step at the centre of the system, consequently we have obtained an increase in the distinctness far out in the system, at the expense of distinctness at the centre. This is precisely what happens in the case which we are considering.

It has been found in every case that the introduction of sodium vapor into one path of the interferometer increases the distinctness of the fringes in a portion of the system which is brought into the field of the instrument by increasing the length of the other path.

We will now consider the case of the helium fringes, which under ordinary circumstances disappear when the path difference is between 1.5 and 2 cms., there being no recurrence of visibility by further increment of path difference, as in the case of sodium light. We must therefore regard the helium (D_3) line as a single line of finite breadth or a close group of lines. In Figure 1 let BC represent the intensity curve of the helium light, C being the edge of shorter wave-length. Immediately above we have a schematic representatation of the fringe system, with its centre at A. Light from the side B of the D_3 line will produce the fringes indicated by the dotted line, which are farther apart than the fringes formed by the light of shorter wave-length coming from the side C of the line. There will, in addition, be an infinite number of other systems formed by light of wave-lengths intermediate between B and C which I have indicated by light shading.

Now suppose sodium vapor to be introduced into one path of the instrument, and the whole system shifted slightly to the left in consequence. Owing to the enormous dispersive power of the vapor, the dotted system (longer λ's) will be shifted more than the other, since the D_3 line lies on the blue side of the sodium absorption-band, and the change in the velocity of the light is greatest for the longest waves, namely, those on the B side of the line. The result of this dispersive action is that the fringes are brought into step at a point D, to the right of the centre, thrown out of step at the centre, and still more out of step to the left of the centre. If we had but the two systems indicated by the solid and dotted lines, it is obvious that the systems would come into step again to the left of the centre, a condition which would occur if D_3 consisted of two infinitely narrow lines very close together. In the actual case the presence of waves of lengths intermediate between those of B and C make such a recurrence of visibility to the left of the centre impossible, and we have distinct fringes to one side only of the original centre of the system. On increasing the density of the sodium vapor, the point D of maximum visibility moves further along to the right, and to keep the fringes in the field it is necessary to turn the screw of the instrument in such a direction as to cause the system to move in *the same direction* as the shift due to the sodium vapor.

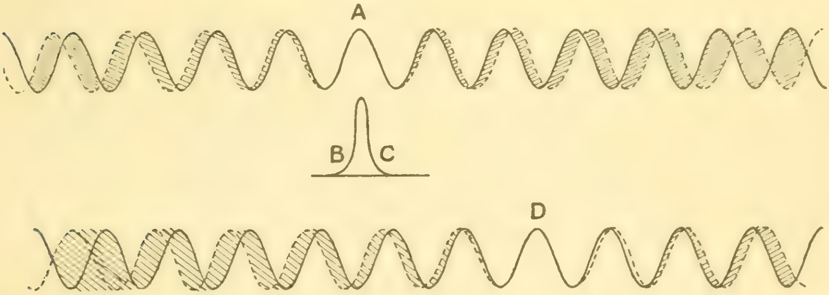

Figure 1.

Now the sodium vapor accelerates the helium light, since its refractive index is less than unity for light of shorter wave-length than that of D_3, consequently the reduced path is less. To shift the fringes in the same direction as that resulting from the shortening of the path through the sodium vapor, we must *lengthen* the other or air-path, which is precisely what was found to be the case, as I have already said.

If the D_3 line lay on the other side of the D lines, the shift would be in the opposite direction, i. e. to the right, and we might at first sight expect the point of maximum visibility to shift to the left of the centre. We must, however, remember that in this case the change of velocity is greatest for the *shortest* waves on the side C of the line; consequently the system indicated by the solid line will suffer the greatest displacement, and we shall have coincidence at D, to the right of the centre, exactly as before. To test this point experimentally, the interferometer was illuminated with light from the monochromatic illuminator, a narrow band on the green side of the D lines being utilized. The formation of sodium vapor in one of the paths gave rise to the same changes as were produced in the case of the helium light, it being necessary to increase the air-path to prevent the fringes from disappearing. If the fringes were made very narrow, so as to occupy only a small portion of the field, the wandering of the system to one side could be easily watched, as the sodium vapor was formed. It must be understood that only a very small actual *displacement* occurs, the wandering of the system being merely a change in position of the region over which fringes can be seen. On repeating the experiment with a band of approximately monochromatic light on the red side of the D lines, a similar drift of the position of maximum visibility was observed, and the direction of the drift was the same as before. In the case of helium light I have been able to increase the path difference to five or six centimeters, or to nearly treble it.

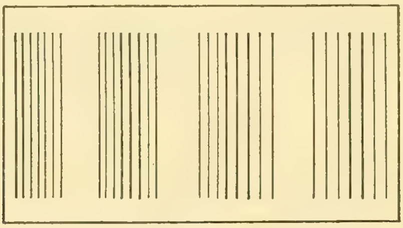

FIGURE 2.

The achromatizing action of the sodium vapor is most beautifully shown if we illumine the interferometer with white light.

Under ordinary conditions only two or three white fringes are seen, bordered on each side by perhaps a dozen rainbow-colored bands, which fade rapidly into a uniform illumination. If sodium vapor is formed in one of the interferometer paths, the colored fringes rapidly achromatize, and increase in number, breaking up, however, into groups, as shown in Figure 2. As the density of the vapor increases the number of groups increases, each group, however, containing fewer fringes. The position of the centre of the grouped system drifts in the same direction, as the point of maximum visibility in the previous experiments.

The explanation of the altered appearance of the fringes in this case is not as simple as in those previously considered. We are dealing with two wide ranges of wave-lengths on opposite sides of the absorption-band. The fringe shifts of the two spectral regions will be in opposite directions, while the drifts of the points of maximum visibility will be in the same direction. It appeared as if this might increase the width of the region over which fringes could be observed, for the red-orange fringes are shifted in one direction and the yellow-green in the opposite. Each set would be more or less perfectly achromatized, and in the region in which they overlapped we should expect a periodic visibility, owing to the difference in the widths of the fringes of the two systems.

To test the point it seemed best to work with a narrow range of the spectrum symmetrical about the D lines. This was obtained by opening the slit of the monochromatic illuminator, bisecting it with a wire, and adjusting the prisms so that the region of the D lines was screened off by the wire. By means of a small screen either of the two narrow portions of the spectrum bordering the D lines could be screened off.

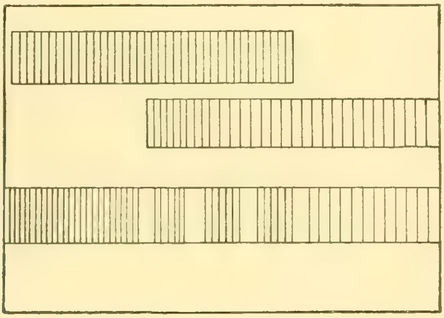

Figure 3.

The effect of the sodium vapor on the fringes formed when the interferometer was illumined by either one or both of the two portions of the spectrum could then be studied at leisure.

It was found that when a considerable amount of the vapor was present the apparent centre of the greenish-yellow fringe system was widely separated from the centre of the orange-yellow system.

When both sorts of light were used at once there was a periodic visibility in the region in which the two systems overlapped, the appearance in the three cases being shown in Figure 3.

The case is a little more complicated when white light, or the entire spectrum, is used, but it does not differ materially from the special case just considered.

Practically the same thing occurs when the interferometer is illuminated with sodium light, except that in this case the density of the sodium vapor in the optical path must be very much smaller. A periodic visibility results even when the light of one of the D lines is removed by the polarizing system described in the previous paper. The case is of course similar to the last-mentioned, for the width of the D line illuminating the instrument is greater than the width of the absorption-band of the rare vapor. We thus have a condition identical with that which we had when the emitting slit of the monochromatic illuminator was bisected with a wire which cut out the D lines from the narrow band of the spectrum which was utilized.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF STRONTIUM.

SECOND PAPER. — THE ANALYSIS OF STRONTIC CHLORIDE.

By Theodore William Richards.

A REVISION OF THE ATOMIC WEIGHT OF STRONTIUM.

SECOND PAPER. — THE ANALYSIS OF STRONTIC CHLORIDE.

By Theodore William Richards.

Presented January 11, 1905. Received January 7, 1905.

About ten years ago the first paper from this laboratory upon the subject of the atomic weight of strontium* showed that the usually accepted value 87.5 was probably about 0.2 per cent too low. This result was not a surprise, for the error had been predicted, upon the basis of a discussion of the faulty methods used by previous experimenters. The new results seemed to be trustworthy, and to represent, as well as a single series of experiments could, the true value of the constant sought.

The determination of an atomic weight by a single method is nevertheless unsatisfactory, no matter how admirable that method may be. Of course the verdict of a good method and a series of good analyses is much reinforced by an experimental proof of the flaws in other discrepant data; but the greatest of certainty is to be obtained only when the subject has been approached from several wholly different points of view. With this idea in mind, therefore, the analysis of strontic chloride was undertaken in order to confirm or disprove, as the case might be, the results of the analysis of strontic bromide described in 1894. The investigation was made in 1895–6, but has remained unpublished because of an incomprehensible discrepancy, now explained.

The balances and weights employed were those used in previous work, to which the reader is referred for all details. All weights were reduced to the vacuum standard as usual. The specific gravity of strontic chloride taken for this purpose was that given by Schroeder, 3.05, which was assumed to be correct, because his determination for baric chloride under similar conditions has often been confirmed. All the care taken in previous work at this laboratory was exercised in this case; it is unnecessary to repeat the several details.

The sample of strontic chloride used for analysis was made from a portion of the strontic carbonate remaining from the third preparation of strontic bromide,* and was therefore no less pure than this specimen. The carbonate had been prepared in the following manner. The so-called "pure" strontic chloride of commerce was dissolved in water, treated with ammonic hydrate and a little carbonate, and filtered from the precipitate containing iron, aluminum, and so forth. To the filtrate was added an excess of sulphuric acid, and the precipitated strontic sulphate was thoroughly washed with dilute sulphuric acid and then with pure water, in the hope of freeing it from magnesium and calcium. When the wash water became neutral to methyl orange the precipitate was treated with enough ammonic carbonate solution to convert about half of it into carbonate, and the mixed precipitate was then washed with water by decantation until only a very small constant trace of sulphuric acid (due to strontic sulphate) was found in the decantate. The carbonate was then decomposed by pure hydrochloric acid, and the solution was allowed to stand in a glass flask for nine months over the undecomposed sulphate, with occasional shaking.† Thus all but a minute trace of the barium was eliminated. The strontic chloride was decanted, the sulphate washed once with water, and the filtered decanted liquid was evaporated in a platinum dish until most of the free hydrochloric acid had been expelled. The dissolved residue was neutralized with ammonia, shaken with a little ammonic carbonate, and then filtered. To the greatly diluted filtrate was added an excess of pure ammonic carbonate, and the precipitate was washed until the wash water was free from chlorine. The strontic carbonate was dissolved in nitric acid which had been twice distilled in platinum, and the nitrate was crystallized twice successively in a platinum dish. Each quantity of crystals was washed with small quantities of water and three or four additions of alcohol. The first mother liquor, upon being fractionally precipitated by means of alcohol, showed distinct traces of calcium in the extreme solution; thus Barthe and Falière's method was not capable of freeing the substance wholly from calcium. The second nitrate mother liquor showed no trace of calcium upon the most careful spectrometric scrutiny, although the test is one of very great sensitiveness; therefore the crystals could have contained none.‡ This effectual elimination of calcium was due no doubt to the difference in crystalline form between the calcium and strontium salts.

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* These Proceedings, **30**, 369 (1894).

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* These Proceedings, **30**, 377 (1894).

† Barthe and Falières, Journ. Chem. Soc., Abs. 1892, p. 1277. Bull. Soc. Chim., [3], **7**, 104.

‡ Richards, These Proceedings, **28**, 7 (1893).

Two hundred grams of the purest crystals, after having been dried at 139°, were dissolved in about a liter of the purest water and filtered into a large platinum dish, into which was passed at first pure ammonia gas and then pure carbon dioxide through a platinum tube. The pure strontic carbonate was washed by decantation eight or ten times, dried on the steam bath, and ignited in a double platinum crucible over a Berzelius lamp. Thus were residual traces of nitrate decomposed.

The mixture of oxide and carbonate obtained in this manner was dissolved in very pure hydrochloric acid, which had been distilled in a platinum still. The chloride was evaporated to dryness, fused in a platinum dish to render silica insoluble, dissolved in purest water, allowed to stand, filtered, acidified, crystallized, and dried. That used for Analysis 1 was crystallized only once, that used for Analysis 4 was crystallized three times, while that used for Analyses 2 and 3 came from the evaporated mother liquors of the last crystallization. The essential agreement of the results from these three fractions shows that if any barium had escaped separation as sulphate, it was eliminated by the first recrystallization as chloride. This is not surprising, since baric chloride is heteromorphous with the strontic salt, these substances crystallizing with two and six molecules of water respectively, at ordinary temperatures. The previous double crystallization as nitrate could not have eliminated barium, since strontic and baric nitrates cystallize in similar anhydrous forms.

The silver used was similar to that used in the work on magnesic chloride.* It was finally prepared for weighing by fusing the electrolytic crystals in a boat of lime in a vacuum. Subsequent work has shown that this silver may have contained as much as $\frac{1}{30000}$ of its weight of oxygen, derived from the decomposition of the included argentic nitrate from which the crystals were deposited; but no correction is made for this trace of impurity, because its amount is uncertain, and because the analytical work can hardly be accurate to within one part in thirty thousand.

Before analysis the strontic chloride was fused in a current of hydrochloric acid and nitrogen, cooled in pure dry nitrogen, and transferred in its platinum boat to the stoppered weighing bottle in pure dry air. For the first determination the apparatus used with the bromide was employed for these operations, but in the last three analyses the improved form devised for the work on magnesic chloride was employed.†

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* Richards and Parker, These Proceedings, **32**, 62 (1896).

† Ibid., **32**, 58 (1896).

These last three determinations were very kindly made by Dr. H. G. Parker immediately after the completion of the magnesium work, and all the precautions discussed in that description were heeded. It is a pleasure to express my indebtedness to Dr. Parker for this assistance.

The analysis of the three fractions of crystallized salt were made by comparison with silver, using solutions at least as dilute as decinormal, and determining the end point by observing with the help of the nephelometer when the supernatant mother liquor contained equal concentrations of silver and chlorine.

A preliminary experiment with salt of less purity need not be detailed. The table below contains the essential data and results of the four consecutive final determinations, the only ones which were made.

The Ratio of Strontic Chloride to Silver.

Number of Determination.	Weight of Fused Anhydrous $SrCl_2$ (in Vacuum).	Weight of Fused Silver (in Vacuum).	Ratio 2 Ag : $SrCl_2$ = 100.000 : x.
	grams.	grams.	$x =$
1	4.2516	5.7864	73.476
2	2.4019	3.2688	73.480
3	3.5184	4.7886	73.475
4	3.0264	4.1189	73.476
			73.477

If silver is assumed to have an atomic weight of 107.93, and chlorine 35.455, according to usual custom, the atomic weight of strontium calculated from these results becomes 87.697. This result, greater by 0.033 than the value found from the bromide, indicated the presence somewhere of an unknown source of error; and the results were left unpublished for ten years because of the doubt therefore attaching to them.

This doubt has now been wholly dispelled by the discovery of an error in the assumed atomic weight of chlorine. It now appears that if silver is taken as 107.93, chlorine must be 35.473;* and the atomic weight of strontium calculated upon this basis becomes 87.661, a value essentially identical with that found from the bromide, 87.663.

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* Richards and Wells, in an investigation now being published by the Carnegie Institution. The somewhat lower value, 35.467, which was first announced as the outcome of this work, was the result of preliminary experiments only.

It must be borne in mind that the standard Ag = 107.930 is probably not precise, if oxygen is taken as 16.000. The recent work just cited has shown that the silver of Stas must have contained weighable traces of impurity, and therefore that silver must have a lower atomic weight than 107.93. Because, however, the exact value is still uncertain, the value 107.930 is still retained, in order to avoid an additional shift from one arbitrary standard to another. An alteration in the assumed value for silver affects that for strontium in direct proportion, and may therefore be applied at any time.

SUMMARY.

In this paper are detailed four determinations of the atomic weight of strontium, by comparison of strontic chloride with silver. Calculated upon the basis of the old incorrect value for chlorine, these results yield a value higher than that obtained from strontic bromide; but on the basis of the new value for chlorine 35.473 (silver being assumed as 107.930) the two different series yield almost identical results. Accordingly the new value for chlorine is confirmed by this investigation. The atomic weight of strontium is thus found as follows (Ag = 107.930).

From $SrBr_2$; $Sr = 87.663$
From $SrCl_2$; $Sr = 87.661$
Average $Sr = 87.662$

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

VISCOSITY OF AIR.

By J. L. Hogg.

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VISCOSITY OF AIR.

By J. L. Hogg.

Presented by John Trowbridge, December 14, 1904. Received January 9, 1905.

From the standpoint of the Kinetic Theory of Gases the viscosity, or internal friction, of a gas is a very important property, and, on this account, it is not surprising that a determination of the coefficient of viscosity of various gases has been the subject of investigation for many scientists. Air has been submitted to investigation probably more frequently than any other gas, but although many careful experiments have been performed and though the results of the more recent measurements show much better agreement than the earlier results showed, yet there is still some lack of agreement in the results obtained by different investigators using the same method as well as in those obtained by different methods. For example, O. E. Meyer * using one method obtained results ranging from 0.000168 to 0.000174 for the value of the coefficient of viscosity at 0° C. Breitenbach,† using the same method, got 0.0001733, which is in good agreement with the result obtained by Schultze ‡ and also with that obtained by Markowski. § By the same method, however, Schneebeli || obtained 0.0001707 and Obermeyer ¶ 0.0001705. Tomlinson,** using a method involving another principle, obtained as a mean result 0.0001715, while F. G. Reynolds,†† by the same general method, obtained 0.0001767.

Since experiments on the relation of viscosity to temperature show a comparatively slow change in viscosity with changing temperature, and since the temperature of the gas is the only property of it which is likely to be subject to even moderate change during the course of an experiment, it would seem that it should be possible to determine the

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* Pogg. Ann., cxlviii.

† Ann. der Phys., **5**, 1901.

‡ Ann. der Phys., **5**, 1901.

§ Ann. der Phys., **9**, 1904.

|| Arch. Sci. Phys. et Nat., **14**, 1885.

¶ Wiener Sitzungsber. Abth., **2**, 1875.

** Phil. Trans., **177**, 1886.

†† Phys. Rev., July, 1904.

coefficient of viscosity with more precision than that indicated by the results hitherto obtained. It would appear that there may be some cause for the discrepancies other than the mere accidental errors which are present in every investigation. A further examination of the subject would, at least, seem justifiable.

Without reference to the definition of viscosity from the point of view of the Theory of Gases, it may be defined as that property of a gas by virtue of which resistance is offered to the motion of one portion of the gas relative to the contiguous portion. The coefficient of viscosity of the gas is defined as the force per unit surface which must be applied parallel to the surface of a plane placed in a mass of gas at unit distance from a fixed plane, parallel to the former, so that the moving plane may have unit velocity in the direction of the force. To measure this coefficient, which is usually denoted by μ, a shearing strain must be produced in the gas and the force necessary to do the shearing measured. The ideal shear would be that obtained by causing one plane in a mass of gas to move with a constant velocity parallel to another fixed plane; but the planes would have to be infinite to avoid the edge effects, so that this method is not practicable.

One of the two general methods of shearing which approach this ideal case is by oscillating a solid body in the gas. The resistance offered by the gas to the motion of the solid is determined by observing the diminution of the amplitude of the oscillations. The quantity actually observed is the arc of swing, and the decrement in the logarithm of this arc is then calculated. In order, then, to determine the coefficient of viscosity, there must be a mathematical expression showing the relation it bears to this logarithmic decrement. The desired relation has been obtained in the case of pendulums having the form of a sphere, an infinitely long circular cylinder, and a circular plate oscillating in its own plane between and parallel to two fixed disks. The cases of the sphere and cylinder were first discussed by Professor Stokes,* while that of the disk was treated by Maxwell. The other method of producing the desired strain, called the transpiration method, consists in causing the gas in question to flow through a tube of narrow bore, a small constant difference of pressure being maintained between the ends of the tube. O. E. Meyer † has discussed the flow of the gas in such circumstances and has deduced an expression for μ in terms of the radius and length of the tube, the mean pressure in the gas, and the excess of the pressure at one end over that at the other.

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* Camb. Phil. Soc. Proc., **9**. Also a note to Tomlinson's paper, Phil., Trans., 1886.

† Pogg. Ann., **127**, 1866.

That there are discrepancies in the results obtained is probably accounted for, as Meyer has pointed out, by the failure of the experimental methods to conform to the conditions imposed in the mathematical analysis. Whatever method is chosen, then, the prime requisite is to be certain that the mathematical conditions are fulfilled.

As has been said, Maxwell gave a theoretical discussion of the case of a circular plate performing oscillations between two fixed parallel disks and parallel to them. He used the formula obtained to get the value of μ, but his mean result is much larger than that obtained by more recent investigators using in some cases the transpiration method and in others the oscillation method. The probable cause of this was discussed by Stokes in a note appended to Tomlinson's paper on the determination of μ. Stokes showed that unless the swinging disk is adjusted so that, as it swings, it deviates very little from the horizontal plane, there will be an appreciable loss of energy, on the part of the moving disk, due to the crowding of the air between the fixed and moving disks.

Tomlinson, in the investigation referred to, by oscillating first a long cylinder and then a short one whose time of swing was the same as that of the long one, obtained, by eliminating in this way the end effects, practically a cylinder of infinite length, and was thus able to make use of Stokes's formula for the case of an infinitely long cylinder, suspended vertically and performing oscillations about its axis. There does not seem to have been any objection raised to the use of this formula for the determination of μ. His most consistent results were those obtained in this way from a single cylinder, but during the course of the investigation he made use of a system of two vertical cylinders, and of one of two spheres suspended side by side.

More recently F. G. Reynolds used a single spherical shell, and also a single cylinder, and he obtained results from the two methods which agree well with each other, but are considerably larger than the mean of Tomlinson's results, and larger than the result obtained by the latter with a single cylinder. For μ at 0° C., Tomlinson got 0.0001715, using the single cylinder, while F. G. Reynolds got 0.0001768.

The seeming completeness and simplicity of the theoretical treatment of the motion of an infinite fluid in which a solid sphere is oscillating about a vertical diameter commends for use in determining μ a pendulum spherical in form, provided it can be shown that, under the given experimental conditions, any quantities neglected in the solution of the differential equations involved are really negligible.

Professor Stokes, in his discussion of the motion of an infinite viscous fluid disturbed by a sphere oscillating about a vertical diameter, neglects, in the general equations expressing the motion, terms which are of the second degree; e. g., the square of the velocity of the fluid at any point is neglected. The assumption that these terms are negligible leads to a comparatively simple expression relating the logarithmic decrement, the density of the fluid, the inertia of the whole pendulum, the time of swing, and μ the coefficient of viscosity. But it has been shown that in the case of the sphere the terms which Stokes has neglected are not by any means always negligible.* These terms, it appears, are of the order of magnitude of $V^2 a \rho / \mu$, where V is the velocity of the fluid at any point, a the radius of the sphere, ρ the density of the fluid, and μ the coefficient of viscosity. In order, then, that Stokes's solution may be valid, the quantity $V a \rho / \mu$ must be small. V and a are the quantities which can be varied in this expression, and if they can be made sufficiently small, the determination of μ should be rendered merely a question of manipulation.

Two very good tests can be applied to determine whether the desired conditions have been obtained. In the first place, if $V^2 a \rho / \mu$ is too great to be neglected, then as V changes so does the relation between the velocity with which the solid body moves and the resistance with which it meets, and therefore the logarithmic decrement must change as the velocity changes. If, therefore, the period of vibration remains constant, the decrement measured when the pendulum is moving through large arcs should be different from that measured when the arcs are small. Thus, if the pendulum be given a certain amplitude of oscillation at the start, and the decrement be measured from arc to arc, there should be a gradual change in its value as the arc of swing diminishes. Again, when the period of oscillation, the radius of the sphere, and the angle through which the sphere moves are known, the maximum value of V can be calculated and compared with its square. Also, by using one of the values of μ, given above, the value of the quantity $V a \rho / \mu$ can be calculated.

Stokes's discussion of the sphere oscillating in an infinite viscous fluid leads to the following expression for the logarithmic decrement:

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* Whitehead, Quar. Jour. Math., **23**, 1889. Lord Rayleigh, Phil. Mag., **36** 1893.

$$\text{Nap. log. dec.} = \frac{\mu' M'}{M K^2} \cdot \left[\frac{v a + 3 + \frac{3}{v a} + \frac{3}{2 v^2 a^2}}{1 + \frac{1}{v a} + \frac{1}{2 v^2 a^2}} \right] T.$$

Where $\mu' = \frac{\mu}{\rho}$, μ is the coefficient of viscosity and ρ the density, $M K^2$ is the moment of inertia of the pendulum, a is the radius of the sphere, T the time of a single swing of the pendulum, M' the mass of the fluid displaced by the sphere, and

$$v = \sqrt{\frac{\pi}{2 \mu' T}} = \sqrt{\frac{\pi \rho}{2 \mu T}}.$$

The decrement here considered is, of course, that due to the resistance of the fluid on the sphere alone.

General Method.

The constancy of the elasticity of quartz fibres commended the use of one of them as a suspension. Since the sphere had to be provided with a stiff wire support to which to fasten the fibre, and on which to support a small mirror to be used in reading the arc of swing, it was necessary to devise a plan by which the friction of the air on the mirror, wire, and fibre, and also the friction in the fibre itself, might be eliminated. The following plan was used. First, a sphere was made of suitable size. The size was accurately measured, and the density of the material of which it was composed determined. Another material was then chosen whose density was two or three times that of the sphere. Two disks of the same dimensions were then made, one of the heavy material and the other of the same material as that of which the sphere was made. The dimensions of the disks were determined by two relations. First, the moment of inertia of the heavy disk about its axis had to be the sum of the moment of inertia of the sphere about a diameter and of the light disk about its axis; and second, the weight of the heavy disk had to be that of the sphere and light disk. Thus, a pendulum formed of the sphere and light disk fixed on a wire passing along the axis of the disk and a diameter of the sphere would have the same period of swing as that of a pendulum consisting of the heavy disk alone mounted as the light disk was. The tension of the supporting fibre would also be the same in the two cases. Two experiments were then performed, one to get the decrement when the vibrator consisted of the sphere and light disk, supported as described on the wire, and the other to get the decrement when it consisted of the heavy disk supported on the wire at the same place as that formerly occupied by the light disk. The difference between these decrements was that due to the resistance of the air on the sphere alone.

Description of Apparatus.

Figure 1 shows the form of the apparatus which was used. A is a glass globe about 14 cm. in diameter to which, above, is connected the tube, B, and below the tube, C. The upper end of B is ground to fit the cap, D, while the lower end of C is ground to fit the mouth of the drying tube, E. The cap, D, is continued above into the tube, F, through which a brass rod surrounded by a rubber tube can just pass. To the lower end of the brass rod the fibre is attached. This method of supporting the fibre enables one to raise or lower the whole suspension very easily. The fibre is about 0.0017 cm. in diameter, and about 35 cm. in length. It is fastened at the lower end by means of shellac to an aluminum wire, G, which bears the mirror, the disk, and the sphere, H. The wire is about 12 cm. long and 0.075 cm. in diameter. The sphere, as finally used, is of glass 1.499 cm. in diameter, the variation in its diameters being less than 0.001 cm. It weighs 4.495 grams. The disk finally used is of glass. It is 1.339 cm. in diameter, 0.364 cm. in thickness, and weighs 1.297 grams. It was made with as great accuracy as the sphere. The sphere and disk are shown at Figure 2. The heavy disk is made of lead. It is of the same size as the glass disk, and weighs 5.793 grams. The mean radius of the lead disk differs only by about 0.001 cm. from that of the glass disk. The difference in thickness is less than 0.001 cm. The disk is fastened on the

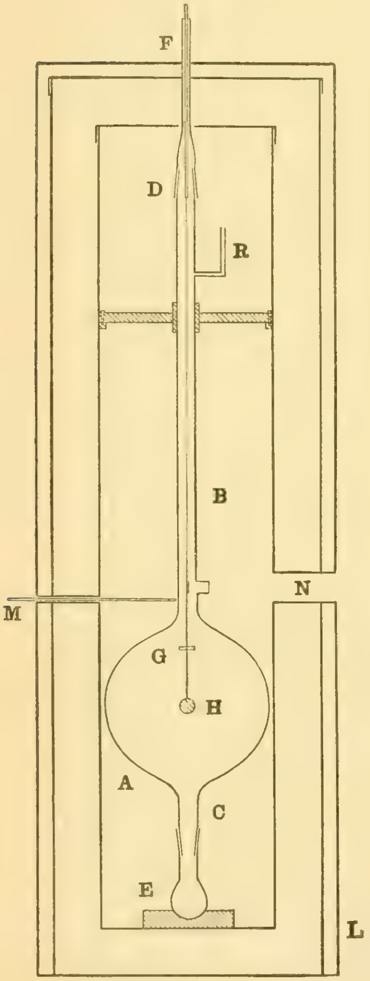

Figure 1.

wire about 5 cm. above the sphere, and is carefully fixed so that the wire is at right angles to its plane. The sphere also is adjusted so that the wire passes along a diameter. The adjustment is maintained by a small quantity of beeswax. The containing vessel, L, the top of which is removable, is a double-walled tin vessel, having an interspace of about 5 cm. This space is filled with water, while a heavy coating of hair felt surrounds the whole. An extra thickness of felt is placed on the top, as here there is no water protection. M is a thermometer. N is the glass-covered opening in the vessel, L, through which the image of a scale can be observed in a telescope by reflection from the mirror. On the top of the containing vessel is a card with its centre at the centre of the lid, and to the prolongation of the cap, D, is fastened an index which marks the zero position of D, i. e., the position in which it must be placed that the front of the mirror may be directed towards the opening, N. By giving D a slight turn a start can be given to the vibrator. A carefully constructed manometer is connected to the tube R, which is also attached to a mercury pump. The connection to the pump has a branch in which is placed a drying tube and through which, if desired, air can be admitted into the apparatus.

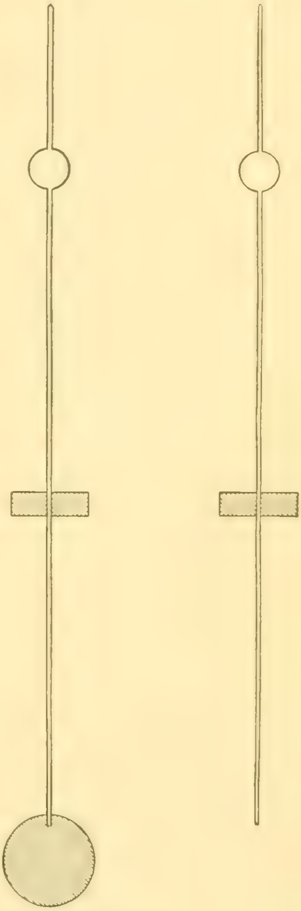

FIGURE 2.

PRELIMINARY EXPERIMENTS.

Preliminary experiments showed clearly that, in order to obtain a regular diminution in the arc of swing, great care was necessary to avoid any convection currents in the apparatus. The water jacket and felt covering described above gave entire satisfaction in this respect and also maintained the temperature of the apparatus sufficiently constant.

A test was now made to determine whether the velocity of the sphere was such as to cause a different logarithmic decrement for large arcs of swing from that obtained for small arcs. The result obtained from the many observations taken was that, for the range which it was proposed to use, the decrement was independent of the amplitude of swing. The velocity of the fluid at any point would therefore seem to have been within the limit set by the mathematical conditions. Moreover, as will appear from the data given later, the maximum angle through which the sphere turned was about 0.015 radians, the time of one complete swing was, in the case of the glass sphere, 83.52 seconds, while the radius of the sphere was 0.7495 cm. The maximum velocity of a point on the equator of the sphere must have been about 0.0011 cm. per second. The square of this quantity is about 0.1 per cent of the quantity itself. Using the value 0.00017 for μ_0 the expression $V a \rho / \mu$ must have been about 0.006.

It will be seen that the plan described above for the elimination of the friction on the fibre, mirror, and wire, as well as the friction in the fibre itself, is satisfactory only if the friction in the fibre remains constant during the experiment. Even with quartz this state of constancy is not reached until the pendulum has continued swinging for some time. With this precaution a very regular value for the decrement was obtainable.

Though the surface of the lead disk seemed to be little, if any, inferior to that of the glass disk, yet to fully justify the replacing of one by the other a wooden disk with a smooth surface was used as a pendulum and the decrement measured. Then the same disk, having its surface blackened with graphite, was used, but no difference in the decrement could be detected. Want of smoothness, merely, should, indeed, affect the decrement only in so far as a rough surface exposes more surface to the air for a given inertia than a smooth surface does.

Conceivably, also, there might be a damping of the swinging of the lead disk due to induced currents in the lead. This was tested by changing the intensity of the magnetic field surrounding the vibrator to two or three times its original value. No change in the speed of damping could be observed.

Method of Experiment.

The main part of the experiment was carried out as follows: After the sphere and disk had been fastened on the wire in the manner already described, the whole suspension was lowered into the containing vessel until the cap, D, rested in its place. When the cap, D, had been adjusted so that the mirror should come to rest in the proper place, and time had been given for the amplitude of the oscillation to diminish to the desired magnitude,* the points of rest of the consecutive arcs were observed. No observations were made until the arc had diminished to about 15 cm. The scale distance was 2.3 metres, so that the largest angle through which the sphere turned from the position of equilibrium was about 0.015 radians. Every point of rest was observed until the amplitude of oscillation had diminished to about 5 cm. A series of twenty consecutive arcs could thus be obtained. By turning the cap, D, a fresh start was given to the vibrator and the same process repeated, care being taken to allow any pendulous motion to subside before any observations were made. Generally, from four such series the mean decrement was obtained. Before removing the sphere from the vessel the time of swing was determined. The pressure was measured at the beginning and end of the experiment, and the temperature observed from time to time during the course of the work.

The suspended system was then taken from the vessel, the sphere and disk removed, and the lead disk fixed on the wire at the same place as that previously occupied by the other disk. When the new system had been returned to the vessel and the swinging of the disk had sufficiently subsided, the same process as was carried out with the sphere was repeated with this new pendulum. The time of swing, the pressure, and temperature were again observed.

These observations gave all the data necessary for the calculation of μ with the exception of the moment of inertia of the vibrator. To get this, three cylinders were made having different radii, and each a length sufficient that its weight should be just that of the lead disk. The weights were made the same to a milligram. Thus the fibre when supporting any one of the cylinders was under the same strain as when it was supporting the sphere and disk. The moment of inertia of each cylinder was calculated, and then a knowledge of the time of swing of each and of that of the wire and mirror when allowed to oscillate alone gave at once the value of the moment of inertia of the mirror and wire. This was added to the moment of inertia of each cylinder, and then from each result the moment of inertia of the vibrator was obtained. The moment of inertia of the vibrator could be calculated, but it seemed better to get it from the time of swing which the vibrator had in the course of the experiment.

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* This process usually took between one and two hours.

Experiment I.

In the first experiment the sphere was made of ivory. Its diameter was 0.8653 cm., and together with the ivory disk weighed 5.460 grams. The moment of inertia of the mirror and wire alone was found to be 0.0016 in C. G. S. units, while that of the vibrator as determined from the different cylinders, which in this case were made of ivory, was 1.6409, 1.6424, and 1.6479, the mean of which is 1.644. The temperature corrected by a standard thermometer was 23°.0, and the pressure 75.65 cm. The logarithmic decrement obtained when the disk was on the wire was 0.00859 and the time of swing 38.83 seconds. The decrement for the sphere and ivory disk was 0.02424, and the time 38.94 seconds. To change the former decrement to what it would be were the time of swing 38.94 seconds the approximate formula $\lambda\,(1 - t_2/t_1)$ was used. It expresses the amount to be added to λ, the decrement when the time is t_1 in order to get what that decrement would become when the time had changed to t_2 and is obtained by considering the pendulum as being damped by a resistance proportional to the angular velocity. It is sufficiently accurate for this correction, which must be small, since the difference in time is small. In this case the correction was −0.0002, so that the corrected decrement for the lead disk was 0.00857 and the decrement due to the resistance to the sphere alone must have been 0.01567. To the base e, this becomes 0.03608.

The following table gives the data necessary for the calculation of μ:

TABLE I.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
23°.0	0.0011871	38.94	0.03608	1.644	0.0032215	0.8653

The insertion of these values in the expression for the logarithmic decrement in Stokes's formula leads to an equation of the fourth degree from which to determine $\mu^{\frac{1}{2}}$. The solution gives

$$\mu_{23^\circ.0} = 0.0001825.$$

Holman * has obtained the formula

$$\mu_t = \mu_0\,(1 + .002751\,t - .0000034\,t^2)$$

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* Phil. Mag., **21**, 1886.

to express the variation of μ with the temperature. This formula has been used to reduce the above result to 0° C.

$$\mu_0 = 0.0001716.$$

Experiment II.

After being in use for some time, the ivory sphere and disk were found to have changed in weight and shape, so that the ivory was replaced by glass. The dimensions of the glass sphere and the requisite disk have been given in the description of the apparatus. The method of performing the experiment was the same in this case as in the previous one. Metal cylinders, however, were used for determining the moment of inertia. The latter was found to be, from the first cylinder, 1.3047, from the second 1.3042, and from the third 1.3029. The mean is 1.3039. The decrement for the sphere and disk was 0.02211, and for the lead disk 0.00862. The time for the lead disk was 41.77 seconds, so that no correction for time was necessary. The results are summarized in Table II.: —

TABLE II.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
18°.6	0.001228	41.795	0.03106	1.3039	0.002166	0.7495

From the data given in this table

$$\mu_{18°.6} = 0.0001795.$$

When reduced to 0° C. this gives

$$\mu_0 = 0.0001708.$$

Experiment III.

In this experiment the temperature of the gas when the sphere was suspended was 15°.6 C., while when the lead disk was suspended it was 14°.8 C. The correction to be applied was obtained by getting the decrement for the lead disk when the temperature was 15°.3 C., and using this as a basis for determining what it would be at 15°.6. In this way the correction was found to be 0.00004. This correction was not quite as satisfactory as might have been desired, as appeared when the decrement was deduced from the observations at 15°.3. The results were not quite so regular as they were in some other cases. This was probably due to having taken observations to be used without having given sufficient training to the fibre. But there can be no serious error involved in the corrected decrement (probably not more than 0.1 per cent) as the correction is necessarily small.

The pressure in this case, as in the last, was also observed to be slightly different when the sphere was used from that when the lead disk was used. But by the same method as that used in the case of the temperature the correction was found to be negligible.

As the time for the lead disk was 41.755, no correction for time was necessary.

Table III is a summary of the data obtained from this experiment.

TABLE III.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0012316	41.76	0.03095	1.3039	0.0021721	0.7495

From the data given in this table

$$\mu_{15^\circ.6} = 0.0001790,$$

and $$\mu_0 = 0.0001716.$$

The value of μ has been determined also for pressures less than atmospheric. The following tables give the results obtained : —

TABLE IV. $p = 36.031$ cm.

Temp	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0005799	41.76	0.03049	1.3039	0.0010227	0.7495

From the data given here,

$$\mu_{15^\circ.6} = 0.0001782,$$

and $$\mu_0 = 0.0001709.$$

TABLE V. $p = 10.014$ cm.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0001612	41.76	0.03037	1.3039	0.0002842	0.7495

Here, $$\mu_{15^\circ.6} = 0.0001786,$$

and $$\mu_0 = 0.0001712.$$

TABLE VI. $p = 2.499$ cm.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.00004022	41.76	0.03035	1.3039	0.00007093	0.7495

In this case $$\mu_{15^\circ.6} = 0.0001786,$$

and $$\mu_0 = 0.0001713.$$

A reference to the description of the apparatus will show that the distance between the fixed and moving surfaces, viz., the inner surface of the glass bulb and the surface of the sphere, was about 6.5 cm., so that no correction need be applied for slip.

In addition to what has been said with regard to the neglect of the squares of velocities it may be urged that, if it can be taken as satisfactorily shown that the value of μ is constant over as large a range of pressure as that over which the foregoing experiments have been made, there is, in the results obtained at the lower pressures, another evidence of the justifiable neglect of quantities of the order of $V^2 a \rho \mu$. For a diminution of ρ diminishes this quantity while not affecting μ, so that, if the value of μ derived from a formula obtained by neglecting the squares of small quantities is the same at atmospheric pressure as it is at a much smaller pressure, 2.5 cm., it may be safely concluded that the use of the formula at atmospheric pressure was justifiable.

The mean of the results obtained at atmospheric pressure is

$$\mu_0 = 0.0001713.$$

This is very nearly the mean of the results obtained by the more recent investigators, some of whom used the transpiration method and others the oscillation method.

Considered from the point of view of the Kinetic Theory of Gases, the viscosity of a gas is measured by the excess of the quantity of momentum transferred in one direction across the mutual boundary of two portions of the gas by the to and fro motions of the particles over the quantity of momentum transferred in the opposite direction. The ions in a gas no doubt play their part in transferring momentum from one layer of the gas to another, but their number is so exceedingly small in comparison with the number of unaffected particles that it is scarcely probable that the transfer of momentum is appreciably affected by the ionization of the gas.

An experiment was performed with the object of ascertaining whether there is any appreciable change in the viscosity of air when in the ionized state. No change in the rate of damping of the oscillation could be observed.

The investigations of Tomlinson and Reynolds, already referred to, seem to be the only ones in which the oscillating body was spherical. A closer examination of these investigations may therefore be permitted. In Tomlinson's experiment two spheres, each 6.3 cm. in diameter, were fixed side by side with their centres 20.78 cm. apart. The system oscillated about an axis, which passed along a vertical line midway between the spheres. The time of one complete swing was 5.76 seconds. The mathematical formula for this case, which was given by Stokes, required a correction for the rotation of the spheres about their respective vertical diameters. When adapted to the case of a single sphere the formula for this correction is that made use of in the present investigation. From the experiment with the spheres Tomlinson found $\mu_0 = 0.0001716$.

The results obtained by the same investigator by means of the two-cylinder method, and by means of the single-cylinder method, showed good agreement with each other and with that given for the spheres. A vibrator in the form of a single cylinder was finally adopted and the two results obtained from it were $\mu_0 = 0.0001716$ and $\mu_0 = 0.0001714$.

Where cylinders are used it would appear that there is not the same limitations for the velocity that there are when spheres are used, and consequently, from considerations of what the velocity of a point on either sphere must have been, the result given for the spheres is perhaps hardly what one might expect. With an amplitude of oscillation, such that the vibrator would move from the point of equilibrium through an angle of 0.02 radians, the maximum velocity of the centre of each sphere must have been about 0.23 cm. per second. The value of $Va\rho/\mu$ would in this case be about 5.5. On this account one would have expected a larger result than that given.

In the experiment performed by F. G. Reynolds the spherical shell which he used had a diameter of nearly 13 cm. It was made to vibrate about a diameter, and the period of oscillation was 16 seconds. The maximum angle, measured from the position of equilibrium, was apparently about 0.15 radians, so that the maximum velocity of a point on the equator of the sphere must have been approximately 0.19 cm. per second. The square of the quantity is not small compared with the quantity itself, and the quantity $V a \rho / \mu$ for this case is about 9.4. There may be in this an explanation of the fact that the result obtained by Reynolds is about 3 per cent greater than the mean of the results given in this paper.

The analogy between the method of the single cylinder and that employed in the present paper is apparent. As has been seen, the result obtained by Tomlinson by this method is almost identical with that obtained in the present investigation; but Reynolds, by this method, obtained $\mu_0 = 0.0001768$. An effort has been made to discover the cause for this discrepancy.

In order to eliminate the effect of friction on the mirror and wire and the viscosity in the wire itself, Reynolds made use of a telescopic arrangement of two hollow cylinders. The decrement was measured, first, with the outer cylinder covering the inner, and then with the inner one pulled out a distance of 25.4 cm. The difference between these decrements he took to be that due to the friction of the air on the inside and outside of a length of 25.4 cm. of the inner cylinder. The inside diameter of the inner cylinder was 4.70 cm., and the inside diameter of the outer was 5.00 cm. The thickness of the metal was 0.04 cm., so that there was a space of 0.11 cm. between the cylinders. In obtaining in the manner described the decrement for the extended portion of the inner cylinder the assumption was made that the friction on the inner surface of a length of 25.4 cm. of the outer cylinder was the same as the friction on the same length of the inside surface of the inner cylinder together with the friction due to a length of 25.4 cm. of the layer of air between the cylinders. This assumption would seem to require justification.

On account of the two distinct operations involved in the method used in the present investigation, it is not well suited for the determination of viscosity at low pressures. Another investigation is in progress, one of whose objects is to determine the law relating viscosity to pressure at pressures where slip is appreciable and, when this is done, to measure very low pressures by measuring viscosity. As in Maxwell's formula for the oscillating disk, account is taken of the friction on and in the fibre, and on the mirror, the sphere has been replaced by a glass disk oscillating between two fixed glass disks. The whole is enclosed in a glass vessel which is in connection with a McLeod Gauge and a mercury pump.

I desire to express my gratitude to Professor Trowbridge for placing at my disposal everything necessary for carrying on the work, and to him and to the other professors of the department for helpful suggestions during the course of the investigation.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

A PQ *PLANE FOR THERMODYNAMIC CYCLIC ANALYSIS.*

By Harvey N. Davis.

With Three Charts and Twenty-one Figures.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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A *PQ* PLANE FOR THERMODYNAMIC CYCLIC ANALYSIS.

By Harvey N. Davis.

Presented by John Trowbridge, December 14, 1904. Received January 9, 1905.

Four years ago, Dr. C. E. Lucke pointed out in an admirable paper * the value of what he calls a "cyclic analysis of heat engines." The variety of processes which are available when the working substance is a gas rather than a vaporized liquid is so great, and the influence of the nature and dimensions of a given cycle, not only upon its efficiency, but upon many other properties, is so complicated, that it is evidently both interesting and important to make a purely theoretical study of cycles as such, and to endeavor to obtain statements of the questions involved in terms of the cycles themselves. And for this purpose, the best methods will almost always be graphical ones, as these are at once the most powerful in investigation and the clearest in exposition. It is desired, in this paper, to reiterate Dr. Lucke's insistence on the value of work of this kind, and to develop a method which shall be more general, and therefore more fruitful, than the one-dimensional graphics which he has proposed.

It is best to begin by inquiring how much must be given to determine completely a cycle of a given type. If the cycle is to be represented on the ordinary $p\,v$ plane of thermodynamics, both its dimensions and its position in the plane must be known. To describe its position will always require two coördinates, and it will be convenient to take for these the pressure (P_a) and the temperature (T_a) of the working substance at the beginning of the cycle, since these quantities are easily measured; often they are simply the pressure and temperature of the earth's atmosphere at the time, and so are not controllable, and for this reason it is best to regard them, not as variables, but as parameters in the equations in which they occur, and to assume for them constant "standard" values. The dependence of the results upon these values will then be a separate, and usually a very simple problem. For the determination of the dimensions of a cycle two more coördinates are usually needed, which may be thought of as describing in some generalized way the height and breadth of the cycle. Thus for a Carnot cycle the temperature and entropy ranges might be used, or the pressure or volume ranges along two adjacent sides, or some less obvious coördinates, as, for instance, the quantities of heat involved in two successive transformations. It is hoped that this paper will show that in a very great number of cases it is advantageous to choose as these two coördinates the ratio of compression (p_b/p_a), which will be denoted by P, and the heat taken in from outside sources by a unit mass of working substance during its passage once around the cycle, which will be denoted by Q. The chief reasons for this choice are to be found in the simplicity of the formulae to which it leads, and in the ease with which the physical interpretation of these formulae can be brought out. It may also be noticed that these quantities are simply and independently controllable and are easily measured.

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* "A Method of Cyclic Analysis of Heat Engines," by Charles E. Lucke. The School of Mines Quarterly, **22** [1901], pp. 223, 329, 411.

If values of Q be taken as abscissae and values of P as ordinates, the result is a $P\,Q$ plane upon which a point represents and completely describes a thermodynamic cycle (when its type and the values of the parameters are known) in exactly the same way that a point on the $p\,v$ plane represents and completely describes a thermodynamic state (when the form of the characteristic equation is known, together with the values of the constants which it involves). Any property (X) of such a cycle can then be plotted along a third rectangular axis, and the resulting three-dimensional surface will give a complete picture of that property for all cycles of the type under consideration. If such a surface be constructed for each different cyclic type to be studied, one can read off from this set of models any desired information whatever about the property to which they correspond. The simplest way to handle such surfaces is to map their contour lines ($X =$ a constant) on the basal $P\,Q$ plane, just exactly as isothermals or adiabatics are mapped on the familiar $p\,v$ plane of thermodynamic states.

We have said that two coördinates will usually be needed to determine the dimensions of a cycle, but in many exceptional cases one is enough. An example is a cycle which consists of an isothermal, an isopiestic, and an adiabatic (Figure 1). The point a, when it is fixed by the parameters, determines the isothermal and the adiabatic upon which it lies, and then a statement of (say) the specific volume at b settles everything. So does a knowledge of P $(\equiv P/P_a)$. So also does a knowledge of Q (that is, the heat that must be supplied to the working substance per unit mass to carry it from b to c). In such cases, there exists a relation, $f(P, Q) = 0$, by means of which either of these quantities can be expressed in terms of the other (and the parameters). On the PQ plane this equation is a curve; and the graphical statement of these facts is that it is impossible to construct a cycle of the given type corresponding to any point of the PQ plane which is not also a point of this curve. Such cycles (which may be called cycles of the first order) fill, not a whole PQ plane, but only a line in such a plane, and the PQX surface is defined only along this line, and degenerates into a curve drawn upon the surface of a cylinder erected on the line as a directrix. In such special cases, it is sometimes best to discard entirely the contour-line method of studying the surface, and to use instead either a development of the cylinder, or some picture of it, for example such an orthographic projection as will eliminate P. This last is exactly what Dr. Lucke did (although from a different point of view) whenever he had a first order cycle to deal with.* The reason that some such

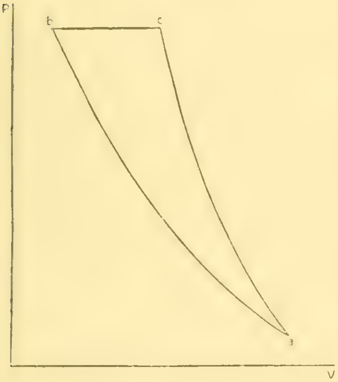

FIGURE 1.

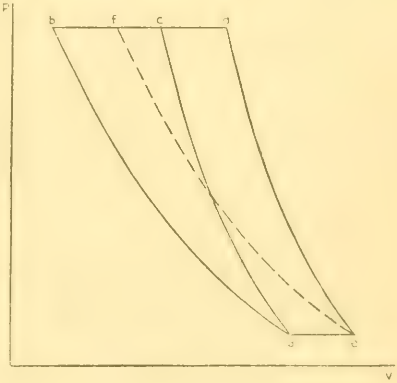

FIGURE 2.

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* Of the cycles of pp. 225–234 of his paper, Numbers I, IC, VI and VIII are of the first order (Cycle VIII being a special case of the cycle discussed above); and the lines marked I and IC in the figures of pp. 418–429 are orthographic projections of the kind described. All the other lines in these figures may be thought of as the intersections of a PQX surface with a plane perpendicular to the PQ plane along either the line $P = 2$ or the line $P = 10$.

proceeding is not always necessary is that cycles of the first order very often may be thought of simply as those special cases of second order cycles which happen to lie on certain curves in the P Q plane. Thus a complete representation of the properties of the cycle of Figure 1 will turn up, quite incidentally, in the last part of this paper.

Perhaps the best way to bring out the scope and power of the graphical method proposed will be to carry through, in some considerable detail, an application of it to a definite problem; an excellent problem for this purpose is presented by the cycles which come up in connection with such turbines as use air instead of steam as the working substance. The simplest case would be that of a turbine of the De Laval type, in which the working substance expands adiabatically from a high pressure to the pressure of the exhaust in a single nozzle so shaped that the available energy in the working substance appears as kinetic energy. The momentum of the moving stream is then used to turn a wheel. The sequence of processes in such a machine * is as follows:

1. Air is taken in at atmospheric pressure and temperature (corresponding to the point *a* of Figure 2) and compressed.

2. It is then heated; this will be done at constant pressure, for the pressure cannot increase if a continuous flow is to be kept up, and if it diminishes during the heating, energy is needlessly lost. (This process is represented by the horizontal line ending at *d*.)

3. There is adiabatic expansion in a nozzle (line *d e*).

4. Either the hot exhaust (point *e*) is thrown away at atmospheric pressure and new air is taken in, or the same working substance is cooled and used over again. (Either process is represented by the line *e a*.)

The only thing not yet described is the nature of the compression, and this may be adiabatic (line *a c*) or isothermal (line *a b*) or anything between. The two simplest cases may be taken as typical; the corresponding cycles will be called

Type A. The Adiabatic or Brayton Cycle (a c d e a of Figure 2).

Type B. The Isothermal Cycle (a b d e a of Figure 2).

Of the non-atmospheric cycles omitted from the list on p. 417 as being not "accurately defined," all but cycle V (of p. 233) are of the third order; such cycles would require three variables and a three-dimensional graphics, which would bear to the P Q plane exactly the same relation which that plane bears to the Q axis of Dr. Lucke's paper. Luckily very few important cycles are of higher order than the second.

* Here, as elsewhere in this paper, only purely ideal conditions are considered.

In a cycle of type A, the temperature of the working substance after compression (point c) is already comparatively high, but in the second cycle this temperature (point b) is still that of the atmosphere, and therefore, in general, much lower than that of the exhaust (point e); the second cycle, therefore, but not in general the first, may be modified by introducing the idea of regeneration. If ef be an isothermal through e, the working substance may be carried from the state b to the state f by means of the heat which the exhaust gives up in passing from the state e to the state a, and this too without any loss of available energy, by simply passing the high pressure air and the exhaust in opposite directions through adjacent passages. A third type of cycle is thus obtained, and may be called

Type C. The Regenerative Cycle ($a\,b\,f\,d\,e\,a$ *of Figure* 2).

Since these cycles themselves and not their applications to engineering are to be studied in this paper, there is no need of ruling out those cases under types B and C in which the point e falls to the left of the point a. Such cycles are figures-of-eight, and the work done by the working substance may perfectly well be zero, or less than zero, but the properties of such degenerate cycles fit on continuously with those of cycles of the ordinary kind, and should be studied with them. In a figure-of-eight case under type C the regenerative process consists in *cooling* the high pressure air to the temperature of the exhaust, and cycles of this kind are of some interest in connection with liquid air machines.

In the study of gas turbine cycles a number of properties should be considered. The most important of these is the temperature (T_e) of the working substance after its expansion (point e), for if this be too high, the blades of the turbine wheel will suffer. Another important property is the velocity (V) of the stream of gas as it leaves the nozzle, and here also there is an upper limit, in that V should bear a definite ratio to the peripheral velocity of the turbine wheel, and this is always limited by structural or other considerations. There is also the efficiency (E) of the cycle; and finally the work (W) which a unit mass of the working substance does in going once around a cycle is of interest because of its bearing on the size of the machinery necessary for a given output. Twelve sets of contour lines are therefore involved, four for each type of cycle; and the proposed problem is to determine by means of them (1) which type of cycle is preferable, and (2) what the dimensions of a cycle of that type should be, it being desirable to make E and W as large as possible subject to the conditions $T_e \leqq \bar{T}_e$ and $V \leqq \bar{V}$, where $\bar{T}_e$ and $\bar{V}$ are given constants.

In obtaining the formulae upon which the following discussion is based, it will be assumed that the working substance is a perfect gas. The analytic expression of this assumption is the two equations

$$p\,v = R\,T\,,$$

where R is a constant, and

$$\frac{C_p}{C_v} = \kappa,$$

where κ is a constant.* Of the four constants R, C_p, C_v, and κ, two must be given to determine the nature of the perfect gas; the two which will be used in the formulae of this paper are C_p and κ. The only other symbols which will be needed are the physical constants g and J, the parameters p_a and T_a, the variables P and Q, and the symbol (T_e, V, E or W) which represents the property under consideration. For convenience of reference the formulae are collected in Table 1; they are easily obtained by the ordinary methods.† It should be noticed that not one of them happens to involve the parameter p_a, a simplification which has resulted from the choice of P as a coordinate. Each of these formulae (except the E-type-A one) can be explicitly solved for Q. In the accompanying figures, values of Q have been calculated with considerable care, usually for some twenty-five different values of P, and it is hoped that the resulting curves, especially on the working charts, are

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* The gas law alone is not enough; if both are given it follows at once that C_p and C_v are themselves constants.

† The T_e formulae for types A and B were obtained by following pressures and temperatures up the compression line, across the top, and down again, by means of the usual equations. T_e for the regenerative type is independent of the parameters. The work was as follows (see Figure 2):

$$T_f = T_e\,; \quad \therefore \quad T_d = T_e + \frac{Q}{C_p}\,. \qquad \text{(a)}$$

But the equation of the adiabatic $d\,e$ gives

$$T_d\,p_d^{\frac{1-\kappa}{\kappa}} = T_e\,p_e^{\frac{1-\kappa}{\kappa}}\,, \quad \text{or} \quad T_d = T_e\,P^{\frac{\kappa-1}{\kappa}}\,; \qquad \text{(b)}$$

therefore, etc. The V equations are special cases of equation (208), p. 151, of Professor Peabody's "Thermodynamics of the Steam Engine" [4th ed. 1900]. The E equations were obtained from the amounts of heat involved in the various transformations, and the W equations are of the form $W = J\,Q\,E$.

TABLE 1.

	Type A. Brayton.	Type B. Isothermal.	Type C. Regenerative.
Final Temperature. T_e	$\frac{Q}{C_p}\frac{1}{P^{\frac{\kappa-1}{\kappa}}}+T_a$	$\left(\frac{Q}{C_p}+T_a\right)\frac{1}{P^{\frac{\kappa-1}{\kappa}}}$	$\frac{Q}{C_p}\frac{1}{P^{\frac{\kappa-1}{\kappa}}-1}$
Velocity. V	$\sqrt{2gJ(Q+C_p T_a P^{\frac{\kappa-1}{\kappa}})(1-P^{\frac{1-\kappa}{\kappa}})}$	$\sqrt{2gJ(Q+C_p T_a)(1-P^{\frac{1-\kappa}{\kappa}})}$	$\sqrt{2gJQ}$
Efficiency. E	$1-P^{\frac{1-\kappa}{\kappa}}$	$\frac{(Q+C_p T_a)(1-P^{\frac{1-\kappa}{\kappa}})-C_p T_a \log_e P^{\frac{\kappa-1}{\kappa}}}{Q}$	$\frac{Q-C_p T_a \log_e P^{\frac{\kappa-1}{\kappa}}}{Q}$
Work. W	$JQ(1-P^{\frac{1-\kappa}{\kappa}})$	$J(Q+C_p T_a)(1-P^{\frac{1-\kappa}{\kappa}})-JC_p T_a \log_e P^{\frac{\kappa-1}{\kappa}}$	$JQ-JC_p T_a \log_e P^{\frac{\kappa-1}{\kappa}}$

sufficiently accurate for any purpose for which such graphs could reasonably be used.*

Type A. The Brayton Cycle. The final temperature curves for this type are represented in Figure 3, and their general shape shows at once the advantage of a high compression ratio. It should be noticed that any final temperature not lower than the initial temperature is possible, it being only necessary to choose a sufficiently small value for Q. That

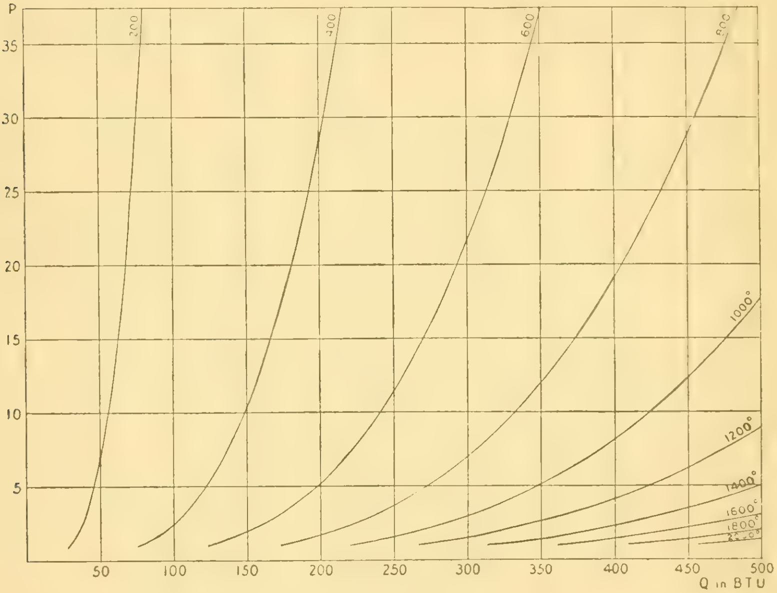

FIGURE 3.

Type A. Final temperature. Showing, for Brayton cycles (adiabatic compression), the temperature of the gas after its expansion.

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* In the numerical work, the following constants (mostly from Peabody) were used:

$C_p = 0.2375$ B. T. U.,
$\kappa = 1.405$,
$g = 32.17$ feet per second,
$J = 778$.

The standard value for the temperature parameter was taken as

$t_a = 80°$ F. or $T_a = 541°$ F. Abs.

The unit of mass used was one pound (gravitational). The resulting T_e is in Fahrenheit degrees absolute, but the results have always been expressed on the ordinary scale. V is in feet per second, and W in foot pounds.

curve of the family for which $T_e = T_a$ coincides with the P axis. All the curves end in the line $P = 1$.

The velocity curves are in Figure 4. Any velocity greater than zero is possible, but here P, as well as Q, must be taken sufficiently small. All the V curves start from finite points on the P axis but run off asymptotically to the line $P = 1$. The velocities in the central parts of the figure are of the same order as those obtainable with steam.

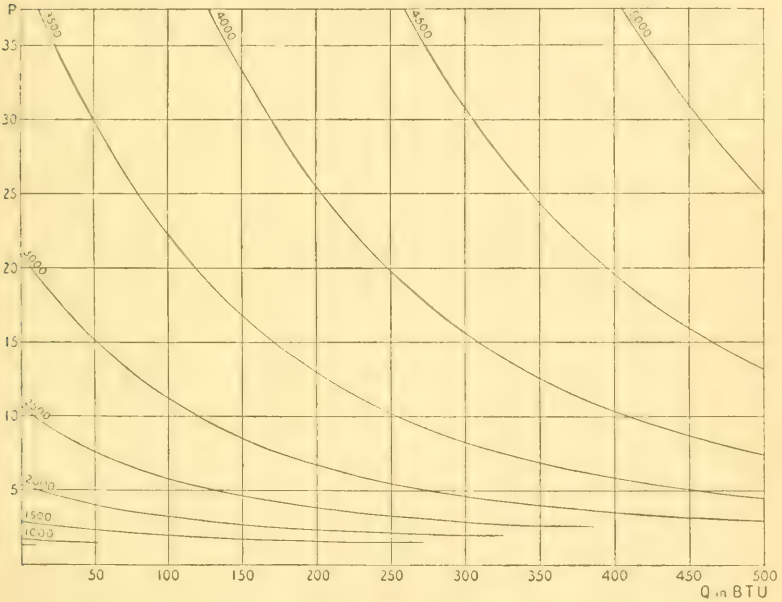

FIGURE 4.

Type A. Velocity. Showing, for Brayton cycles, the velocity in feet per second of the stream of gas as it strikes the blades of the turbine.

In the problem proposed, upper limits were assigned both to T_e and to V, and we have called these limits $\bar{T}_e$ and $\bar{V}$. That is, speaking graphically, no cycle is available whose point lies beyond either of the curves $T_e = \bar{T}_e$ or $V = \bar{V}$, and the remote parts of the $P\,Q$ plane are to be left out of consideration. There remains a finite portion of the plane (the shaded area of Figure 5*) within which an available cycle must lie. It may be called the $\bar{V}\,\bar{T}_e$ area, or, less explicitly, a $\bar{V}\,\bar{T}$ area.

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* The limits represented in Figure 5 are $\bar{t}_e = 400°\ F.$ (that is $\bar{T}_e = 861°\ F.$ Abs.) and $\bar{V} = 4000$ ft. per sec.

The second part of the problem proposed for this discussion is equivalent to the question, which point in a VT area is best for the purposes in hand. The answer is given by the other two sets of curves.

The E curves are shown in Figure 6, and are straight lines, the efficiency of a cycle of this type being independent of its breadth. Figure 7 shows the W curves. The line $P = 1$ is a part of the curve $W = 0$ because it is also the curve $E = 0$. The P axis is also a part of the curve $W = 0$, because a type A cycle whose point is on that axis, has zero area. Each of the other curves has these lines as asymptotes.

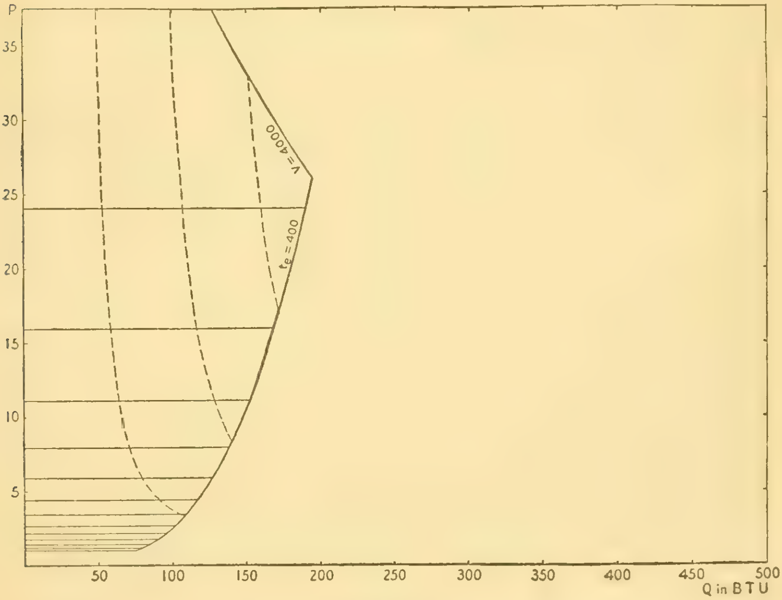

FIGURE 5.

Type A. VT area. This area (of which only the lower part is included within the limits of the figure) contains all the adiabatic cycles which satisfy the conditions $T_e \leq T_e$ and $V \leq V$. The curves within give the efficiency and work of these available cycles and enable one to choose between them.

In Figure 5 the full lines are efficiency curves, and the dotted lines are work curves. If efficiency is the important thing, the best point is evidently as far up in the area as possible, that is at the intersection of the curve $V = V$ with the P axis. The corresponding cycle is a null cycle ($Q = W = 0$) and must be regarded as a limiting case in the direction of increasing efficiency. If, on the other hand, the important thing is to decrease the amount of air handled, as will often be the case,* then the best point in the area is the intersection of the curve $V = \bar{V}$ with the curve $T_e = \bar{T}_e$. Under such circumstances there is a perfectly definite and comparatively low ratio of compression which should not be exceeded. The intersection of the curves $V = V$ and $T_e = \bar{T}_e$ may be called for convenience the VT_e corner, or, less explicitly, a VT corner.

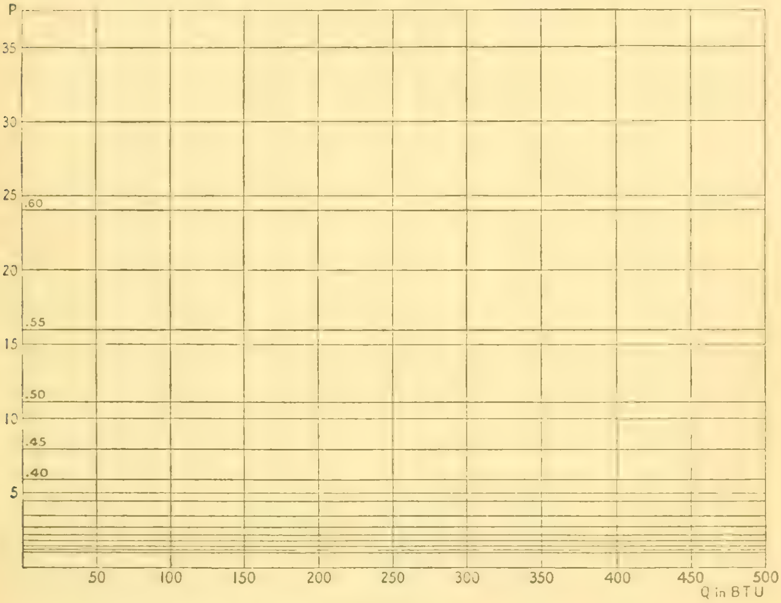

FIGURE 6.

Type A. Efficiency. The efficiency of a Brayton cycle is independent of its breadth.

All four of these families of curves are plotted together for reference on the first working chart, the T_e and V curves as lines, the E curves in dots, and the W curves in dashes. Of course the result is a complicated plane; but it is believed that a study of the figures in the text will make this chart intelligible and that the information of various kinds which it can afford will repay one for this trouble. To avoid unnecessary confusion, the efficiency lines between $E = .00$ and $E = .25$

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* Both the size of the necessary machinery and the inefficiency of air compressors point that way. Highly compressed air is sometimes a more expensive commodity than fuel.

have been omitted from this and the two following charts, as being unimportant.

Type B. The Isothermal Cycle. The T_e and V curves for this type are shown in Figures 8 and 9. They are the corresponding curves for the adiabatic case, sheared horizontally in such a way that the $T_e = T_a$ curve, which formerly lay along the P axis, has now a position indicated by the dotted line of Figure 8.* All cycles whose points lie on this line

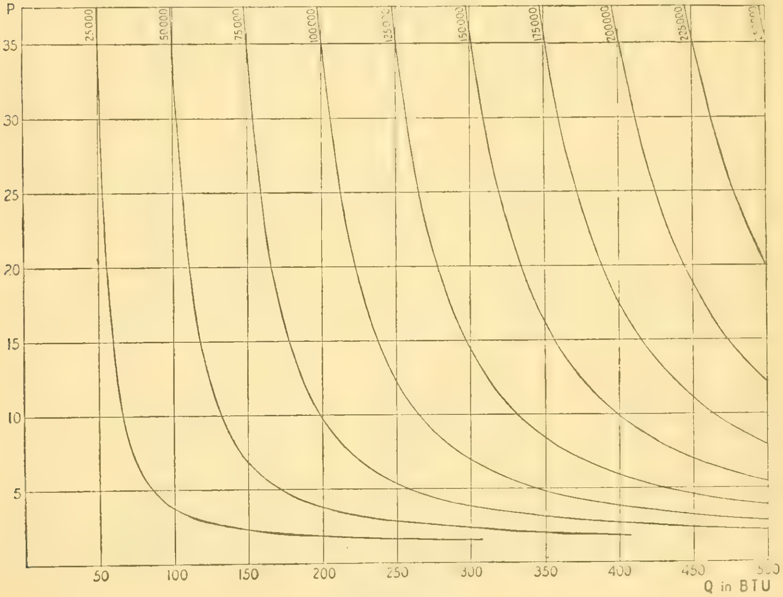

FIGURE 7.

Type A. Work. These curves give the net work in foot pounds which one pound of gas can do while making one complete circuit of a Brayton cycle.

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* For if the T_e formulae (types A and B) be solved for Q, they give

$$_{T_e}Q_A = C_p\,(T_e - T_a)\,P^{\frac{\kappa-1}{\kappa}}, \qquad \text{(a)}$$

where $_{T_e}Q_A$ stands for the Q of any point of a T_e curve for the type A case ; also

$$_{T_e}Q_B = C_p\,(T_e\,P^{\frac{\kappa-1}{\kappa}} - T_a) \qquad \text{(b)}$$

$$= C_p\,(T_e - T_a)\,P^{\frac{\kappa-1}{\kappa}} + C_p\,(T_a\,P^{\frac{\kappa-1}{\kappa}} - T_a)$$

$$= {}_{T_e}Q_A + ({}_{T_e}Q_B)_{T_e = T_a}.$$

are peculiar in that for them the point e of Figure 2 coincides with the point a of that figure; in other words this line ($T_e = T_a$) is a locus of first order cycles of the type of Figure 1, which thus appear as special cases of the second order cycles of type B. It is evident that the abscissa of any point of this locus represents the amount of heat that must be supplied to a unit mass of isothermally compressed air, to raise its temperature to that of adiabatically compressed air at the same pressure;

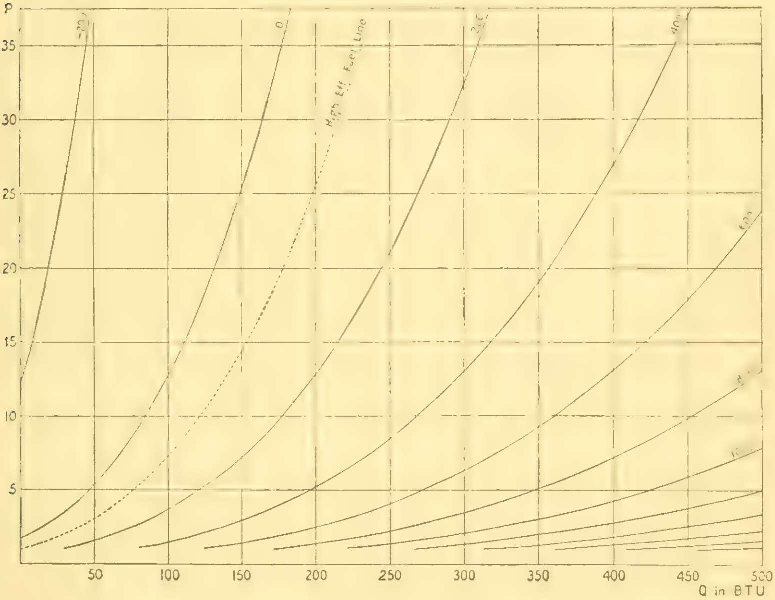


FIGURE 8.

Type B. Final temperature. For cycles with isothermal compression. (Cf. Figure 3.)

and therefore, to bring such air to a given thermodynamic state requires an amount of heat which is the sum of this preliminary Q' and the Q_A of the corresponding adiabatic cycle. This explains the shearing of the adiabatic plane. Points to the left of this locus represent figure-of-eight cycles, so that, for the engineer, this line is a sort of natural boundary for the isothermal plane.

The $V\,T$ area for the same values of $\bar{V}$ and $\bar{T}_c$ as were used before, now takes the form indicated in Figure 10. Its boundaries have simply been subjected to the horizontal shear already described, so that the P

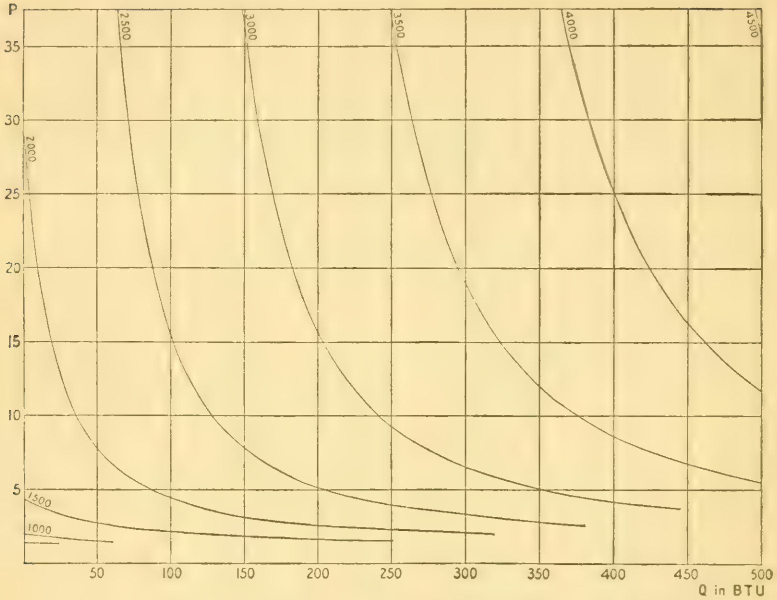


FIGURE 9. *Type B. Velocity.* For isothermal cycles. (Cf. Figure 4.)

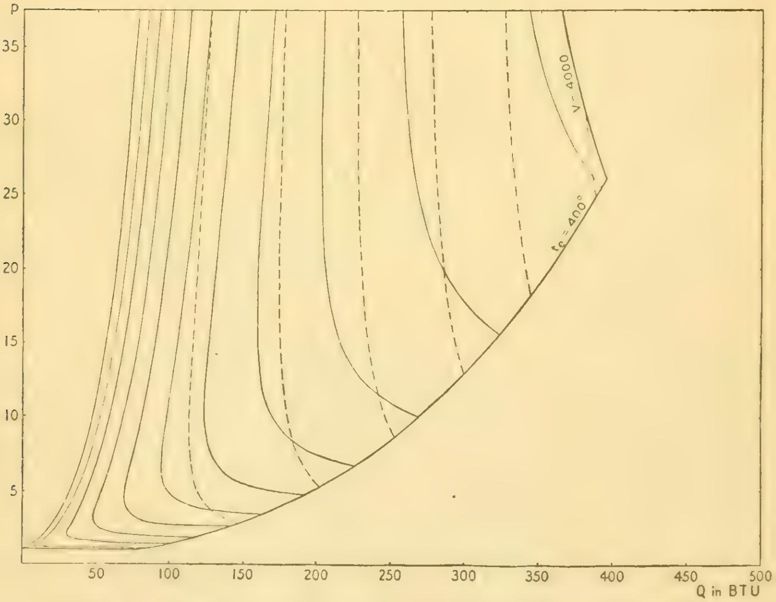


FIGURE 10. *Type B. VT area.* For isothermal cycles. (Cf. Figure 5.)

of the VT corner is the same as before; but the lines within are entirely changed.

The efficiency lines for type B are shown in Figure 11. The line $P = 1$ is still a part of the curve $E = 0$, but the other part of this curve is now well out in the plane. It is the locus of those figure-of-eight cycles whose (algebraic) area is zero. Cycles to the left of it have negative areas, and any negative efficiency whatever is possible for any value of P ($P > 1$). An extraordinary property of the curves of this family is that any vertical line cuts each of them twice if at all; their turning points lie on the above mentioned locus of first order cycles.* As P increases indefinitely, Q also increases indefinitely, logarithmically. The other end of each curve has the corresponding type A efficiency line as an asymptote. This may be explained by regarding an isothermal cycle as made up of an adiabatic cycle together with a first order cycle of the type of Figure 1 (see Figure 2). The efficiency of the type A part is independent of Q and is very much higher than that of the triangular part that goes with it. Therefore, for small values of Q, the efficiency of the whole cycle is small, because a great part of the heat supplied is made use of under the unfavorable conditions of the first order cycle. But as Q increases, P being kept constant, this low efficiency part of the heat supplied becomes less and less important in comparison with the rest, and the efficiency of the cycle approaches as

---

* For the E-type-B formula can be written

$$ {}_EQ_B = C_p\,T_z\left(\frac{\log_e P^{\frac{\kappa-1}{\kappa}} - 1 + P^{\frac{1-\kappa}{\kappa}}}{1 - P^{\frac{1-\kappa}{\kappa}} - E}\right). \qquad \text{(a)} $$

Differentiating and eliminating E, gives as the derivative

$$ \frac{\partial Q}{\partial P} = \frac{\kappa-1}{\kappa}\,\frac{Q}{P}\,\frac{1 - P^{\frac{1-\kappa}{\kappa}} - P^{\frac{1-\kappa}{\kappa}}\,\frac{Q}{C_p\,T_z}}{\log_e P^{\frac{\kappa-1}{\kappa}} - 1 + P^{\frac{1-\kappa}{\kappa}}}\,. \qquad \text{(b)} $$

The condition that this vanish is

$$ 1 - P^{\frac{1-\kappa}{\kappa}} - P^{\frac{1-\kappa}{\kappa}}\,\frac{Q}{C_p\,T_z} = 0. $$

or

$Q = C_p\,T_z\,(P^{\frac{\kappa-1}{\kappa}} - 1) = C_p\,(T_c\,P^{\frac{\kappa-1}{\kappa}} - T_z) = ({}_TQ_B)_{T_c = T_z}$ (see eq. b of note on p. 640).

a limit that of an adiabatic cycle with the same ratio of compression.* From this point of view, the locus of these first order cycles may be called the "high efficiency fuel line" as it indicates the beginning of the high efficiency fuel for a given value of P.

The work curves for this type are shown in Figure 12, and they, like the efficiency curves, have turning points on the high efficiency fuel line. The curve $W = 0$ is, as always, the same as the curve $E = 0$. The other lines go off to the right with $P = 1$ as an asymptote.

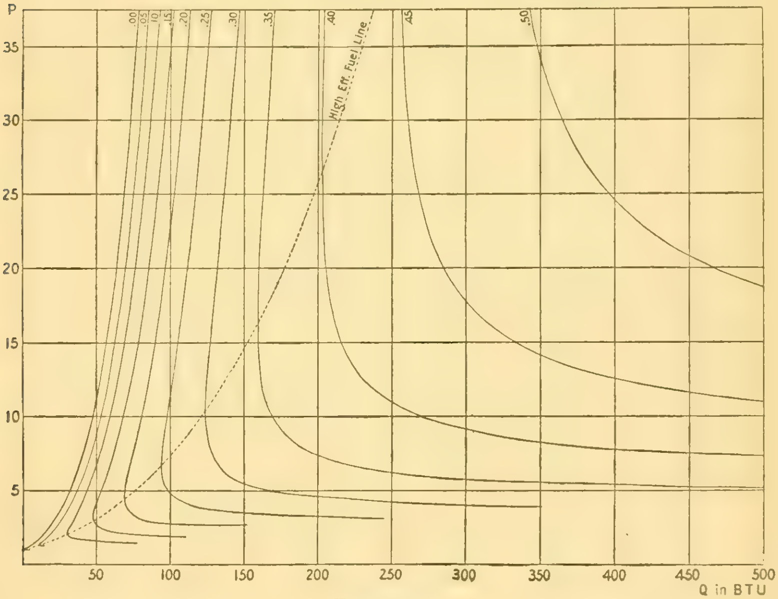


FIGURE 11.

Type B. Efficiency. For isothermal cycles. The turning points of these curves lie on the "high efficiency fuel line." (Cf. Figure 6.)

An inspection of Figure 10 shows that, as far as efficiency is concerned, it pays, in this particular case, to keep at least as far up the right hand

---

* The formula can be put in the form

$$E_B = \left(1 - P^{\frac{1-\kappa}{\kappa}}\right) - \frac{C_p T_a}{Q} \left(\log_e P^{\frac{\kappa-1}{\kappa}} - 1 + P^{\frac{1-\kappa}{\kappa}}\right)$$

$$= E_A - \frac{\phi(P)}{Q},$$

where $$\lim_{Q=\infty} \frac{\phi(P)}{Q} = 0.$$

boundary of the $V\,T$ area as the top of the figure. In general, an efficiency curve comes in along the Q axis more horizontally than the V curves, and if followed upward far enough, swings completely back to $Q = \infty$ again, cutting each V curve twice if at all. There will always be one efficiency curve that just touches a given velocity curve. Call the point of tangency S. Its cycle has evidently a higher efficiency than any other cycle giving the same final velocity, for the efficiency decreases as a point moves away from S in either direction along the given V curve. Therefore, if the point S of the curve $V = V$ lies on the boundary of a given $\bar{V}\,\bar{T}_e$ area, it is the best point of the area for efficiency. But if T_e is so small as to bring the corresponding $V\,T_e$ corner farther up the V curve than the point S for that curve, then the $V\,T_e$ corner itself is the best point of the area for efficiency, as it lies nearest S. The locus of the point S for all possible values of V is the line drawn with long dashes in Figure 13. It looks somewhat like one of the T_e family, but its equation is quite different.*

---

* The analytic work is as follows. The equation of the V curves is

$$_VQ_B = \frac{V^2}{2gJ\left(1 - P^{\frac{1-\kappa}{\kappa}}\right)} - C_p T_i$$

Differentiating and eliminating V, gives as their slope

$$\frac{\partial Q_V}{\partial P} = -\frac{\kappa - 1}{\kappa}\,\frac{1}{P}\,\frac{Q + C_p T_i}{P^{\frac{\kappa-1}{\kappa}} - 1}.$$

The slope of the E curves has been found (note on p. 643) to be

$$\frac{\partial Q_E}{\partial P} = \frac{\kappa - 1}{\kappa}\,\frac{Q}{P}\,\frac{1 - P^{\frac{1-\kappa}{\kappa}} - P^{\frac{1-\kappa}{\kappa}}\,\dfrac{Q}{C_p T_i}}{\log_e P^{\frac{\kappa-1}{\kappa}} - 1 + P^{\frac{1-\kappa}{\kappa}}}.$$

The necessary and sufficient condition for a point of tangency is $\frac{\partial Q_V}{\partial P} = \frac{\partial Q_E}{\partial P}$, and this can be put in the form

$$\left(1 - P^{\frac{1-\kappa}{\kappa}}\right)\left(\frac{Q}{C_p T_a}\right)^2 - \left(\log_e P^{\frac{\kappa-1}{\kappa}} + P^{\frac{\kappa-1}{\kappa}} - 3 - 2P^{\frac{1-\kappa}{\kappa}}\right)\left(\frac{Q}{C_p T_a}\right)$$
$$- \left(\log_e P^{\frac{\kappa-1}{\kappa}} - 1 + P^{\frac{1-\kappa}{\kappa}}\right) = 0.$$

that is $$F_1(P)\,x^2 - F_2(P)\,x - F_3(P) = 0.$$

where $x \equiv \frac{Q}{C_p T_a}$. This is the equation of the desired locus. Each of the functions F is positive for values of P greater than 1, so that, of the two real roots of the equation, one is always less than zero (by Descartes' rule of signs). The other root gives the line plotted in Figure 13.

If, on the other hand, it is desirable to make W rather than E as great as possible, the best point of a VT area is always, as in the case of the Brayton cycle, the VT corner, and the best ratio of compression is the same as before. In the case of each of these types it is necessary to choose between efficiency and work, but the two best cycles are now much nearer together than they were before.

The type B working chart is similar to that for type A, except for the presence of the high efficiency fuel line and the locus of points S which are drawn with long dashes.

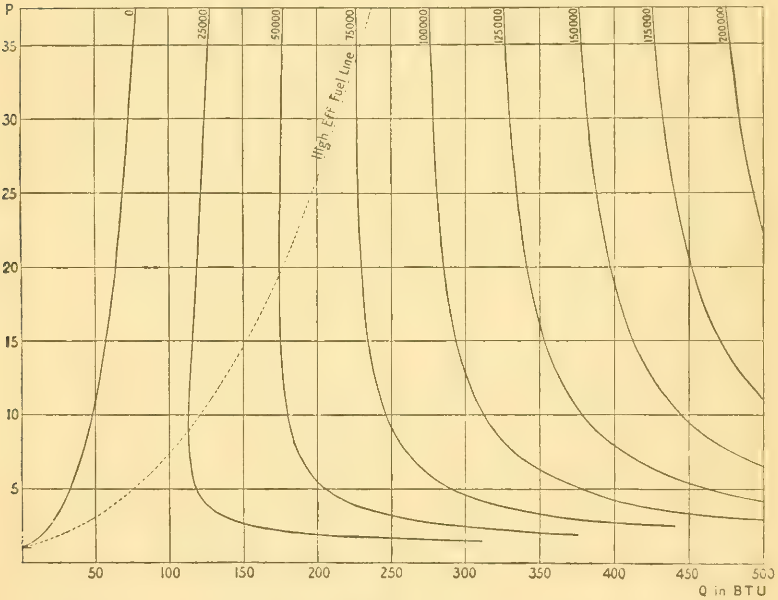


FIGURE 12.

Type B. Work. For isothermal cycles. These curves also have turning points on the "high efficiency fuel line." (Cf. Figure 7.)

Type C. The Regenerative Cycle. The final temperature curves for type C are shown in Figure 14; they are the T_e curves for type B displaced horizontally but without distortion, until their lower ends, which were scattered along the line $P = 1$, are all brought to the single point $P = 1$, $Q = 0$. This is because T_e is (by definition) constant along one of these curves, and so, T_a being fixed, the heat $C_p(T_e - T_a)$ that can be obtained from the exhaust is the same for all cycles on that curve; consequently the Q of each point of the curve as drawn on the type B

plane will be diminished by the same amount. To the point $P = 1$, $Q = 0$ (and indeed to the whole of the lines $P = 1$ and $Q = 0$) no physical meaning can be attached; and any final temperature from absolute zero to infinity may be found in any neighborhood of this singular point.

The V curves of Figure 15 are much simpler than those for the other two types, the final velocity depending only on the "quality of the mixture." The E curves are represented in Figure 17, except that portions of all of them near their common intersection at the singular point have been omitted. Any efficiency from $-\infty$ to $+1$ is possible in any neighborhood of this point. The W curves (Figure 18) are all alike, and are the $E = 0$ curve indefinitely repeated at constant intervals. The $V\,T_c$ area (Figure 16) differs from those for the other types chiefly in that it extends upward indefinitely. Fortunately the $V\,T$ corner is always the best point in the area, not only for work, but for efficiency as well.

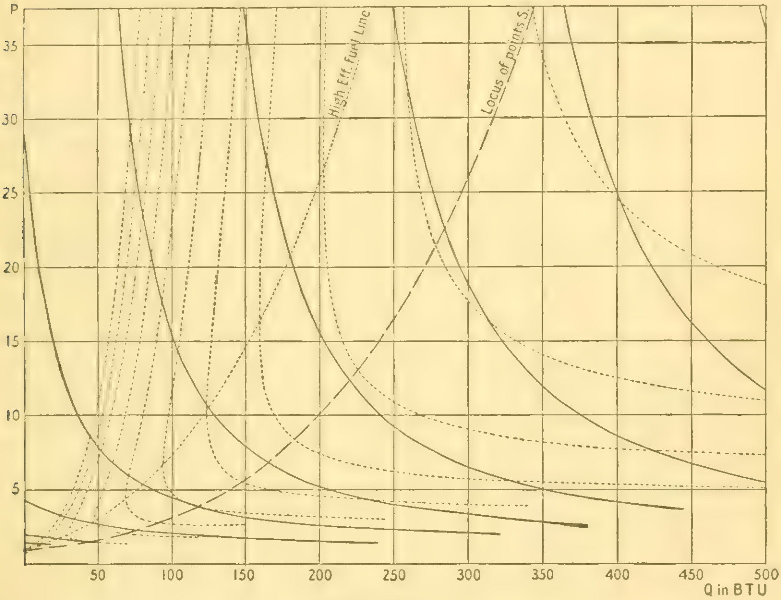


FIGURE 13.

Type B. Locus of points S. The full lines are velocity curves and the dotted lines are efficiency curves. The points of tangency lie on the locus indicated.

A glance at the working chart for this type shows that the whole type *B* plane has, as it were, shrunk horizontally toward its high efficiency fuel line. That line has many of the same interesting properties on the type *C* plane that it had before, in that here also it is the locus of the same set of first order cycles, and is, for the engineer, a natural boundary. It is interesting to notice that any line of any family of curves whatever crosses this high efficiency fuel line at exactly the same point in both the type *B* and the type *C* planes, although the general character of the line may be totally altered.

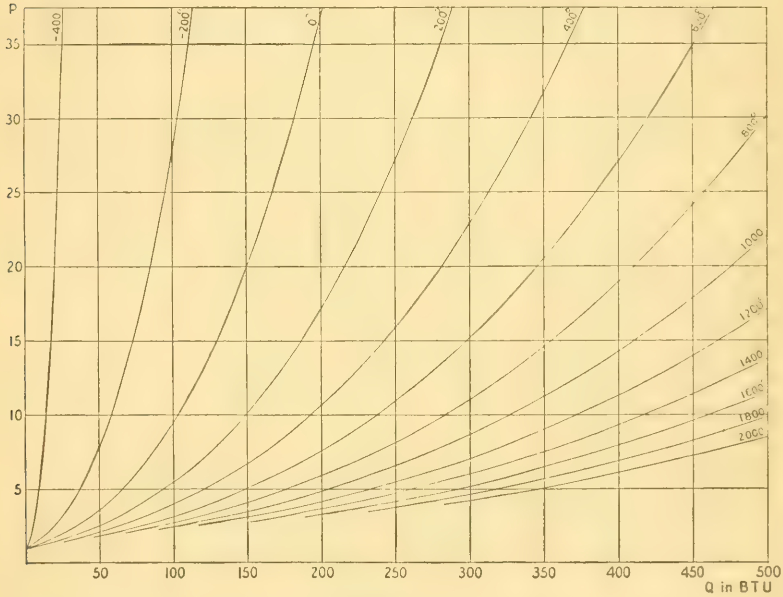


FIGURE 14.

Type C. Final temperature. These curves are for cycles with isothermal compression and a subsequent transfer of heat from the exhaust. They all start from the point $Q = 0$, $P = 1$, and there are an infinite number of them in the lower part of the plane. (Cf. Figures 3 and 8.)

As has already been suggested, the dependence of the properties of these cycles upon the values of the parameters p_a and T_a is very simple. In no case is p_a involved at all. The efficiency and work of a Brayton cycle and the final temperature and velocity of a regenerative cycle are independent of T_a also, and the curves of Figures 6, 7, 14, and 15 may be used whatever the initial conditions. In the isothermal case, the T_a and

V equations, when solved for Q, involve T_a in an additive constant, so that the effect of changing it is to shift the whole PQ plane along the Q axis without distortion ; the curves of Figures 8 and 9 may be used for the general case provided only that all readings of Q are increased or decreased by a suitable constant. The efficiency equations for types B and C, when solved for Q, involve T_a as a factor, so that the curves of Figures 11 and 17 can always be used by multiplying values of Q by a suitable constant. And in the other cases, the dependence upon T_a, while slightly less simple, can be easily followed out. None of the re-

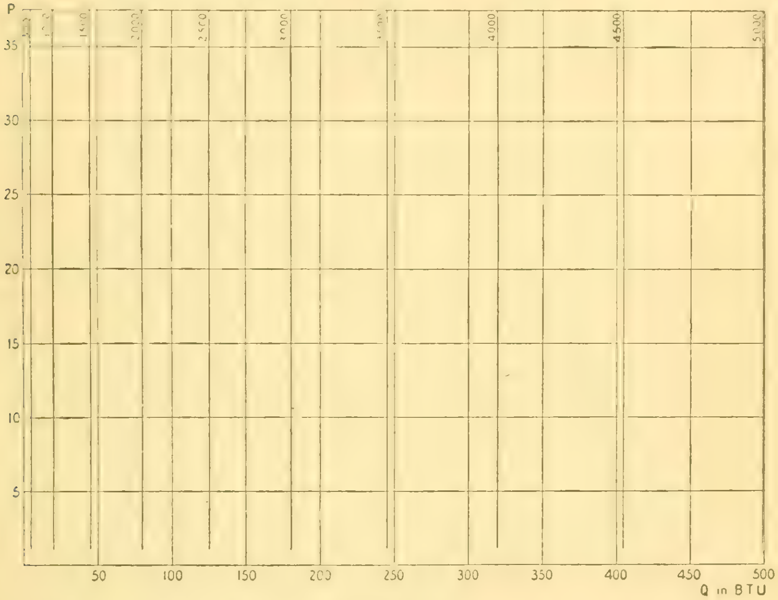


FIGURE 15.

Type C. Velocity. The velocity which a regenerative cycle produces depends only on the "quality of the mixture." (Cf. Figures 4 and 9.)

TABLE 2.

	Brayton.				Isothermal.				Regenerative.			
	p_a	T_a	P	Q	p_a	T_a	P	Q	p_a	T_a	P	Q
T_e		•	•	•		•	•	•			•	•
V		•	•	•		•	•	•				•
E			•			•	•	•		•	•	•
W			•	•		•	•	•		•	•	•

sults of this paper are essentially affected by a change in T_a. The dependence of the principal properties upon the four coördinates is summarized in Table 2, a blank space indicating that the coördinate at the top of the column is not involved in the corresponding equation.

The second of the questions proposed at the beginning of this investigation has now been answered for each of the three cyclic types; the

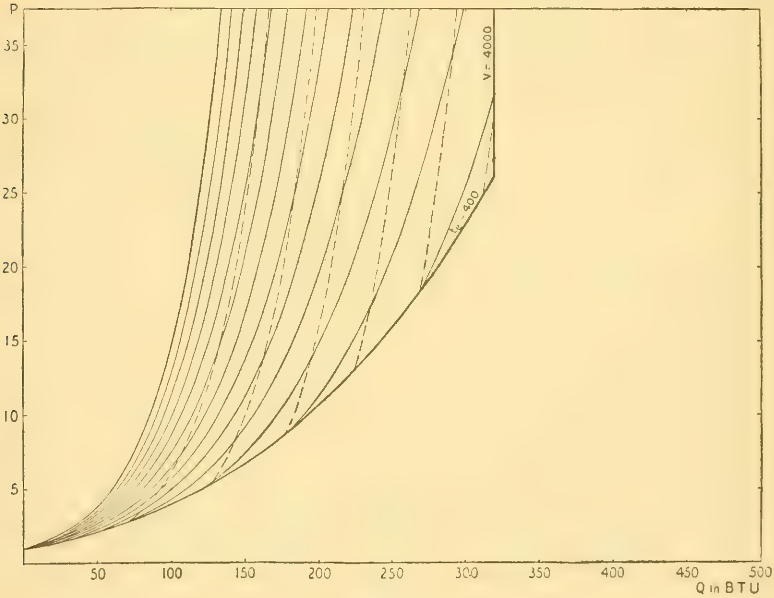


FIGURE 16.

Type C. VT area. For regenerative cycles. (Cf. Figures 5 and 10.)

TABLE 3. THE BEST POINTS.

	For Efficiency.	For Work.
Type A. Brayton . .	The top of the VT area, on the P axis (a null cycle).	The VT corner.
Type B. Isothermal .	The point S.	The VT corner.
Type C. Regenerative	The VT corner.	The VT corner.

first question involves simply a comparison of the cycles thus selected. These are distributed in the $V\,T$ area as is indicated in Table 3, and their properties are summarized in Tables 4 and 5. The highest efficiency is that of a cycle of the Brayton type, but it is a null cycle and any attempt to approach it for the sake of high efficiencies is at the cost of a high compression ratio and a very small value for W. If this cycle be left out of account, the best of those remaining, from every point of view, is

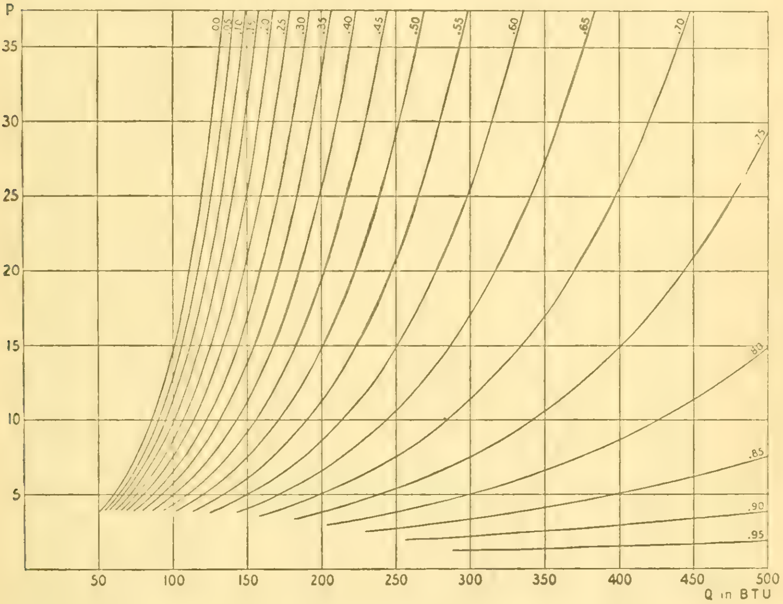


FIGURE 17.

Type C. Efficiency. For regenerative cycles. These curves, like those of Figure 14, converge toward $Q = 0$, $P = 1$. The line $P = 1$ corresponds to $E = 1.00$. (Cf. Figures 6 and 11.)

TABLE 4. THE EFFICIENCY CYCLES.

	P	Q	t_e	V	E	W
Type A. Brayton . . .	75.5	0	80	4000	.713	0
Type B. Isothermal. .	41.3	358	239	4000	.509	142000
Type C. Regenerative	26.2	320	400	4000	.621	154000

that regenerative cycle which corresponds to the $\bar{V}\,\bar{T}_e$ corner. Its efficiency and work are both as high as any in the tables, and its compression ratio is comparatively low. Even the isothermal cycles are usually to be preferred to any of the Brayton type on account of the much larger values of W which they involve.

The results to which this investigation has led are: (1) that of the three types considered, the regenerative cycle is, theoretically, the best for gas turbines, and (2) that a definite compression ratio as well as a

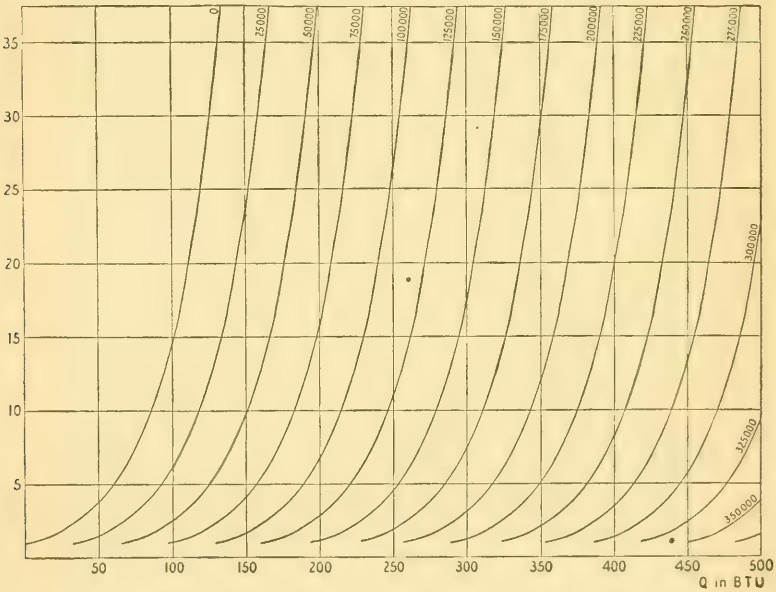


FIGURE 18.

Type C. Work. For regenerative cycles. (Cf. Figures 7 and 12.)

TABLE 5. THE WORK CYCLES.

	P	Q	t_e	V	E	W
Type A. Brayton . . .	26.2	195	400	4000	.610	92000
Type B. Isothermal .	26.2	396	400	4000	.502	154000
Type C. Regenerative	26.2	320	400	4000	.621	154000

definite consumption of fuel is determined by the conditions imposed, and should not be exceeded.

Many other interesting cyclic properties can be studied by means of the PQ plane. As an example, the curves of highest temperature (T_d) have been plotted for types A, B, and C, and are shown in Figures 19, 20, and 21. This quantity T_d is important, not only structurally, but also because the nature of a gas is found to change very materially by reason of dissociation when the temperature is raised above a certain limit. On this account it may sometimes be necessary to consider, in addition to the two limiting values $\bar{T}_e$ and $\bar{V}$, a third upper limit $\bar{T}_d$, which the highest temperature should not exceed, and to rule out a part of the $\bar{V}\bar{T}_e$ area on account of the new condition $T_d \leq \bar{T}_d$. In all three cases the part of the area surrounding the $\bar{V}\bar{T}_e$ corner would be the first to become unavailable on this account, so that the simple results obtained above would have to be somewhat modified. The graphical methods of this paper would, however, lead directly to the desired results in

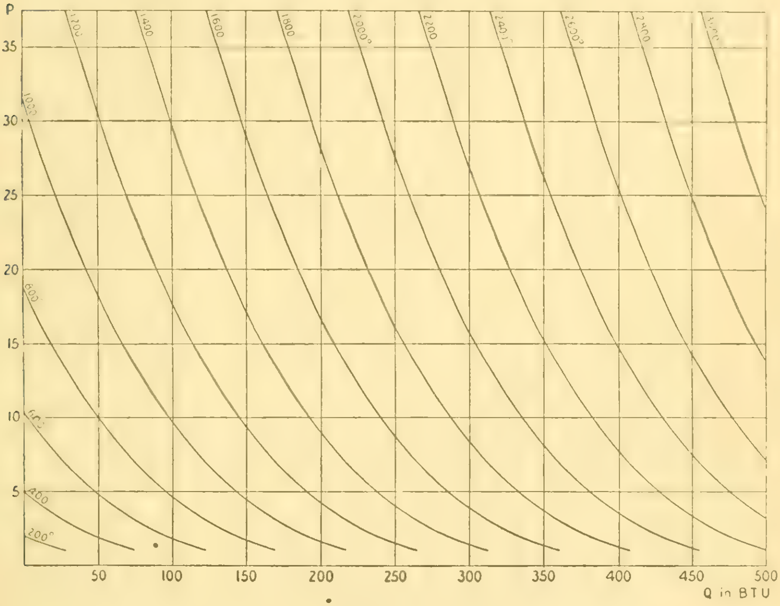


FIGURE 19.

Type A. Highest temperature. For adiabatic cycles.

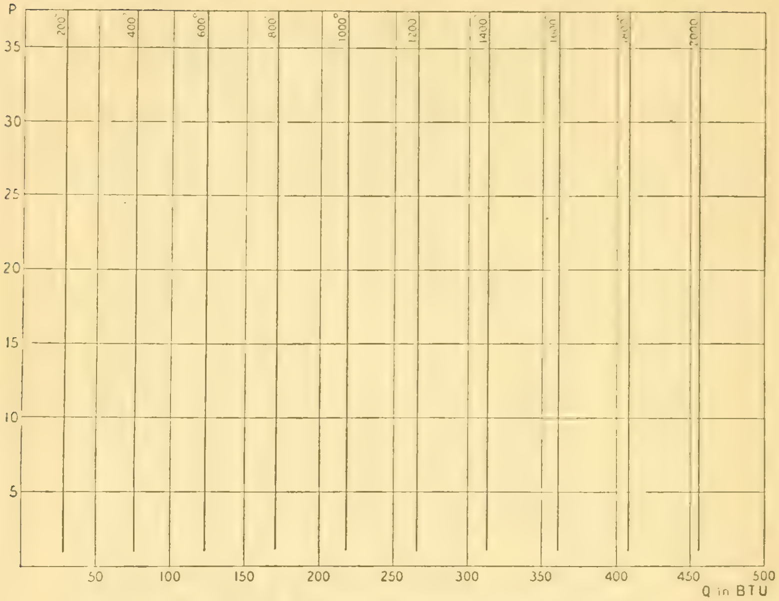


FIGURE 20. *Type B. Highest temperature.* For isothermal cycles.

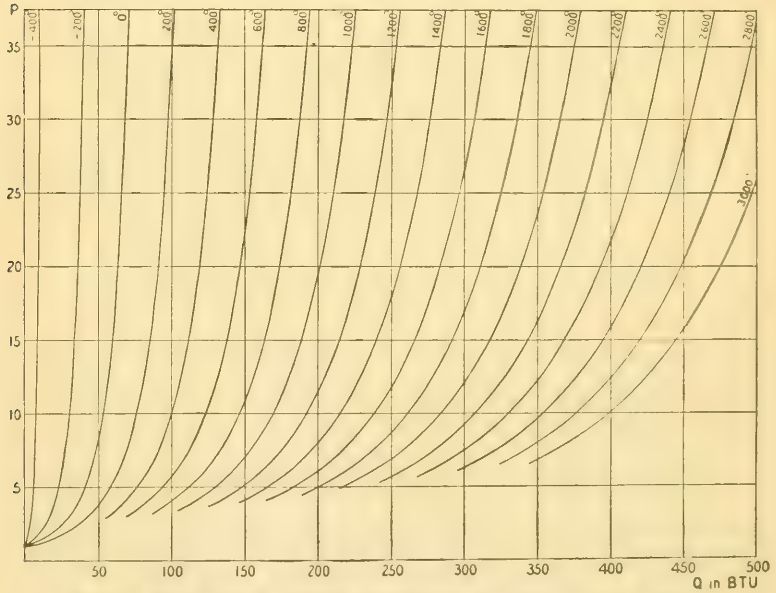
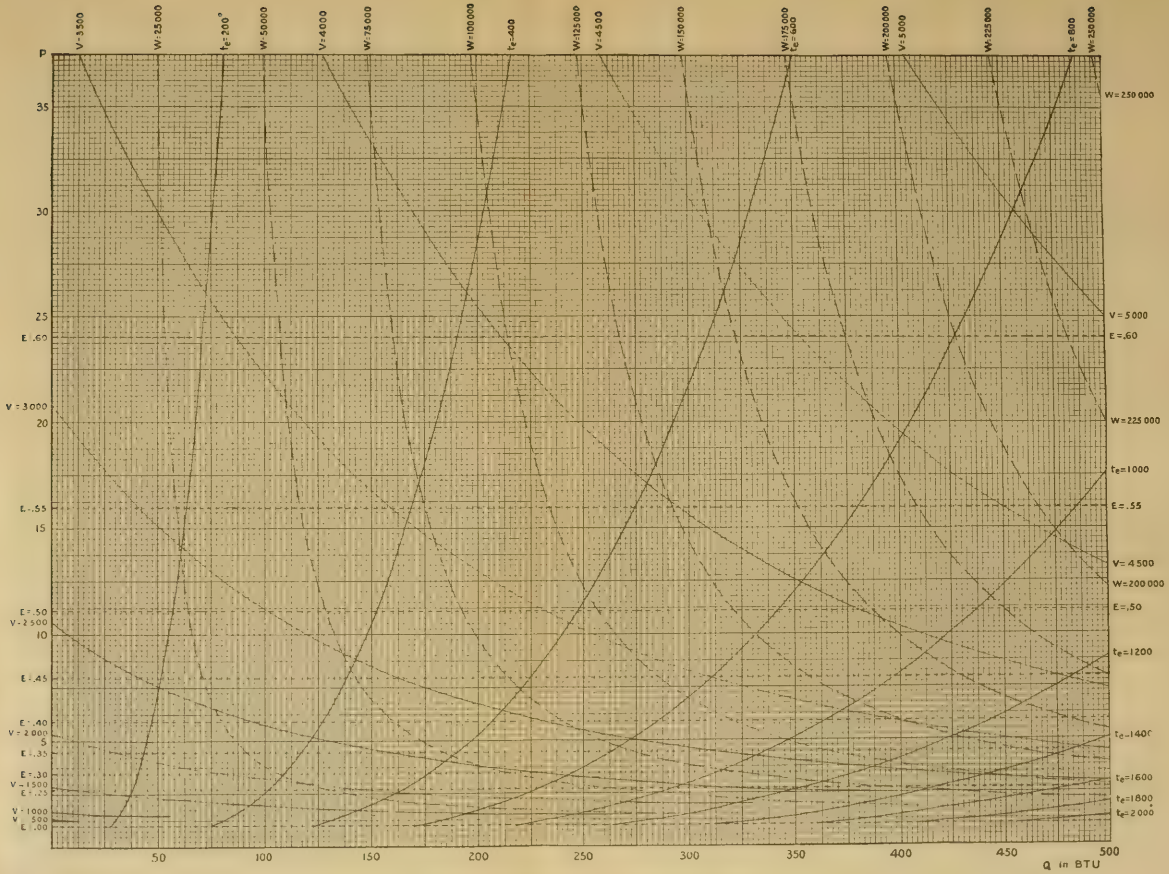
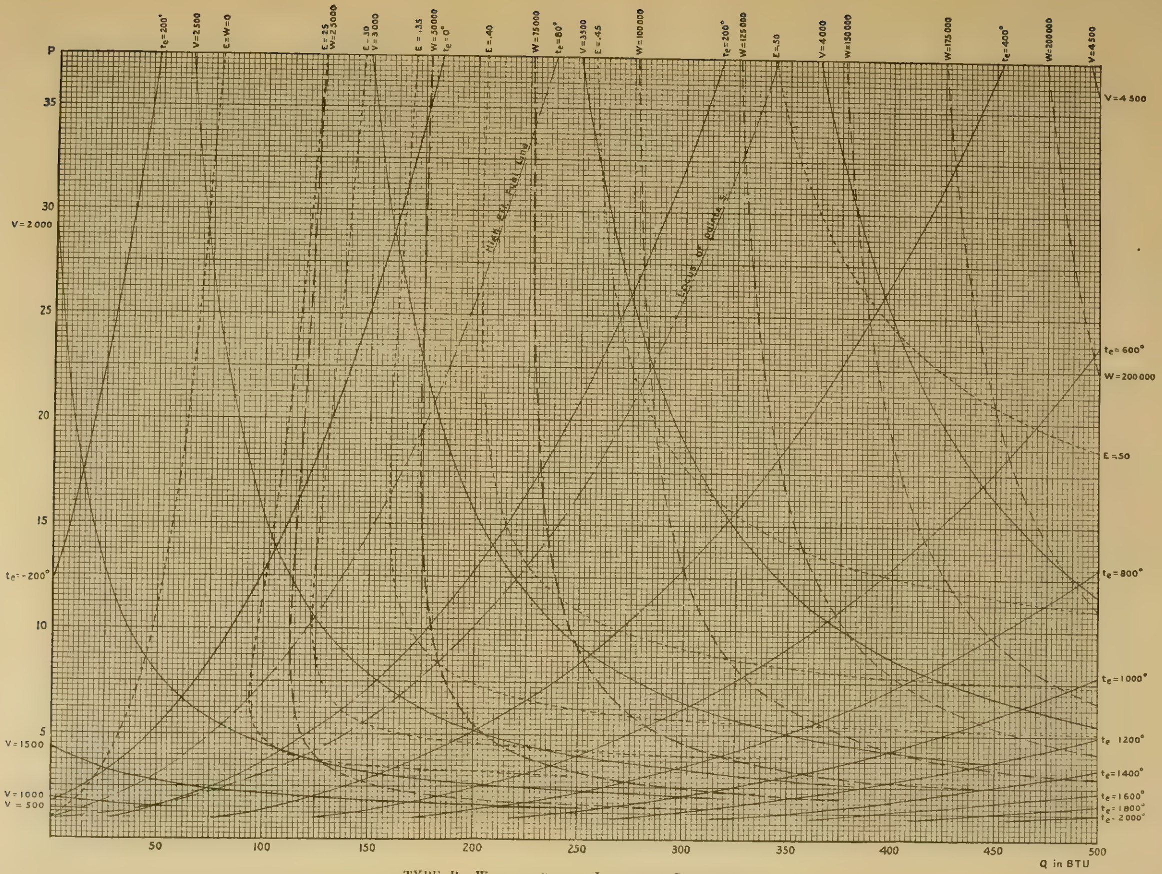


FIGURE 21. *Type C. Highest temperature.* For regenerative cycles.

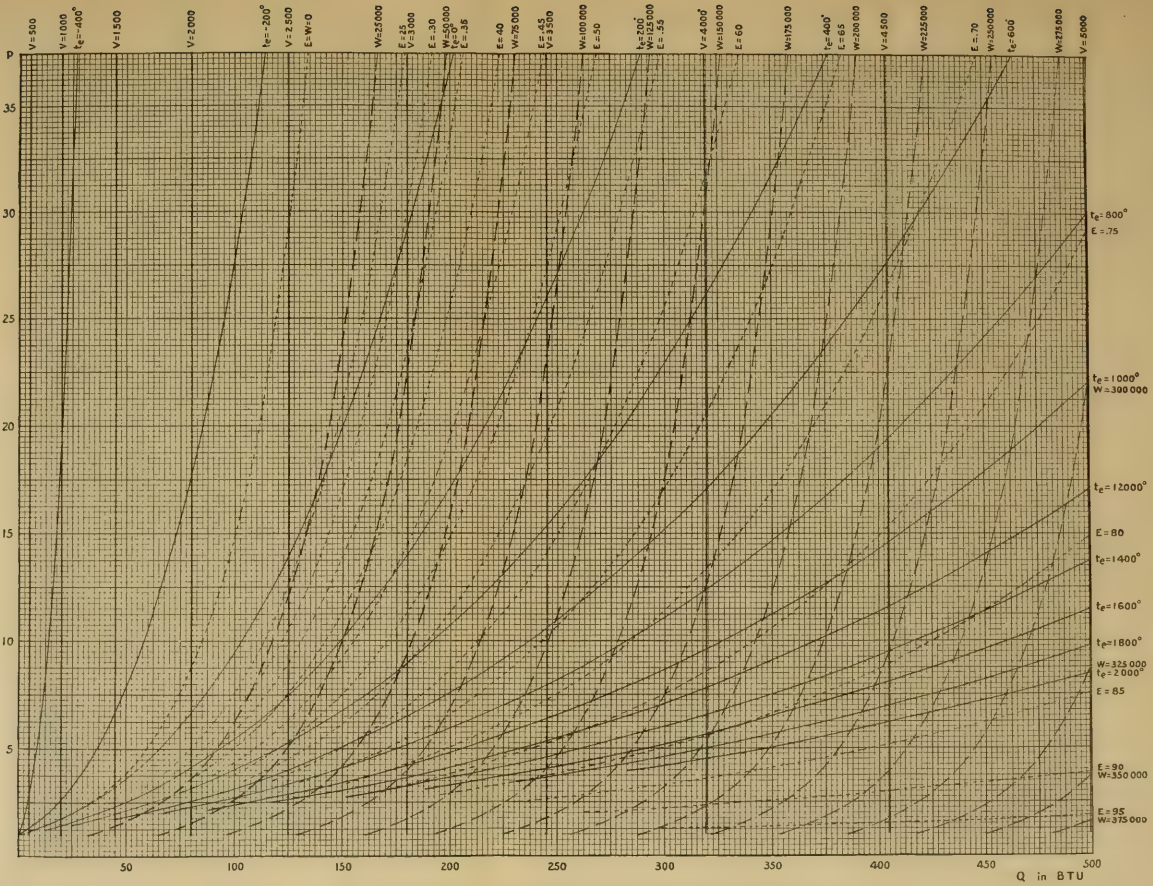
any given case, and probably also to a general theory, so that a prolongation of the present investigation is unnecessary. It is hoped that these methods will be found applicable to a great variety of problems in cyclic analysis, and that they will help in the development of this interesting subject.



TYPE A. Working Chart. Adiabatic Compression.



TYPE B. Working Chart Isothermal Compression.



TYPE C. Working Chart. Isothermal Compression with Regeneration.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

NEW METHODS OF DETERMINING THE SPECIFIC HEAT AND THE REACTION-HEAT OF LIQUIDS.

By Theodore William Richards and Arthur Becket Lamb.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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Presented March 8, 1905. Received January 27, 1905.

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Introduction.

The determination of the specific heats of solutions is a matter of importance, not only because these values are a necessary part of all thermochemical calculation, but also because of the probable connection between change of heat capacity and the "bound energy" of a reacting system.* Moreover, the irregular relations apparently existing between the values themselves still lack adequate explanation — and no satisfactory explanation can be given until the facts are more definitely and exactly known.

These considerations induced us to undertake the study of the problem, and the present paper recounts as briefly as possible the devices which were found to be necessary in order to eliminate the most serious possibilities of error.

The study of the heat capacity of liquids led directly, as will be seen, to a study of the heat of reaction evolved by mixing liquids; and for this purpose also new devices were found to be serviceable and conducive to precision. Because the two investigations are closely allied, they are given here together.

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* Richards, These Proceedings, **38**, 293 (1902); Van't Hoff, Drude's Annalen, Boltzmann.

PREVIOUS DIRECT METHODS OF MEASURING THE SPECIFIC HEATS OF SALT SOLUTIONS.

The brief discussion of the methods which previous investigators have used in measuring the specific heats of salt solutions will serve both to emphasize the most obstinate difficulties in these measurements and to furnish the basis for a more intelligent criticism of the new methods to be described.

Person,* in 1851, made a few rather inaccurate determinations in this field. Later, Schüller † carried out a somewhat more extended series of measurements. Both these investigators used the simple method, which Andrews had devised in 1845, for the determination of the specific heats of liquids. A weighed amount of water was inclosed in a glass vessel, into which a thermometer fitted. This served as a "calorifer." It was heated to a given temperature, and then plunged into the solution and used as a stirrer. The temperature of the solution was measured at frequent intervals until it had become constant, or was regularly falling, and the procedure of Regnault employed to correct for the radiation of heat. The method of Andrews, as they employed it, had little to recommend it beyond simplicity. The correction for radiation was uncertain, while it is quite impossible for the thermometer in the calorifer accurately to measure its temperature.

Julius Thomsen ‡ was the first to make a protracted study of the specific heats of solutions. He pointed out the inaccuracies in the method just mentioned, and elaborated a very original method to overcome them. A known quantity of heat was furnished to the solutions by burning successively equal measured quantities of hydrogen in a platinum vessel, which they successively surrounded. Since the pressure of the hydrogen was kept accurately constant by an automatic device, the heating effect was directly proportional to the time, and therefore by choosing an initial temperature as much below the room temperature as the final was above it, no correction for radiation was necessary. The burning hydrogen is certainly superior to the calorifer of Person and Schüller; but the necessary apparatus is cumbrous, and the accuracy of the method is limited by the accuracy of the volumetric measurement of the hydrogen. The doubtful accuracy of the correction for radiation sets a further limit to the refinement of the method. In the skilled hands of Thomsen, results were obtained which were concordant to within one to two-tenths of a per cent.

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* Person, Ann. de Chim. et de Phys. (3), **33**, 437 (1851).

† Schüller, Pogg. Ann., **136**, 70 (1869).

‡ Thomsen, Thermochemische Untersuchungen (1882).

Almost simultaneously with Thomsen, Pfaundler* published measurements of the specific heat of sulphuric acid at various dilutions, and Marignac published an extensive series of measurements of the specific heats of salt solutions, duplicating most of Thomsen's, and including many others. Both of these investigators employed the method of Andrews, but in an improved form. They interposed a blank experiment upon water between every experiment upon an unknown solution. In this way they were able to correct, both for the heat exchange with the surroundings, and for the lag of the thermometer in the calorifer. The efficacy of the former correction is, as was recognized, dependent on the constancy of the external conditions. The published results of Marignac are means of a number of determinations, and for these he claims an accuracy of from one to two one-tenths of a per cent. They agree, on the average, with the measurements of Thomsen to within about two or three one-tenths of a per cent.

Pfaundler,† before this, had devised an electrical method for the measurement of the specific heats of liquids. Here a known amount of heat is furnished to the liquids by the conversion of electrical energy into heat. Two wires of the same resistance were prepared, and one coiled up in a calorimeter containing the liquid of unknown specific heat. These coils were arranged in series, and a current passed through them. The rise in temperature in the two liquids was of course inversely proportional to the heat capacity. If these capacities were nearly the same, the correction for radiation practically canceled out. If they differed widely, the resistances were so altered that the temperature rise was nearly the same with the same bulk of liquid.

For use with electrolytes, the heating coils must of course be insulated, and Pfaundler was unable to find a satisfactory coating for them. He finally ‡ resorted to coiled glass capillaries containing mercury. The resistance of mercury, however, has so large a temperature-coefficient that he was obliged to measure the resistance of the coils at frequent intervals during a determination. His results were not especially accurate. In the few results which he published the error is at least as great as one one-tenth of a per cent, and this in spite of the fact that, by means of a delicate thermopile, which measured the difference between the temperatures of the two solutions, he had reduced the thermometric error to a low value.

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* Pfaundler, Sitzungsber. d. Akad. d. Wissen., Wien, **62**-2, 379 (1870).

† Pfaundler, Ibid. **59**-2, 145 (1869).

‡ Pfaundler, Ibid., **100**-2a, 352 (1891).

The method of Pfaundler has since been somewhat modified and used effectively by Magie, in whose hands it has attained very considerable accuracy.*

The method of Andrews, of Thomsen, of Marignac, and to a less extent the electrical method of Pfaundler and Magie, involve a more or less fortuitous correction for the heat exchange with the surrounding. All of them further involve the thermometric difficulty of accurately measuring a rapidly rising or falling temperature. Both these errors seem at first sight unavoidable in specific heat determinations. In 1887 Hesehus † attempted to avoid these difficulties by making the final temperature that of the surrounding air. This he accomplished in the following very simple fashion. After the hot body, whose specific heat was to be determined, had been brought into the calorimeter, he added cold water, little by little, at such a rate that the temperature in the calorimeter remained constant. Knowing the initial temperatures of the hot body, and the cold water, and the temperature in the calorimeter, and knowing the weight of the hot body and of the water added, he could evidently calculate the specific heat. Since there was no temperature change in the calorimeter, there was no heat exchange to correct for, and since the temperature of the hot body, the cold water, and the water in the calorimeter were stationary, the temperature measurements could be made under the most favorable conditions. Hesehus determined the specific heat of brass in this way, and Waterman,‡ after improving the technique of the method, made extended determinations of the specific heats of metals. His results showed excellent concordance (0.05 per cent) and established the efficacy of the method.

The method of Hesehus and Waterman is a great improvement over that of Andrews, because the final medium temperature at least is measured accurately. This medium temperature in specific heat determinations is of more importance than either the higher or the lower temperature, since an error in it is magnified in the calculation. Nevertheless, the cooling of the warm body just before the mixing, and the warming in transit of the cold water gradually added, bring in here also the old difficulty of the dissipation of heat in a new form.

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* W. F. Magie, Physical Review, **9**, 65 (1899); **13**, 91 (1901); **14**, 193 (1902); **17**, 105 (1903).

† Jour. de la Soc. Phys. Chem. Russ., Nov., 1887; Jour. de Physique, **7**, 489 (1888).

‡ Phys. Review, **4**, 161 (1896).

A New Method.

In devising a new method for determining the specific heats of liquids, the endeavor was made to retain the advantages of the method of Hesehus, while minimizing the disadvantages. The important objects to be accomplished were the following: —

First, the thermometers must all be read while stationary, not with a moving thread of mercury.

Secondly, as little time as possible must be allowed for the cooling of the hot liquid or the warming of the cold liquid.

Thirdly, conditions must be so adjusted that these two last-named errors, each made as small as possible, should cancel one another.

The first of these objects was attained by keeping both the hot and the cold liquids in very accurate thermostats before the moment of mixing, and by mixing them in a calorimeter surrounded by a thermostat maintained at the final temperature of the mixture. Thus all the temperatures were measured accurately, without the possibility of thermometric lag.

The second object was attained by delivering the two liquids into the mixing calorimeter simultaneously with great rapidity, having previously so adjusted their relative weights, and their relative temperatures, that their equilibrium temperature would be nearly identical with that in the water jacket of the calorimeter.

The third object was attained by making all the delivering apparatus as similar as possible in the two cases, and by having the temperature of the hot liquid as much above the final temperature as that of the cold liquid was below it.

The quickest adjustment of the final temperature is attained if the liquids are allowed actually to mix, not merely to equalize their temperatures through a heat-conducting septum. But if two liquids dissolve one another, they almost invariably evolve or absorb heat in the process. This extra heat adds algebraically to the heat-capacity effect, and must be subtracted in order to obtain this alone. Fortunately the heat of reaction is easily found in the present case by making additional experiments with the same apparatus, maintaining all the thermostats at the same temperature; or still more simply by means of another device described on page 675.

With these data, knowing the specific heat of one of the liquids within the given range, it is easy to calculate both the specific heat of the other liquid and that of the mixture, as will be shown.

The Hot Thermostat.

As has been said, the new method requires the two solutions should mix quickly and completely. To accomplish this, glass pipettes, each of about 0.23 liter capacity, were constructed to contain the liquids, and provided with ground-glass stoppers inside the lower ends of their bulbs, as illustrated below. These stoppers could be pulled out simultaneously by means of long silver wires. The entrance tubes were made of large enough bore to admit easily both the silver wires and large Beckmann thermometers. The exit tubes had bores of 10 mm. and therefore permitted a very rapid outflow of the liquids. These tubes were bent at such an angle that, when the pipettes were in place, the issuing streams met before striking the walls and floor of the receiving vessel in the calorimeter.

The method further required that the solutions in these pipettes should be kept at constant temperatures, one above, the other below, that of the room. The temperature of the solutions must be known to within the thousandth of a degree, and the development and construction of thermostats which would permit this was the main difficulty of the research.

Thermostats maintaining a temperature above that of the room, with the required constancy, have been constructed by Bradley,* Geer,† and others. In the present research, that of Bradley was chosen, as offering the fewest difficulties of construction. The action of the thermostat depends on the automatic regulation of an inflowing stream of hot water. This regulation is accomplished by the alternate contraction and expansion of a suitable liquid, inclosed in a long spiral tube, whereby a column of mercury is raised or lowered, and thus made to open or close the end of a capillary tube through which the warm water enters.

The thermostat, as used here, is partially illustrated in the diagram.‡ It differs in several respects from the instrument as described by Bradley.

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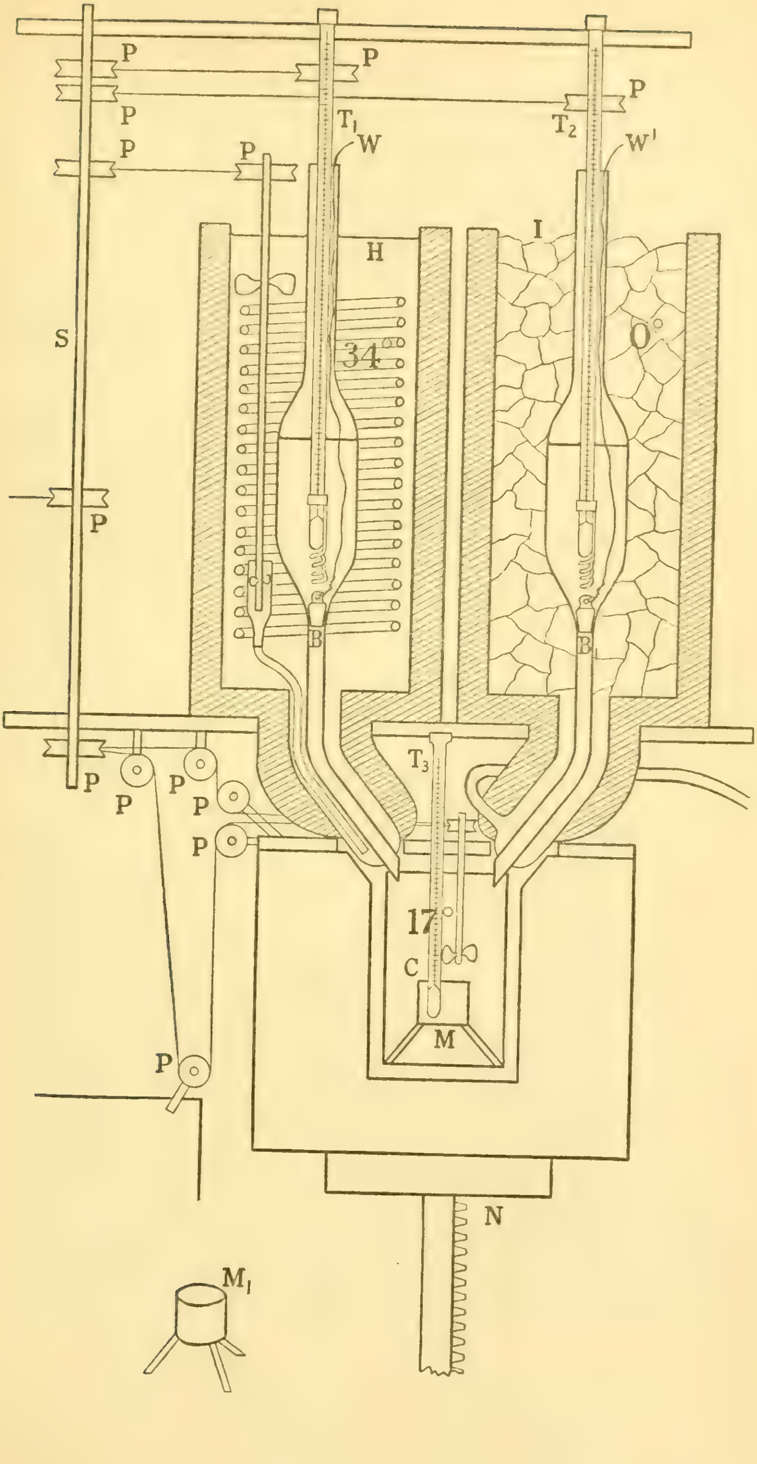
* Journ. Phys. Chem., **6**, p. 118 (1902).

† *Ibid.*, p. 85 (1902).

‡ The following designations will assist in the understanding of the diagram: —

B, B′. Stoppers in pipettes.
C. Calorimeter for mixing.
H. Hot thermostat.
I. Cold thermostat.
M. Platinum ring and tripod to prevent spattering.
M′. Perspective view of same, removed.
N. Vertically movable supporting stand for calorimeter.
P, P. Pulleys for driving stirrers.
S. Motor shafting.
T_1, T_2, T_3. Thermometers.
W, W_1. Wires for pulling stoppers (B, B′).


P
P
P
P
P
P
T1
W
T2
W'
I
H
34°
0°
S
P
B
B1
T3
P
P
P
P
P
P
17°
C
M
P
N
M1

In place of a glass spiral to contain the expanding liquid, a metal one was substituted. This was made from several lengths of thin-walled steel tubing, 1 centimetre in diameter. Its total length was about 30 feet and its volume about 0.45 litre. . The joints were brazed and re-inforced, and the whole thoroughly galvanized and painted. The change introduced the inconvenience of joining the glass cut-off arrangement to the steel tube. This, however, was accomplished without difficulty, by means of a commercial cement called de Kotinsky Laboratory Cement. The spiral responded with great rapidity to temperature changes. To prevent leakage through the glass stopcock* we bent that end of the steel tube into a U and filled it with mercury. Evidently a much less efficient stopcook is required with mercury than if benzol (the expanding liquid) were in direct contact with it. Further, bulbs were provided in both arms of the U-tube of the glass cut-off, to afford greater latitude of temperature changes. An additional inlet provided with a stopcock was inserted to admit when desirable large amounts of hot water. This proved of convenience in heating the thermostat rapidly to about the right temperature.

Ether was first employed as an expanding liquid. In many respects it is an ideal substance for such a purpose. Its coefficient of cubical expansion and its relative conductivity are greater than those of any of the commoner liquids. Its specific heat, too, is very small. Unfortunately, not only is its boiling point inconveniently low, but, in standing, bubbles invariably appeared in the spiral tube, even when the method of filling precluded the possibility of any bubbles having been initially present. After many trials, therefore, ether was abandoned, in spite of its exceptional advantages. After ether there is little choice between a number of organic liquids, such as benzol, toluol, chloroform, and acetone, as the following table shows. The alcohols were quite unsuited to our use, for beside having rather high specific heats they exerted a strong solvent action on the cement. Benzol was chosen.

To fill the spiral, benzol was poured in at one end; by proper tilting most of 'the air was forced out at the other. The remainder was removed by boiling the liquid under reduced pressure. This was carried out by attaching the end of the spiral to a glass tube, which reached nearly to the bottom of a filtering flask, which in turn was connected

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* This glass stopcock is not shown in the diagram; it was at the opposite end of the steel coil from the cut-off arrangement, and served to admit the proper amount of liquid to the spiral. In order to understand the details of the apparatus, Bradley's paper should be consulted, for only the differences are emphasized here.

Liquid.	Coefficient of Cubical Expansion.	Relative Conductivity ($H_2O = 1$).	Specific Heat
$(C_2H_5)_2O$	0.0021	0.33	0.40
C_6H_6	0.0014	0.22	0.36
$CHCl_3$	0.0014	0.29	0.35
$C_6H_5CH_3$	0.0014	. . .	0.43
$CaCl_2 + H_2O$ (46%)	0.00045	0.95	0.83

with a water pump. The pump was started and the liquid in the spiral kept boiling until most of it had boiled or been ejected over into the filter flask. The pump was then disconnected, and the liquid, free from air, was now pressed by the atmosphere back into the spiral. Enough liquid was always previously placed in the filter flask, so that at the end no air would go back with the liquid. This process was repeated several times at each end of the spiral. The bubbles were thus removed with such certainty that the opacity of the metal tube is not, as Bradley feared, a disadvantage.

The shape of the thermostat was of course governed by the shape and dimensions of the pipette. Further, it had to be provided with a protruding jacket or snout surrounding the lower tips of the pipette, else the temperature of the liquids would be altered in running out. The water in the thermostat was vigorously stirred by means of a small, but rapidly revolving, propeller. This did not have much effect on the water in the snout, and a second smaller propeller was attached to the same shaft and made to fit quite closely *inside* of a glass tube. From the end of this glass tube a rubber tube extended down to the very tip of the snout. This second propeller acted as a pump, and kept up a vigorous circulation of the water in the snout.

The hot water was furnished by a large copper thermostat heated by gas. Its temperature was kept constant by means of an ordinary Ostwald regulator at about 50°, that is, about 16° above that of the hot pipette-thermostat. A regulator is scarcely necessary, as large fluctuations in temperature seem to have no effect on the constancy of the hot thermostat. The water, as it left the heater, was filtered through a piece of fine silk cloth.

As long as the mercury and the glass cut-off were fairly clean, this thermostat proved admirably suited to the purpose. When equilibrium had been established, often for hours at a time no change could be detected on a Beckmann thermometer which could easily be read to thousandths of a degree. In the latter part of the work the mercury occasionally caught in the annular space outside the capillary inlet tube, causing sudden and vexatious fluctuations of temperature. This would perhaps be obviated by a different ratio between the size of the capillary entrance tube, the size of the annular space, and the head of the water. It was found more convenient, however, to discard altogether the method of cutting off the flow of hot water directly by means of the mercury column, and to use the mercury as a means of making and breaking a feeble galvanic current. When made, this current actuated a relay which in turn sent a stronger current through an electro-magnetic arrangement for cutting off the supply of hot water. This apparatus worked admirably, and was used in a few of the last experiments.

The Cold Thermostat.

It was endeavored, at first, to maintain a constant temperature below that of the room, by passing a rapid stream of tap water around the pipette containing the solution. This gave surprising constancy at times, but was usually far too variable for the purpose. In the hope of increasing its constancy, the tap water was then made to pass first through a large mixing vessel, itself well insulated by an outside jacket of running water. This was a slight, but inadequate improvement. A special apparatus was therefore devised, but this apparatus was finally supplanted by a much simpler one, and therefore need not be described in detail. It utilized the tap water as a cooling agent, being available whenever this is tolerably constant, and consisted essentially of two water jackets, one inside the other, insulated from one another with cotton wool. The inner one contained the pipette, wherein the solutions were placed. The liquid within the pipette has been kept in this device for over an hour without suffering the slightest change in the temperature as observed by means of a very sensitive Beckmann thermometer.

As has been said, this method also was discarded for the much simpler one of packing the pipette in ice, — a proceeding which would have been adopted in the first place had it not at first seemed advisable to maintain a temperature somewhat above the freezing point. By using pure ice, mixed with previously chilled, boiled, distilled water, we were able to do away entirely with the measurement of the temperature of the cold solution, because it was exactly zero. This of course was a distinct gain.

Here as in the hot thermostat, the tip of the pipette had to be maintained at the same temperature as the bulb. The similar glass jacket or snout provided for this purpose could not be so well insulated as the body of the thermostat, and therefore was provided with an outlet tube by means of which ice-cold water could be run through it from above. Thirty seconds before the solution was run out from the pipette, this current of ice-cold water was started, and thus the tip of the pipette, already very cold, was brought to zero.

The Calorimeter and Remaining Apparatus.

The construction of a calorimeter in which the hot and cold liquid should mix presented no especial difficulty. The receiving vessel itself was a platinum can weighing some 100 grams, and having a capacity of 0.5 litre. This was supported on corks in a somewhat larger can of tinned iron, which in turn was sunk in a large jacket of water. It was found important to have the annular space between the two cans as small as possible, in order to make sure that the platinum can was at the same temperature as water in the jacket. To make this doubly sure the annular space was packed around the top with a little wool. The whole was covered with a tight-fitting wooden cover.

The liquids fell into the platinum can with considerable violence. This was necessary, if the pipettes were to be emptied in the brief space of a couple of seconds. Much difficulty was at first experienced in the spattering which thereby results, — an evil which was finally remedied by a very simple cylindrical device of platinum foil resting on the floor of the platinum can. It is pictured in the diagram. As a further security and to assist in preventing evaporation, a thin lacquered-iron cover and an outer wooden cover were fitted over the top of the platinum can. In order to shorten as much as possible the distance through which the liquids must pass while exposed to the air, two holes were provided in the covers of the calorimeter large enough to admit the snouts of the two thermostats. In this way the tips of the pipettes were brought directly into the platinum can, and the exposure of the jets was very slight. These holes, when not in use, were closed tightly with corks. It was further important that the ends of the snouts should not remain inside the calorimeter any longer than necessary, although the involved error would have been small, since one snout was as hot as the other was cold. In order to accomplish this easy removal, the calorimeter was placed on an adjustable stand, which by means of a rack and pinion could be raised and lowered with rapidity and certainty.

The liquids must evidently be stirred both before and after mixing. Because of the small available space in the entrance tubes of the thermostat-pipettes the thermometers were utilized for stirring the initial hot and cold liquids. For this purpose small spiral-shaped pieces of silver foil were attached to the lower end of the thermometers projecting somewhat below the bulb. The upper end of each thermometer was provided with a bearing from which it hung, and which permitted the thermometer to rotate about its long axis. A thin glass tube was snugly fitted over the shaft of each thermometer, and this, rubbing against the upper tube of the pipette, furnished the second bearing. The spiral-shaped piece of foil acted like a screw, and produced circulation of the liquids in the pipettes.

The stirring in the cold solution was discontinued as soon as the thermometer immersed in it showed that the constant temperature of zero had been reached. The mixed liquids in the platinum can were stirred by means of an inverted glass T-tube, provided with holes along its vertical arm, and rotating at a moderate speed. The belting which ran this was so arranged as not to interfere with the free vertical motion of the calorimeter (see diagram). A simple catch was also provided, such that a single motion sufficed to start or stop this stirrer.

Unfortunately a few drops of the liquid always fell from the pipettes after the main body of the liquids had rushed out. We could not be at all sure that these drops were at the same temperature as the rest of the liquid. For that reason an arm was constructed which carried two glass cups, and which could be swung beneath the ends of the pipettes the moment the calorimeter had been lowered.

For results accurate only to between 0.1 and 0.2 per cent, it would be possible to neglect both these drippings and the liquid remaining in the pipettes. For more accurate measurements, the amounts of these drainings and drippings must be determined. At first it was planned to weigh them, having collected the liquid left in the pipettes in suitable absorption tubes after evaporation by means of a current of dry air. This proved to be a very tedious process.

A much more expeditious and satisfactory method was finally adopted with water or aqueous solutions. Instead of dry air, the pipettes were rinsed with dry acetone, the drippings added to this, and the whole made up to 50 millilitres at 25°. The water in the mixture was then determined by means of specific gravity measurements, and was subtracted from the total weight of water originally in the pipettes in order to find the amount which actually found its way into the calorimeter.

For the specific-gravity determinations a thermostat was constructed which maintained a constant temperature of 25° within the one one-hundredth of a degree. It was heated by means of an incandescent electric-light bulb, and regulated by a simple regulator filled with alcohol and a little mercury, which, instead of cutting off a current of gas, made and broke an electric relay current, which in turn opened or closed the lighting circuit. A glass float was then made which was adjusted so that it neither sank nor floated in a known mixture of acetone and water containing a slightly greater percentage of water than was usually present in the unknown mixtures. After each mixture had remained an hour in the above thermostat the float was put in and water added from a burette until the float just sank. Acetone was then added until equilibrium was reached. Since the mixture to which the float was adjusted contained about 3 per cent of water, evidently the volume of the water needed to be known with much greater accuracy than that of acetone. The water burette was therefore very narrow, and was graduated to hundredths of a millilitre. The float was easily sensitive to less than one one-hundredth of a gram of water, and since the total amount of water usually amounted to somewhat more than 170 grams, the error in this method affected the final result by less than one one-hundredth of a per cent.

When the solutes in the solutions whose specific heats are being determined are insoluble in acetone, this method can be used for them also. This was the case with copper sulphate. If a given solute is soluble in acetone the pipettes can be rinsed with water, and the amount of the solute in this determined by chemical methods. This was the case with silver nitrate, for example.

Beckmann thermometers were used to measure the temperature of the hot liquid, the cold liquid, and the mixture. The scales were all divided into one-hundredths of a degree, and by means of a small lens could be read to within a one-thousandth of a degree with certainty. A set of Baudin thermometers, described elsewhere, was used to standardize these.* Comparisons were made in all cases under experimental conditions, and the zero point of the standard was afterwards taken.

The apparatus having been described, it is desirable to emphasize its advantages.

In the first place, all error from thermometric lag is avoided. Again, such slight errors from cooling as may be present during transference is compensated by an equal and contrary heating effect, for while the liquids are falling from the ends of the pipettes into the mixing vessel, the absorption of heat from the surroundings, by the cold liquid, exactly cancels the radiation of heat from the warm liquid. Even the cooling and warming due to evaporation and condensation are also practically identical, since the air through which the liquids pass is saturated with water vapor. It will be seen, in the description of the apparatus, that not only do these errors cancel, but the errors themselves have been almost eliminated, by making it possible to raise the calorimeter, just before the liquids are to run out, to such a height that the tips of the pipettes are within the calorimeter.

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* Richards, Am. Jour. Sci. (4), 6, 201 (1898).

The possibility of an unequal radiation or absorption of heat by the two liquids after transference must however be considered. Such a thing could happen only if the mixing was tardy, — for example, if the cold solution chanced to remain for some time close to the walls of the mixing vessel, while the warm water, for the most part, occupied a more central position. The following consideration will show how remote such a contingency must be. In the first place, the stoppers, which released the liquids, were pulled out within $\frac{1}{3}$ to $\frac{1}{5}$ of a second of one another. The exit tubes were identical in form and length, and were so directed that the two issuing streams met before striking the walls and floor of the mixing vessel. Further, the very moment that all the liquid had run out (and this required only about two seconds), active stirring of the mixed liquids was begun. In less than 90 seconds more, the precise final temperature was established, It is evident that the great difference of temperature must have been equalized within a very few seconds after mixing. We have seen, then, that not only is an unsymmetrical distribution of the hot and cold liquids unlikely, but if this should occur its duration must be exceedingly brief.

Because the errors due to radiation after, before, and during mixing were thus eliminated, it is evident that the accuracy of this method must depend first, on the accuracy with which the relative amounts of the two liquids are known, and second, on the refinement of the temperature measurements. The initial weights of the two liquids can be known with all the desired accuracy. By taking care to avoid evaporation and by determining the amounts of liquids left in the tubes, we can be almost equally sure regarding the amounts of the liquids which actually mix. It appears, then, that the accuracy of the method must depend chiefly on the refinement of the temperature measurements.

Of the temperature measurements, that of the *mixture* is the most important; because an error in it has the greatest effect on the final result. The reason for this is, that we determine the fall in temperature of the warm liquid by subtracting this equilibrium temperature of the mixture from the initial temperature of the warm liquid; while we also determine the rise in temperature of the cold liquid by subtracting its initial temperature from the temperature of the mixture. Any error, then, in this latter temperature measurement has a double effect on the specific heat.

The accuracy of the thermometry, of course, depends primarily on the thermometers employed. It will suffice to say here, that, by comparing the thermometers with standards after every experiment, and taking the zero point of these latter, and making the correction when necessary for exposed thread and hydrostatic pressure, we have usually felt sure of the equilibrium temperature to within two or three one-thousandths of a degree. This, with the usual fall of temperature of 17 degrees, would correspond to a percentage error of $\frac{2 \times .0025}{17.000}$, or about 0.03 per cent.

The stationary condition of the thermometer is so favorable to precise measurement of temperature that it seems by no means impossible that, using still greater precautions in the temperature measurements, by means of platinum resistance thermometers, one might be sure of the temperature of the mixture to within less than a single one-thousandth of a degree, which, with the same temperature fall of seventeen degrees, would correspond to an uncertainty in the specific heat of less than 0.01 per cent. This would far exceed the accuracy with which the varying specific heats of water are now known.

As we have just implied, the accuracy of the method is directly proportional to the temperature fall of the warm liquid, or the temperature rise of the cold one. But the specific heat which is measured is the *mean* specific heat over the temperature interval employed, and since we cannot be sure that the change of the specific heat with the temperature is linear, it follows that the greater the temperature interval, the less the certainty with which the specific heat at any given temperature is known. The temperature interval here employed was about seventeen degrees.

The rise in temperature due to the work done by the actual fall of the liquids from the pipettes can be easily calculated. Thus, the total weight of the falling liquids was about 350 grams; the length of the fall about 40 cm. The kinetic energy converted into heat, therefore, amounted to 40 × 350 × 981, or 13,700,000 ergs, which corresponds very nearly to one-third of a calorie. This would have heated the 350 grams of mixture nearly a one-thousandth of a degree. Further, the stirring of the mixture, though lasting for only a minute, must have exerted a warming effect. This also was found to amount to about one one-thousandth of a degree.

These two constant effects, the further possibility of the cold liquid not being exactly at zero degrees, constant thermometric errors, and other hiddden errors, were completely eliminated by making parallel determinations, using water in each pipette. Had it not been for these small constant errors, we should of course have obtained relative specific heats of the hot and cold water whose ratio corresponded to the ratio of the known mean specific heats of water between these temperatures, provided, that is, that the known value also is free from such errors. As a matter of fact, we found that these ratios were not exactly equal, but that they differed by a small constant amount, — less than 0.05 per cent.* The small magnitude of this difference is equivalent to proof that the apparatus just described gives about the same relation between the specific heats of the warm and cold water as has the work of others, and forms an excellent test of the efficiency of the present apparatus.

The computation with water in both pipettes is very simple; but, as has been said, the simple heat capacity of a solution is not determined by this method, for the heat of dilution is superposed when the cold solution is mixed with warm water. It is necessary, therefore, to make a separate measurement of the heat of dilution. In spite of the ease with which this latter measurement can be made, this might, under some conditions, be considered a drawback, and for that reason it was originally planned to have the liquids equalize their temperature without uniting. For this purpose there was devised a cylindrical vessel divided into two compartments by means of a thin, greatly convoluted partition. Since in the present work the heat of dilution was also of interest and value, as will be shown, the determination of the heat of dilution at this time involved no extra labor. For this reason the specific heats of solutions were determined by actual mixing, and corrected for the heat of dilution.

In passing, it may be noted that this apparatus could be used for determining the heat of reaction of solutions upon one another, when thermometric lag is to be excluded. One of the solutions could be placed in the calorimeter at the temperature of the room, and the other in one of the thermostat-pipettes at a temperature just low enough to counterbalance the heat of the reaction, so that the mixture also would be at the temperature of the room. This application of the device will be tried in the near future.

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* The standard of comparison was the table chosen by Ostwald and Luther as representing the true values (Phys. Chem. Mess., p 195 [1902]). Our value for the ratio, 1 : 1.0059, was between the value derived from this table and the lower value to be derived from the work of Callendar and Barnes (Zeitschr. phys. Chem., **32**, 161 [1900]).

THE HEAT OF DILUTION.

For the determination of the heat of dilution the same apparatus could have been used, adjusting all the thermostats to the same temperature. It was, however, much more convenient to use for this purpose a simpler and probably more accurate device, described below.

This device consisted merely of a beaker divided into equal right and left-hand compartments, by means of a detachable partition of silver foil. The partition was held in place by a narrow U-shaped ledge of glass cemented to the walls and floor of the beaker. To this ledge the silver foil was fastened, water-tight, by means of hard paraffine. A glass rod was securely attached by wires and cement to the partition, and by its means the partition could be easily detached from its supports. Water was placed in one compartment and the solution in the other. Each was provided with a stirrer and a sensitive Beckmann thermometer. The whole was then placed in a calorimeter at the desired temperature. As a rule, it was more expeditious to adjust the temperature of the liquids on either side of the partition to within a few one-hundredths of a degree of each other, before they were placed in the calorimeter. When the liquids in each compartment had reached practically the same temperature and were either stationary or very slowly rising in temperature, readings were made of the thermometers at the minute intervals. At a given moment, having made a final reading of the thermometers, the partition was removed, active stirring continued, and the thermometers read at intervals of a minute, until the same regular rate of warming or cooling had established itself. This seldom required more than two or three minutes. It was not difficult to make the necessary corrections for radiation over so short an interval. The *average* change recorded in both thermometers evidently represents the actual temperature change — since both liquids were not usually at exactly the same temperature.

Since the heat of dilution of these concentrations only amounts to a few one-thousandths of a degree, there is evidently no need of great accuracy in determining the quantity of liquids used. We therefore measured out equal volumes, using a pipette of 0.173 litre capacity. This reproduced very nearly the actual weights of water and solution used in the above determinations of specific heats. The heat capacity of the beaker amounted to 110 × 0.2 = 22, or 92 mayers; about one-sixteenth of the total heat capacity of the mixture. In most cases this was negligible.

This simple apparatus is so efficient in its operation as to be worthy of use also in other thermochemical operations where great accuracy and small range of temperature is needed. The method in which the mixing is accomplished is certainly preferable to any method which demands the pouring of one of the liquids from one vessel into another.

RESULTS.

A few actual results with the apparatus may now be discussed in order to show the accuracy which has been obtained. It is possible to calculate the specific heat of a solution by mixing water in this apparatus in two ways. The value for the solution over the range 0° to 17° may be based either upon the heat communicated to it by the warm water as calculated from the average specific heat of the warm water, and its fall of temperature in mixing, or else upon the value for water between 0° and 17° as found by mixing this cold water with exactly the same amount of hot water in parallel experiments. The two methods of calculating give almost the same value, the difference being less than 0.05 per cent, as was implied in previous statements concerning the quantitative mixing of hot and cold water made on page 674.

The cause of such slight difference as exists need not be at present discussed — it is enough to emphasize again the fact that reference of the cold solution to cold water measured in parallel experiments over the same range of temperature is by far the best, since thereby any possible constant errors of experimentation are eliminated, being canceled by appearing in each series. The results below with cupric sulphate and argentic nitrate are computed in this way. The heats of dilution were subtracted before the calculations were made. The calculations for the concentration $CuSO_4 + 200H_2O$ were made on the assumption that very slight dilution evolves only a negligible quantity of heat, which was shown to be true.

The "probable error" of this result, computed by the method of least squares, is only about 0.01 per cent. Constant errors must have been eliminated by the method of determination which compared the solution with water under exactly the same conditions, in parallel experiments. Hence the result is probably not far from the truth, although the series is claimed to be only preliminary, made to demonstrate the practicability of the method. Because there are no other determinations of such a solution between 0° and 17°, comparison with others is impossible. By

SPECIFIC HEAT OF CUPRIC SULPHATE SOLUTIONS BETWEEN 0° AND 17°.

Actual Composition of Solution.	Specific Heat compared with Water over the same Range.	Specific Heat of $CuSO_4+200H_2O$ (calculated).
$CuSO_4 + 216\ H_2O$	0.9469	0.9430
"	0.9469	0.9430
$CuSO_4 + 201.6\ H_2O$	0.9444	0.9440
"	0.9438	0.9434
"	0.9434	0.9430
		0.9433

extrapolating linearly the results of Marignac between 18° and 52° a result of 0.949 instead of the value 0.943 may be obtained, but this extrapolation is too uncertain to form a satisfactory basis of comparison.

In the same way, allowing for the heat of dilution, the specific heat of a solution $AgNO_3 + 196.7H_2O$ was found to be 0.9476. This has never been determined before at any temperature.

As has been said, the heat of dilution was found before calculating the above results. The results were as follows:

The rise of temperature on dilution of the cupric sulphate solution with an equal volume of water was found, in two experiments at 17° with the divided beaker-calorimeter, to be +0.008° and +0.010°, or +0.009° in average. This is then a warming effect.

The rise of temperature on dilution of the argentic nitrate was found in the same way, in three experiments to be at 16° —0.007°, —0.005°, and —0.006°; or —0.006° in average. This is a cooling effect, which was duly subtracted from the final temperature of mixing in the specific heat experiments.

In these determinations the purity of the salts is not of paramount importance, for the specific heats of the impurities would differ but slightly from that of the pure salt; nevertheless, pains was taken to secure high purity. An accurate knowledge of the concentration of the solution is of more importance, and the solutions were all carefully analyzed. Even this, however, does not require the greatest refinement for the specific heats of the solution are so nearly equal at this dilution to that of water that they change but slightly with changing concentration.

An Indirect Method of Measuring the Specific Heats of more Dilute Solutions.

Having once determined accurately the specific heat of a given solution, it is easy without much more labor to determine that of any more dilute solution of the same substance. The method for accomplishing this purpose, depending upon the change of the heat of dilution with temperature, was first suggested by Berthelot,* but seems never to have been used in actual practice. Probably the reason for this neglect was the difficulty of determining accurately with any ordinary apparatus the very small heats of dilution involved — but this difficulty is now removed by the very simple and convenient divided beaker-calorimeter, described above. Therefore it seems worth while to explain the method and report a few experiments demonstrating its value.

In principle, this method depends upon the well-known thermodynamic relation usually known as Kirchhoff's law, namely

$$K_1 - K = \frac{U_T - U_{T+t}}{t},$$

where K and K_1 represent heat capacities of factors and products respectively, and U_T and U_{T+t} heats of reaction (in this case of dilution) at the temperatures T and $T + t$. Having measured U_T and U_{T+t} by means of our apparatus, the change of heat capacity of the system during the reaction is very simply obtained; and if the heat capacity of the concentrated solution and the water are both known at first, the heat capacity of the more dilute solution is at once found. The dilute solution can then be once more diluted, and so on indefinitely.

This is a very accurate process, the error being divided instead of being multiplied in the operation.

The heat of dilution of the solutions at 16° or 17° had already been determined. Lack of time and the high temperature of summer weather (June, 1904) prevented us from carrying out the dilution of one of the previous solutions at 0° as would have been desirable, — for $t°$ should cover the range over which the specific heats have been studied. Nevertheless, a few determinations of the heat of dilution of the argentic nitrate made at 24° will show the mode of applying the problem and afford a fairly accurate result. In four experiments, the temperature-changes on the dilution of the argentic nitrate solution, with an equal bulk of water at 24°, were respectively −0.002, −0.005, −0.006, −0.006; or on the average $-0.005 = U_{T+t}$. At 16° this had previously been found to be $-0.006 = U_T$. Then $K_1 - K = \frac{(-0.006^\circ + 0.005^\circ)\,7060}{8^\circ} = -0.9$. The total heat capacity having been in the first place $0.9476\,(170 + 196.7 \times 18) + 196.7 \times 18 = 7056.8$ (that is, the sum of the separate heat capacities of the argentic nitrate solution and the water) the heat capacity of the mixture will be $7056.8 - 0.9 = 7055.9$, and the specific heat will be this quantity divided by the heat capacity of an equal weight of water, or $(170 + 196.7 \times 18) + 196.7 \times 18 = 7251.2$.* The specific heat of the dilute solution is thus 0.9731.

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* Mech. Chim., I, p. 278 (1879).

Further determinations of many substances by all these methods will soon be made in this Laboratory. The determinations above given are only preliminary, as has been said; and they are recorded here rather to indicate the working of the methods under consideration than to fix the values in question. Much more accurate work could undoubtedly be done with the method if more pains was taken with the thermometry, without question the least certain of the measurements involved.

We take pleasure in acknowledging our indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for generous pecuniary help in the construction of the somewhat complicated apparatus.

SUMMARY.

1. A new method of determining the specific heat of solutions is described.

2. A new device for determining accurately the heat of dilution of a solution is described.

3. Both these methods involve only the reading of stationary thermometers, and therefore avoid error from thermometric lag.

4. In both methods error from uncompensated cooling is avoided as much as possible.

5. Each of these methods may be used for certain types of chemical reaction, as well as for the physico-chemical measurements above mentioned.

6. The method proposed by Berthelot for determining from the change

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* It will be easily seen that the rounded-off values of the atomic weights suffice in this calculation, if a result no nearer than 0.01 per cent is needed.

of the heat of dilution with temperature the specific heats of increasingly dilute solutions is tested for the first time.

7. Several devices and methods of experimentation are suggested and tested; especially a simple method for determining the water adhering to glass vessels, and the construction of a very sensitive thermostat.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF
HARVARD UNIVERSITY.

New Series. — No. XXIX.

NEW PLANTS FROM THE ISLANDS OF MARGARITA AND COCHE, VENEZUELA.

By J. R. Johnston.

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NEW PLANTS FROM THE ISLANDS OF MARGARITA AND COCHE, VENEZUELA.

By J. R. Johnston.

Presented by B. L. Robinson, February 8, 1905. Received February 4,1905.

As there will necessarily be a delay in the publication of a Flora of the Islands of Margarita and Coche which is in preparation by the writer, it seems advisable at this time to put on record the following new species. All localities not otherwise specified are on Margarita Island.

Bactris falcata, n. sp. Slender tree; trunk 10 m. high, armed; bark gray; rings of spines about 3 cm. apart; spines as much as 7 cm. long, irregularly arranged and numerous. leaves pinnate, 25 to 45 cm. long, falcate, abruptly caudate-acuminate, with cross veins inconspicuous and numerous; upper pinnae broadly lanceolate, narrowing little at the base, with about 8 main ribs; lower pinnae linear, narrowing at base, 3 cm. wide, with 1 main rib and 8 secondary and as many tertiary ones; rachis keeled on the upper side, convex below, slightly rufescent: peduncle and spathe densely clothed with black shining spines 2 cm. in length; peduncle recurved; spathe 15 cm. long: spadix racemosely branched, 1 dm. long; branches as much as 6 cm. in length, slender, numerous: calyx of the pistillate flower circular, 4 mm. in diameter; corolla circular, quite regular, 1 cm. broad; drupe yellowish red, about 2 cm. in diameter, a little broader than long, turbinate. — Rio Asuncion, in heavy woods near Juan Griego trail, *Johnston*, no. 220, July 16, 1903.

Renealmia lutea, n. sp. Plant 12 to 15 dm. high: leaves distichous, linear-lanceolate, 23 cm. long and 4 cm. wide; apex attenuate; base narrowing into a short petiole; vagina truncate at the top; surface more or less covered with fine white scales: inflorescence simply racemose; scape arising from base of stem; bracts of the scape sheathing, 12 cm. long; those subtending the flowers, 1- to 2-flowered; lower ones as much as 3 cm. long; tubular bractlet equalling the calyx, nearly 1 cm. long, externally pubescent at base and tip: calyx 3-lobed; lobes unequal, broadly truncate, 5 to 7 mm. long: corolla 3-fid, 1 cm. long; lobes obovate, rounded at apex, concave; labellum rotund, shortly 5-lobed, possessing a long claw. — Rio Asuncion, in damp woods along Juan Griego trail, *Johnston*, no. 298, Aug. 12–15, 1903.

Elleanthus attenuatus, n. sp. Roots few, fleshy, fibrous, densely pubescent: stems 2 to 4, ascending, 6 to 9 dm. high, cylindrical; base covered with two or three sheathing leaves; lower stem naked or retaining the vaginas of old leaves; upper stem leafy: leaves glabrous, narrowly lanceolate, 15 to 20 cm. long and 3 to 4 cm. wide, with 9 to 11 veins, depressed above, plicate below, entire, long-acuminate; base cuneate, constricted, broadening into vagina; sheath cleft anteriorly, about 4.5 cm. long; uppermost leaves bractlike: inflorescence spicate, cylindrical, elongate, about 7 cm. long; rachis puberulent; bracts slightly exceeding pedicel, about 1 cm. long, two-thirds the length of the flower, lanceolate to ovate, acute or acutish, reddish, striate: lateral sepals triangular, lanceolate, acuminate; 1 posterior sepal narrowly lanceolate, acute: 2 lateral petals linear-lanceolate, acute, equalling other parts of perianth; labellum broadly ovate, 1 cm. long, obtuse; margin crenate, with 2 elongated separate callosities at base: all parts of plant sometimes minutely puberulent. — San Juan mt., in wet woods at the summit, alt. 640 m., *Miller & Johnston*, no. 270, July 30, 1901, and *Johnston*, no. 233, July 6, 1903. Allied to *E. furfuraceus*, Reichb. f., from which it differs in having larger, more attenuate, more ribbed leaves, and in its longer inflorescence.

Epidendrum Johnstoni, Ames, n. sp. "Plants 1 to 4 cm. high, diphyllous. Leaves narrowly elliptic-oblong, obtuse, sometimes stained or suffused with madder-purple, 2 to 4 cm. long, 6 to 10 mm. wide; raceme terminal, lax, shorter than the leaves, 2 to 4-flowered, subtended by a conduplicate sheath, 5 to 10 mm. long; lateral sepals elliptic, subacute, 9 mm. long, 4.5 mm. wide, upper sepal similar; petals narrowly elliptic, obtuse, 8 mm. long, 2 mm. wide; lip adnate to the column, 8.5 mm. long, 3-lobed, lobes subequal, the lateral ones semicircular; in front of the column two erect callosities which converge obscurely in a faintly tubercled hemispherical crest; column 4.5 mm. long, stained with madder-purple; flowers apparently greenish-yellow, suffused with madder-purple. — San Juan mt., alt. 600 m., *Johnston*, no. 236, July 2, 1903. Plants similar in habit to dwarf forms of *E. conopseum*, R. Br., to which species this one bears a slight resemblance. The main points of difference are in the hemispherical callous, in the length of the column, and in the broader segments of the perianth."

Peperomia Victoriana, var. margaritana, C. DC., var. nov.; "limbis quam in specie minoribus, usque ad 26 mm. longis et ad 17 mm. latis; caulibus, foliis, et spicis in sicco flavescentibus." — Along the ledges of the cliffs, alt. 300 m., *Johnston*, no. 18, July 22, 1903.

Piper Johnstoni, C.DC., n. sp. (§ CARPUNYA) "foliis breviter petiolatis, ellipticis, basi aequilatera acutis, apice acuminatis, utrinque glabris, nervo centrali ultra ½ longitudinis suae nervos adscendentes utrinque 4 mittente; petiolo glabro basi vaginante; pedunculo glabro; spica florente limbi dimidium vix aequante, tenui, apice obtusa; bracteae pelta triangulari margine pedicelloque angusto hirsutis; antheris ovatis quam filamenta multo brevioribus; ovario ovato glabro; stigmatibus linearibus.

Ramuli glabri, graciles; spicigeri 2 mm. crassi, collenchymate subcontinuo libriformi; fasciculis intramedullaribus sub-2-seriatis, nempe 2 inter periphericos et alios intramedullares adjunctis. Limbi in sicco membranacei creberrime pellucido-punctulati, in specimine florente usque ad 19 cm. longi et ad 8¾ cm. lati, nervis arcuatis. Petioli 11 mm., pedunculi 6 mm. longi. Spicae florentes fere 9 cm. longae, 2 mm. crassae. Stamina 3 filamentis in sicco laciniosis. Stigmata 3, sessilia." — Juan Griego trail, in damp woods, alt. 500 m. *J. R. Johnston*, no. 19, July 11, 1903. "Species *P. brachypodo* C.DC. Prodr. xvi. pt. 1, 327, proxima."

Piper margaritanum, C.DC., n. sp. (§ STEFFENSIA) "foliis breviter petiolatis, oblongo-lanceolatis, basi leviter inaequilatera acutis, apice acute et sat longe acuminatis, supra sparsim pilosis, subtus ad nervos et nervulos hirsutis; nervo centrali usque ad ⅓ longitudinis suae nervos adscendentes alternos utrinque 4 mittente; petiolo adpresse hirsuto, basi vaginante; pedunculo parce hirtello petiolum fere aequante; spica limbi dimidium vix aequante, apice obtusa; bracteae pelta triangulari margine pedicelloque angusto hirsutis; bacca obpyramidato-trigona glabra; stigmatibus linearibus.

"Frutex 24 dm. altus, viridis, gracilis. Ramuli parce hirtelli, spiciferi vix 2 mm. crassi, collenchymate continuo zona interna libriformi, fasciculis intramedullaribus 1-seriatis, canali vacuo centrali. Limbi in sicco membranacei pellucido-punctulati, circiter 15 cm. longi, 4 cm. lati. Petioli sub limbo 5½ mm., inter limbi latera 2½ mm. longi. Spica baccifera fere 3 mm. crassa. Stamina 4. Baccae in sicco nigrae ¾ mm. longae. Stigmata 3, sessilia."

San Juan mt., alt. 700 m. *Johnston*, no. 20, July 11, 1903. "Species *P. salicariaefolio* Kunth proxima."

Coccoloba Ernstii, n. sp. Stem branching, wrinkled in dry speci-

men: leaf broadly oval, entire, 5.5 to 7 cm. wide and 7 to 9.5 cm. long, glabrous, membranous; veins subprominent, secondary veins inconspicuous and the reticulation scarcely visible; base truncate or rounded, shortly decurrent into petiole; apex shortly obtuse or acute; petiole glabrous, 2 to 2.5 cm. long; ochrea as much as 1 cm. long, oval, with an acute or split apex, glabrous; ochrea scar encircling the stem, conspicuous; leaf scar large or small, one-third way from base of ochrea, sometimes covered by a short spine: inflorescence racemose, a single ascending raceme to a node; lateral, 5 to 10 cm. long; peduncle short; rachis minutely puberulent; nodes of rachis with two minute bracts, 1- to 2-flowered; pedicel 1.5 mm. long: fruit lacking. — El Valle, South hill, *Johnston*, no. 250, Aug. 31, 1903. Probably allied to *C. Cruegeri*, Lindau.

Capparis collina, n. sp. (§ Cynophallophora.) Small tree: leaves alternate, green, paler beneath, glabrous, dull, narrowly elliptical or lanceolate, 6 to 9 cm. long, and 3 to 4.5 cm. wide, obtuse or rounded at the base, acute, blunt or rarely emarginate at the apex, membranous; upper surface smooth, lower with prominulous veins: inflorescence terminal, corymbose; bud globular: sepals 4, valvate, rotund, one-third the length of corolla: petals obovate, 1.5 cm. long: stamens between 50 and 60, exceeding the corolla scarcely twice: ovary 5 to 8 mm. long, cylindrical, truncate, on gynophore 3 to 4 cm. long: fruit smooth, long and slender, very slightly torulose, equalling or exceeding gynophore, 6 to 8 cm. long, 3 mm. wide. — Tree 5 m. high on hillside north of El Valle, *Johnston*, no. 10, July 15, 1903. This species differs from *C. cynophallophora*, L., in its leaves which are smaller, more narrow, and of a different shape; in its smaller flowers and slender, smooth fruit.

Calliandra panlosia, n. sp. Small tree, 3 to 5 m. high, branching copiously: leaves bipinnate, pinnae 4- to 5-jugate; pinnules 15 to 30 in number, 5 to 7 mm. long, 1.5 mm. wide, everywhere pilose, paler on the under side; petiole 4 to 8 cm. long; petiolules 3 to 5 cm. long; young stem and petioles rufescent; stipules geminate, lanceolate, 6 mm. long: inflorescence axillary, 2 to 3 peduncles of unequal length in each axil, at maturity about 2 cm. long: flowers shortly pedicellate: calyx 1 mm. long, pubescent: corolla 6 mm. in length, 5-dentate, appressed-pubescent: young fruit with reddish-brown pubescence, cylindrical. — Abundant on hills at altitude of 300 to 600 m., El Valle to Juan Griego, *Miller & Johnston*, no. 58, July 22, 1901, and *Johnston*, no. 27, July 2, 1903.

Caesalpinia acutifolia, n. sp. Tree 3 to 5 m. high: leaves bipinnate; pinnae 4 to 6 in number; pinnules 12 to 14 in number, nearly opposite, narrowly oval, 3 cm. long and 0.8 to 1.2 cm. wide, pellucid-punctate, ciliate at the margin, obliquely obtuse at the base, acute or rarely obtuse at the apex, puberulent, paler on the under side; petiole 2.5 to 5 cm. long; petiolules 1.5 mm. long: inflorescence lateral; racemes simple, 6 to 10 cm. long; pedicels jointed beneath the calyx, as much as 8 mm. long: calyx yellowish-pubescent and covered with stipitate glands, 5-parted; 4 lobes, subequal, elliptical; the fifth (lower) lobe exceeding the others twice, 13 mm. in length, pectinate, concave in bud and capping the others: petals 5, glandular-punctate; 4 subequal, rotund, with a distinct claw nearly half as long as the blade; fifth (upper) petal narrow, edges incurved, 8 mm. long, reflexed, pubescent at the base: stamens 10, declined, inclosed in the corolla; filaments broad at the base: ovary stipitate, included within the corolla; stigma sessile, lower than the anthers: fruit unknown. — On the hillside, El Valle, *Johnston*, no. 33, Aug. 3, 1903.

Gliricidia lutea, n. sp. Shrub 9 to 15 dm. high, bark whitish: leaflets 5 in number, 2 to 4 cm. long and 0.6 to 1.5 cm. wide; narrowly elliptical, above glabrous, below glabrescent or when young sericeous, base and apex cuneate; petiole 4 cm. long, petiolule 2 mm. long, both rufescent: inflorescence racemose, precocious; pedicels 3 mm. long, geminate, equalling the yellowish-green calyx which has turbinate base and truncate, slightly 5-dentate margin: petals yellowish-white; standard orbicular, emarginate; base narrowing sharply, slightly pubescent on upper side above calyx and on lower side at apex, 1 cm. long; wings oblong, shortly auriculate, 11 mm. long; both wings and keel possessing long, slender claws: ovary stipitate, narrow, slightly compressed, 2-valved, pubescent; 3 to 5 reniform ovules; style terminal, recurved, glabrous; stigma minute: pod dry, flat, tipped with a short point, 3 cm. long, 1 cm. wide. — On dry hillside, El Valle, *Miller & Johnston*, no. 246, 1901, and *Johnston*, no. 34, Aug. 15, 1903.

The yellow corolla and few leaflets readily distinguish this species from the others of the genus, which have roseate corollas and many leaflets. These distinctions, although marked, are not considered of sufficient importance in the groups allied to *Gliricidia* to warrant the proposal of a new genus.

Inga macrantha, n. sp. Shrub branching copiously; young stems rufescent: leaves 6- to 10-foliolate; leaflets lanceolate to elliptical, somewhat pubescent on both sides, especially at the ribs, with the upper surface shining and the lower dull; base obtuse; apex acute or acuminate, commonly mucronate; larger leaflets 7 cm. long and 3.5 cm. wide; glands scutellate, located on the upper side of the petiole between the two opposite leaflets: inflorescence pedunculate; peduncles 1.5 to 4 cm. long; pedicels none; calyx rufescent, striate, 8 to 9 mm. long: corolla about 2 cm. long, with long appressed pubescence: stamens exceeding corolla twice: style a little longer than stamens: pod flat, velvety, brown, 5 to 10 cm. long, about 2 cm. wide; the margins thickened and channeled. — Alt. 400 m., on Juan Griego trail from El Valle, *Johnston*, no. 25, July 11, 1903.

Machaerium striatum, n. sp. Shrub 2 m. high: leaves alternate; leaflets 5, subopposite, lanceolate, subcoriaceous, glabrous, with a rounded or cordate base, and acuminate apex, entire, 4 to 8 cm. long, and 1.5 to 4 cm. wide; petiole and petiolule puberulent; petiolule 2 to 3 mm. long; petiole 3 to 4 cm. long; stipule broad, striate, puberulent, 3 mm. long: inflorescence axillary, a compound raceme, rufescent; bracts leathery, small, ovate; bracteoles 2, concave, orbicular, wide, the two meeting around the calyx: calyx tubular, 2 to 3 mm. long, 5-dentate, the teeth short, rounded; standard oval, obtuse, with a shortly unguiculate base, externally pubescent; wings free, oblique, with a claw nearly one-third the length of entire petal; keel barely exceeded by wings, with its parts cohering, falcate, deeply auriculate, slightly pubescent on the outside: stamens monadelphous: ovary pubescent, stipitate, a deep cup surrounding the stipe; ovules lateral, 1 or 2: fruit unknown. — North hill, El Valle, *Johnston*, no. 124, Aug. 8, 1903.

Securidaca cordata, n. sp. Shrub 1 to 2 m. high: leaves alternate, ovate, 4 to 7 cm. long and 2.5 to 5 cm. wide, glabrous on both sides, slightly cordate at the base, retuse or obtuse at the apex, entire; petiole glabrous, 3 mm. long: racemes lateral, at or between the nodes, simple, bearing a dozen or less flowers; pedicels puberulent or glabrous, 8 mm. long: bud elongated, oval, acute: calyx glabrous, its margin ciliate; lobes oval, unequal, 2 inner ones being 11 mm. long, almost twice the length of the outer, shortly unguiculate: the two petals of the keel falcate, 8 mm. long; lip elliptical, the end folded, the posterior fold dentate: stamens 8; stamineal tube cleft dorsally, saccate and pubescent at the base toward the keel, otherwise glabrous: ovary glabrous, flattened; style a little longer than stamens; stigma arcuate, two globular bodies at centre: fruit ovoid; wing on the upper posterior portion about 2.5 cm. long, and 1 to 1.2 cm. wide; anterior dorsal portion short and oblique, surface of fruit lightly veined. — North hill, El Valle, alt. 250 m., *Johnston*, no. 60, Aug. 8, 1903.

Argithamnia cochensis, n. sp. Shrubby, deep red or purple throughout: leaves alternate, lanceolate, appressed-pubescent, 1.5 to 4 cm.

long, and 0.5 to 1.5 cm. wide; penninerved; margin serrulate to entire; base acute; apex acute, often mucronate; petiole 2 mm. long; stipules scarcely 1 mm. long, setaceous: inflorescence monoecious: flowers sessile or subsessile; bracts minute, ovate: *staminate flower:* — sepals 5, narrowly lanceolate, united slightly at base, with acute apex, white-dotted lobes, externally pubescent; petals externally sparingly long-pubescent, broadly lanceolate, acuminate, cuneate at the base, shortly unguiculate; glands alternate with petals, glabrous, slightly adherent to petals and stamineal tube, fleshy, triangular, prolonged-acuminate; stamens 10, 3-whorled, the uppermost whorl bearing no anthers: corolla slightly exceeding the light-green calyx which is 3.5 mm. long: *pistillate flower:* — sepals 5, lanceolate, oblique from the middle up, with base narrowing but slightly, and apex attenuate, both sides sparingly long-pubescent, 4.5 mm. long, slightly coherent at the base; corolla 1.2 length of calyx, 5-lobed; lobes cohering but slightly at the shortly unguiculate base, lanceolate, with an acuminate apex; petals white-dotted; glands between the petals small, triangular; ovary pilose; the three styles united half way, each 2-lobed. — Coche, in the interior, *Johnston*, no. 12, Aug. 5, 1903.

Argithamnia erubescens, n. sp. Shrub 3 to 6 dm. high: leaves often becoming purple, fasciculate, lanceolate to obovate, 4 cm. long and 1 cm. wide, subsessile, minutely and remotely serrate; both sides covered by a long white pubescence; apex obtuse or acute, often mucronate; base attenuate: inflorescence dioecious, fasciculate, 1 to 4 flowers in a fascicle: *pistillate flower* appressed-pubescent; pedicel 4 mm. long; bracts 2, minute, oval, acute; calyx 5-parted; sepals lanceolate, 8 mm. long and 3 mm. wide; petals 5, subequal, slightly exceeding the sepals, minutely pubescent toward the base, elliptical, or narrowly obovate, shortly unguiculate, glands between petals conspicuous; ovary pubescent, style united above the middle, stigmas bilobed: *staminate flower* shortly pedicellate; sepals 5, narrowly lanceolate, acute; petals obovate, narrow at the base; stamens about 10, 2-whorled; the stamineal column somewhat exserted. — El Valle, *Miller & Johnston*, no. 213, July 22, 1901, and *Johnston*, no. 58, Aug. 21, 1903.

Croton margaritensis, n. sp. Shrub 6 to 9 dm. high: leaves alternate, long-petioled, stipulate, narrowly to broadly lanceolate, 1 to 3 cm. wide, 4 to 6 cm. long, sinuately serrate, the teeth bearing glands; upper surface remotely stellate-pubescent; under side similar, but pale green; apex of blade acute or acuminate; base rounded or cordate; petiole slender, 2 to 3 cm. long; stipules very small, setaceous, bearing at apex a single gland: inflorescence monoecious, in a simple raceme:

pistillate flowers lowermost on the rachis; pedicels as much as 2 mm. long; bracts small, not equalling the pedicel; sepals 5, slightly pubescent, becoming at least 6 mm. long, possessing marginal stipitate glands; petals lacking; ovary slightly pubescent, with its styles twice dichotomously divided into unequal branches: *staminate flower* smaller than the pistillate; sepals elliptical, equalling the shortest petal; petals 5, unequal, from unguiculate to spatulate or obovate, largest nearly twice the length of the short obovate one; stamens 6, staminodia (?) 4; filaments at base long-pubescent; capsule glabrous or with few stellate hairs. — Among shrubs near summit of San Juan mt., alt. 700 m., *Johnston*, no. 50, Aug. 28, 1903. The upper stems possess a scattered loosely stellate pubescence, and some leaves are a dingy brown, while others are dark brown. This species differs from *C. populifolius*, Mill., in having bracts and stipules only one-third as long, in its lanceolate leaves, and in the absence of petals in the pistillate flower.

Croton Milleri, n. sp. Shrub 20 to 25 dm. high: leaves alternate, stipulate, narrowly to broadly lanceolate, 3 to 7 cm. long, 1 to 2.5 cm. wide, above brownish green, glabrous, minutely black-punctate, underneath shining silvery-lepidote, a few scales with brown centers; margin of leaf subentire; apex acute; base obtuse or narrowly cordate; petiole 3 to 7 mm. long; stipules setaceous, caducous, 2 mm. long: inflorescence terminal on short lateral leafy branches; simple spike sessile, 2 to 10 cm. long, often recurving; flowers pedicellate; pedicels 2 to 4 mm. long; bracts setaceous, scarcely equalling the 1-flowered pedicel; *pistillate flowers* lowermost, apetalous; calyx 5-fid; lobes ovate, 3 mm. wide, margin slightly reflexed; ovary lepidote; style bifurcate 3 times: *staminate flowers* with sepals narrower than those of pistillate, with 5 petals, obovate, white, densely pubescent within; stamens about 16 with filaments pubescent at base; no rudiment of gynoecium present in staminate flower. — El Valle, *Miller & Johnston*, no. 229, July 18, 1901, and *Johnston*, no. 48, Aug. 10, 1903. Common on the low plains between El Valle and Punta Mosquito. This species resembles *C. elaeagnoides*, Wats.

Pavonia cochensis, n. sp. Shrubby, prostrate, branching: branches as long as 55 cm., stellate-pubescent: leaves long-petiolate, velvety stellate-pubescent on both sides, ovate, cordate, 3 cm. long and 2.5 cm. wide, some as broad as long, with an acute or obtuse apex; petiole 2.5 cm. long; stipules setaceous, 2 mm. long: inflorescence axillary, solitary; pedicel 1.5 cm. long: involucre about 17-leaved; leaves setaceous, 1 cm. in length, pubescent: calyx deeply 5-fid; lobes 4 mm. long, lanceolate, glabrous on the inside near the base, with an acute apex: corolla and stamens unknown: carpels shortly 3-aristate, minutely puberulent, back convex, areolate; sides flat; base narrowed; seeds triangular-reniform, apex rounded, base acute, puberulent. — San Pedro, growing in the deep sand near the graveyard, Coche, *Johnston*, no. 18, Aug. 5, 1903. Allied to *P. humifusa*, A. St. Hil.

Casearia spiralis, n. sp. Tree 5 m. high: leaves alternate, elliptical, 4 to 10 cm. long and 3 to 5 cm. wide, membranous, punctate with pellucid dots and lines, crenate-serrate, indentations glanduliferous; apex shortly acute; base obtuse or truncate; petiole glabrous, 1 cm. long; stipule lanceolate-setaceous, 4 mm. long; inflorescence fasciculate, fascicles sessile or shortly pedunculate, compound, the branches bracteate at the base and at the nodes; bracts and bracteoles glabrous, as much as 1.5 cm. long; calyx corolline, 5- to 7-lobed; lobes unequal, imbricated, the smaller outside, pellucid-punctate, glabrous; corolla none; stamens about 22, slightly perigynous; connective of anthers produced into a short acumen; single, very short staminodia clothed with a whitish pubescence alternating with the stamens: ovary ovoid, pubescent; the three parietal placentas having many ovules; style short, thick; stigma 3-lobed; fruit unknown. — El Valle: river trail, *Johnston*, no. 283, Aug. 30, 1903.

The spiral arrangement of the sepals distinguishes this plant at once from all other *Caseariae* so far as can be determined from their descriptions. According to Warburg in Engler and Prantl's Pflanzenfamilien, III. 6a, 13 this spiral condition would place this plant in the *Erythospermeae*, a group consisting of four African genera and a monotypic Chilian genus. Though the plants of these genera have no staminodia, some of them have scale-like appendages at the base of the inside sepals. The plant under consideration, *Casearia spiralis*, has none of these appendages to the floral envelope, but has the staminodia characteristic of the genus *Casearia*. In such groups as these in which the calyx is imbricated and varying in the number of its parts, from 4 to 6, the spiral or cyclic character would seem to be a less distinctive characteristic than the presence or absence of calyx-appendages or of staminodia. Thus the plant seems to be more nearly affiliated to the *Casereae* than to any of the *Erythospermeae*, though showing affinities to both. As to whether it is actually a *Casearia* may be a question.

None of the species of *Casearia*, so far as described, show any marked irregularity of sepals or any definite spiral arrangement. Nor have any of them as many as seven lobes and very few as many as 22 stamens, which is the case in *C. spiralis*. Nevertheless, the form of anthers, the single staminodium alternating with each stamen, the amount of adnation between androecium and calyx, all suggest affinities of the plant under consideration with the genus *Casearia* rather than any other. As it does not seem advisable to base a new genus upon this one character, the spiral calyx, this new plant may stand as a *Casearia*, for the present at least.

Passiflora monticola, n. sp. Climbing, glabrous; tendrils at every node: leaves alternate, broadly ovate; blade 4 to 6 cm. long and 5 to 8 cm. wide, 3-lobed, glabrous, glaucescent on under side, subpeltate; base retuse; apex of lobes acute; margin of lobes entire; usually 2 separate glands between each two lobes; petiole glabrous, 2.5 to 3 cm. long; glands 1 to 3, unequal; stipules obliquely lanceolate, mucronate, about 2.5 cm. long and 12 mm. wide: inflorescence solitary, axillary; pedicels exceeding petiole, 2 to 4 cm. long; bud elongated, elliptical, 2 to 2.5 cm. long; bracts 3, lanceolate, acuminate, 4 mm. long, a little below the flower: sepals and petals 5 each; corona in at least two series, the outer series large and purple, the inner one-third as long and light-colored; 5 stamens adnate to gynoecium just below the ovary: ovary glabrous; styles 3, separate nearly to the ovary; stigmas peltate; fruit glabrous, ellipsoidal, 4 cm. long, 3 cm. wide; carpophore 2.5 cm. long; seeds many, flattened, elliptical in outline, areolate, with an abrupt point at the end opposite to the funiculus. — Climbing over low shrubs at the mountain top, alt. 700 to 795 m., San Juan mt., *Johnston*, no. 64, July 11, 1903. Probably belongs in section *Granadilla*, although its bracts are rather small. The corona is not membranous, a fact which tends to exclude the plant from the only other possible section, *Murucuia*.

Passiflora nitens, n. sp. (§ GRANADILLA.) Climbing, tendril-bearing; tendrils simple: leaf 3-lobed, 3-nerved, 9 to 12 cm. long and 7 to 10 cm. wide; lobes serrulate, usually with marginal glands between them; middle lobe with a much narrowed base; base truncate, shortly decurrent; upper surface shining; veins finely puberulent; under side covered with a white, glistening pubescence; petiole puberulent, 2.5 cm. long; 2 sessile glands above the base, often 1 or 2 near apex; stipules setaceous, unequally incised, about 4 mm. long: inflorescence solitary; pedicels puberulent, 3.5 cm. long; bracts 3, enclosing the flower, elliptical, acute, finely puberulent, 5.5 cm. long, 2.5 cm. wide; flower about 3.5 cm. long: calyx delicate, apparently similar to corolla, both (?) purple: corona 3-serial, lowest series short, membranous, middle with edges fimbricate, and upper with long, broad filaments: stamens 5; filaments flat, united part way to gynophore: styles 3 to 4, free to the ovary;

ovary glabrous, ellipsoidal, at least 1.8 cm. long. — El Valle, South hill, *Johnston*, no. 65, Aug. 31, 1903.

Cereus margaritensis, n. sp. Stem columnar, erect, 5 to 8 m. high, branching 1 to 2 m. from ground; branches ascending, usually 8-angled, clothed with a thin gray waxy coat; areolae oval or circular, about 6 mm. across, scarcely 1.5 cm. apart, with 11 short spines and a central longer one, the lower ones a little longer than the upper; often a few more small spines on the upper side which usually possesses a small brown woolly cushion; middle and lower part of areolae usually destitute of wool; central and lower spines recurved, longest almost 2 cm. long; all bulbous at base, and flattened horizontally: flower about 6 cm. long: calyx-tube slender, its lower part with few scales, and destitute of wool and spines; lobes broadened at top, acute or acutish: corolla lobes narrow, acute: fruit ellipsoidal, narrow at each end, glabrous or with few scales. — El Valle, on the hillside, *Johnston*, no. 344, July 27, 1903. Allied to *C. candelabrum*, Web., and to *C. eburneus*, Salm.

Blakea monticola, n. sp. (§ PYXIDANTHA.) Shrub 1 to 2 m. high, branching copiously: leaves alternate, obovate, 4 to 7 cm. long, 2 to 5 cm. wide, 5 main nerves, with inconspicuous cross nerves, which average 1 mm. apart, entire, with apex abruptly acute or with a blunt point and with base cuneate, on both sides minutely punctate; petiole 0.75 to 1.75 cm. long; inflorescence axillary, solitary; pedicels about 2 cm. long: bud globose, glabrate, minutely apiculate: bracts 4, decussate, outer ones 13 mm. long, inner 10 mm., orbicular, slightly apiculate: calyx 8 mm. long, slightly sinuate-lobed: corolla white; petals 6, orbicular, 12 mm. long: stamens 12; anthers biporous, produced into a subulate appendage: style 12 mm. long; stigma minute. — Abundant on the exposed top of San Juan mt., altitude 700 to 795 m., *Johnston*, no. 51, July 6, 1903. Differs from *B. Schlimii*, Triana, in its smaller leaves and shorter petioles.

Jacquinia petiolata, n. sp. Shrub 5 m. high: leaves fasciculate, 1 to 4 in a fascicle, obovate, puberulent when young, glabrate when older; apex rounded, obtuse, or emarginate; base cuneate, continuing into petiole; midrib conspicuous; lateral veins plain, nearly parallel, arched and connecting near margin, which is slightly or not at all revolute; new leaves 1.5 cm. wide and 2 cm. long, older ones often 2 cm. wide and 4 cm. long; petiole 5 to 7 mm. long, puberulent: inflorescence axillary or terminal, fasciculate, 1 to 3 flowers in a fascicle; pedicel about 4 mm. long, whitish pubescent as all the new parts: calyx 5-lobed; lobes orbicular; margin slightly crenate: corolla white; petals elliptical, apex rounded; corona 5-parted, parts triangular, apex obtuse, margin slightly fimbriate, a little shorter than the corolla; base fleshy, united into a tube with the bases of filaments: filaments equalling petals; anthers small, sagittate, apex narrowly emarginate: ovary pubescent when young; style persistent, 1.5 mm. long, tapering; stigma minute; fruit one-celled with few seeds, fleshy, 2 mm. in diameter, capped by a small circular often pubescent disk subtending the style; fruit glabrous. — Punta Mosquito, east of Laguna Chica, *Johnston*, no. 273, Aug. 10, 1903, and Coche, Aug. 5, 1903.

Ditassa subulata, n. sp. Procumbent, pubescent: leaves oval to elliptical, varying from 2 by 0.8 cm. to 1 by 0.4 cm., subglabrous, often brown above, green below, revolute, with base obtuse, and with apex acute and mucronate by the prolongation of the one central nerve, about 7 to 10 pairs of lateral nerves parallel to each other and at an angle of 75° to 80° to the midrib; petiole pilose, 2 to 3 mm. long: inflorescence nodose, 3- to 5-flowered, subsessile, pilose; pedicels slender, equalling the petiole: calyx deeply lobed; lobes lance-ovate, narrowly obtuse or acute; 5 subulate glands at base of calyx alternating with the sepals: corolla white, deeply lobed; lobes narrowly lanceolate, subvalvate, externally glabrous, coarsely pilose within, 3 mm. long; corona 2-serial, adnate at base of column; outer series subulate, exceeding the stigma nearly twice, two-thirds as long as the corolla; inner series subulate, barely exceeding stigma, nearly two-thirds as long as outer series: appendages of anthers hyaline, orbicular, emarginate: ovary glabrous; stigma pentagonal with a smaller convex cap: fruit unknown. — San Juan mt., climbing over the low shrubs at the summit, alt. 740 m., *Johnston*, no. 262, July 6, 1903.

Evolvulus arenicola, n. sp. (§ ANAGALLOIDEI.) Annual; stem short; branches several, simple, 5 to 10 cm. long, silky-pubescent: leaves distichous, subsessile, elliptical, silky-pubescent all over, 0.6 to 1 cm. long and 0.2 to 0.4 cm. wide; base attenuate; apex acuminate: flowers axillary, solitary; pedicels 2 mm long with two minute bracts at the base: calyx externally pubescent, 2 mm. long, 5-lobed; lobes narrow, acute: corolla pale blue, 3 times as long as calyx: ovary glabrous, containing 4 dull brown seeds. — Along the trail from Porlamar to San Antonio, *Miller & Johnston*, no. 218, Aug. 2, 1901. Allied to *E. incanus*, Pers. and *E. holosericeus*, HBK.

Bassovia ciliata, n. sp. Shrub 15 to 21 dm. high, branching: leaves alternate or fasciculate, 2 to 3 in a fascicle, unequal, lanceolate, as much as 7 cm. long and 3 cm. wide, attenuate at both ends; margin entire, ciliate; upper side green, remotely pubescent; under side paler, finely pubescent; petioles 1 cm. long: inflorescence fasciculate, as many as 8 flowers in a fascicle; flowers pedicellate; pedicels 1.5 cm. long, slender, puberulent: calyx truncate, slightly puberulent, not enlarging at fructification, possessing 5 to 6 setae arising laterally at the margin, about 1.5 mm. long, little more than one-half the depth of the calyx cup: corolla 6 mm. long, deeply 5-lobed; lobes ovate, acute, the margin finely glandular or papillate: stamens 5, affixed to base of corolla; anthers oblong, twice the length of filaments; filaments a little broadened at the base: ovary ovoid, 2-celled, with central placentation; placenta a little thickened, not divided, several-ovuled: fruit globular, several-seeded. — El Valle, river trail, *Miller & Johnston*, no. 255, July 15, 1901, and *Johnston*, no. 75, Aug. 30, 1903.

Solanum margaritense, n. sp. Slender shrub, 2 to 3 m. high; prickles stout, straight, as much as 7 mm. in length; stem rusty stellate-pubescent: leaves 2- to 4-clustered, unequal, above slightly stellate-pubescent, below paler, ovate, entire, or slightly repand, a few spines arising from the midrib on both sides; base often unequal, a little more attenuate than the apex which is often acute or shortly acuminate; blade of the larger leaves 5.5 cm. long and 3.5 cm. wide; petiole pubescent, as much as 1 cm. long: inflorescence fascicled, 2 to 4 flowers in a fascicle; pedicel slender, pubescent, 1.2 cm. long: calyx pubescent, 5-fid, segments narrowly lanceolate, two-thirds length of corolla: corolla 5-fid, lobes narrowly lanceolate, pubescent without, glabrous within, white, 0.7 cm. long: stamens just included within the corolla; filaments one-third as long as anthers: ovary 2-celled, puberulent; ovules numerous; style filiform, slightly exceeding corolla. — El Valle, a single ascending stem branching copiously near the top, at the side of the river trail, *Johnston*, no. 315, Aug. 20, 1903. Allied to *S. lanceaefolium*, Jacq., from which it differs in having straight spines, more deeply lobed calyx, and in general a less dense pubescence.

Solanum umbratile, n. sp. Shrub 10 to 15 dm. high, unarmed; stem and leaves clothed with brown stellate pubescence: leaves alternate, ovate-lanceolate; base attenuate, decurrent into the petiole: apex acuminate; lower surface paler and more densely pubescent than the upper surface, minutely punctate, and having 4 to 6 pairs of prominent lateral veins; larger leaves 3.5 cm. wide and 8 cm. long; petiole 0.5 cm. long: inflorescence axillary; peduncle 2 mm. or less long, bearing 1 to 6 clustered flowers; pedicels usually simple, slender, as much as 8 mm. in length: calyx 5-fid, stellate-pubescent, segments narrowly ovate, stellate without, reflexed at maturity: stamens erect; filaments one-third length of anthers, broad at base; anthers a little shorter than corolla, broad; apical pores the width of cell, at maturity splitting laterally: ovary glabrous, ovoid, gradually passing into the filiform style; stigma minute, shortly 2-lobed; fruit ovoid, 7 mm. long; seeds numerous, flat, reniform, 2 to 3 mm. long. — Rio Asuncion, in the heavy woods along the Juan Griego trail, *Johnston*, no. 321, July 22, 1903. Allied to *S. gracillima*, Sendtn. of Brazil, it differs in having broader leaves, a very short peduncle, a stouter pedicel, and a larger calyx, the lobes of which are rounded at the apex.

Bignonia acuminata, n. sp. (§ Conjugatae.) Shrubby, scandent: leaves opposite, 3-foliolate; leaflets oval with rounded base, and with acuminate apex, entire or repand, on both surfaces covered with a glistening white pubescence, less dense above than below, in dry state nearly black above and gray below, 4 to 6 cm. long and 2 to 3 cm. wide; middle leaflet often giving place to a simple stout tendril; petiole 3 to 4 cm. long; lateral petiolules 0.5 cm. in length, middle one twice as long; stipules not apparent: inflorescence axillary, short-pedunculate, racemose-paniculate; pedicels slender, short, 3 to 5 mm. long; rachis and pedicels pubescent; bracts lanceolate, acute, 3 mm. long; bractlets similar, slightly smaller: calyx 2-lipped, pubescent or glabrous, 7 mm. deep, rarely unequally 3-lobed, each lobe rounded, very shortly apiculate: corolla purple, 3.5 cm. long, slightly 2-lipped, 5-lobed; lobes rounded: stamens pilose at the base, 4, didynamous; staminodium short: disk broad, pulvinate; ovary puberulent, 2-celled, axial placentation, with 2 rows of ovules in each cell; style slender, glabrous; stigma clavate, 2-cleft: fruit linear, compressed, 7 mm. wide, about 10 cm. long. — El Valle, *Johnston*, no. 345, 1903.

Tabebuia rufescens, n. sp. Arborescent: leaves opposite, palmately compound, 5-foliolate; young leaflets elliptical, acute or acuminate, stellate-pubescent, 1 cm. wide and 2.5 cm. long, probably larger at maturity; petiole 3.5 cm. long, and petiolule 2 to 6 mm. long, all tawny stellate-pubescent; inflorescence terminal, cymose, pubescent with the exception of the corolla, precocious; pedicel 1 cm. long; bracts minute, subulate: calyx tubular, 5-ribbed, 5-lobed; lobes acute: corolla yellow, 5 to 7 cm. long, glabrous without, villous within on under side; staminodium long, glabrous: style persistent; stigma clavate, 2-lobed: young pods brownish, velvety-pubescent; pod 2-celled, cylindrical, recurved. — San Juan mt., alt. 500 m., *Johnston*, no. 79, July 2, 1903.

Chiococca micrantha, n. sp. Shrubby; stems decumbent, 3 m.

long, slender, straight: leaves opposite, narrowly lanceolate, attenuate at both ends, acuminate, glabrous, shortly petiolate, including petiole 3 to 5 cm. long and 0.5 to 1.5 cm. wide; stipules produced into an awn the length of stipular sheath: inflorescence lateral, about 3-flowered; bracts subulate, entire, 2 mm. long; pedicels 5 mm. long; calyx 4- to 5-fid, lobes acute, minutely puberulent, accrescent in fruit; corolla greenish, 4- to 5-lobed, twice the length of calyx; anthers broad, filaments short: fruit flattened, circular in outline, 3 mm. in diameter. — San Juan mt., in woods above South hill, alt. 400 m., *Johnston*, no. 115, July 27, 1903. This species differs from *C. racemosa*, Jacq., in having smaller acuminate leaves, entire bracts, more slender calyx-lobes, smaller corolla, more slender habit, and a much reduced raceme; from *C. brachiata*, R. & P., in smaller, narrow leaves, in much reduced simple raceme, and slightly smaller flower.

ANGURIOPSIS, nov. gen. Flowers dioecious, fasciculate-spicate. Staminate flowers sessile or shortly pedicellate: calyx short, campanulate, 5-lobed. Corolla rotate, 5-parted, segments ovate. Stamens 2, free, affixed to the throat of the calyx-tube; filaments short; anthers oblong, bilocular; cells linear, straight or slightly curved. Rudimentary ovary none. Pistillate flowers sessile or subsessile on the short fascicled spikes. Staminodia 2. Ovary oblong; placentas 2; style bifid, stigmas 2, bifid; ovules few, horizontal. Fruit oblong, irregular in shape, glabrous, 1- to 4-seeded. Seeds ovoid with a broad neck at one end.

Stem woody, climbing. Leaves lobed, tomentose. Tendrils simple. Flowers very small.

Cucurbitaceous and allied to *Anguria* and *Gurania*. It is strikingly different, however, in having the pistillate and staminate flowers of unequal size as in *Melothria*, and in having a distinctly spicate inflorescence, a character rare or entirely lacking in other *Cucurbitaceae*.

A. margaritensis, n. sp. Stem climbing, somewhat woody; tendrils simple, as much as 15 cm. long; leaves clustered at the nodes; young leaves orbicular or slightly 5-angled; apex rounded; base widely cordate; margin crenate-dentate; upper surface green, lightly hispid; under surface densely velvety-pubescent, whitish; the largest leaves varying from 12 mm. wide and 18 mm. long to 15 mm. wide and 12 mm. long, undoubtedly becoming larger; petiole pubescent, 1 cm. long: inflorescence shortly spicate, the spikes single or fasciculate, from 2 or 3 mm. to 1 cm. long, rugose and whitish pubescent: *staminate flowers* sessile or shortly pedicellate, about 2 mm. long; calyx green, campanulate, 5-lobed, lobes ovate, glabrous, with a plain midrib; corolla 5-partite, lobes ovate, spreading, acute, internally villous; stamens 2, free, inserted upon the throat of the calyx-tube; anthers oblong, 2-celled, straight or slightly curved, 1 mm. long; filament short, thick; rudiment of gynoecium none: *pistillate flower* 5 mm. long, sessile or shortly pedicellate, pedicel pubescent; calyx 5-parted, green, lobes ovate, glabrous; corolla rotate, 5-lobed, lobes acute, internally pubescent; staminodia 2, minute; ovary with two placentas and about 6 ovules horizontally arranged; style short, filiform, bifid at apex, stigmas 2, bifid, villose; fruit 1- to 4-seeded, glabrous, ovoid, about 7 mm. long. — Along the roadside from El Valle to San Antonio, climbing over such shrubs as *Lycium salsum*, R. & P., and *Cereus eburneus*, Salm., *Johnston*, no. 286, Aug. 8, 1903.

Vernonia Milleri, n. sp. Shrub 12 to 18 dm. high: leaves petiolate, alternate, 1.5 to 3 cm. long and 0.5 to 2 cm. wide, with acuminate apex and attenuate base, slightly mealy-pubescent above, the lower surface densely whitish-pubescent, minutely black-dotted, and reticulate veiny; petiole 0.5 cm. long; pedicels 0.2 cm. long: inflorescence irregular, densely cymose, the close aggregation of heads 4 cm. in diameter; heads 3- to 4-flowered, 0.6 cm. long, 0.3 cm. broad; bracts 3- to 4-serial, outer setaceous, the inner ones becoming broadly lanceolate, 0.3 cm. long, three times the length of the outer: corolla 5-lobed, lobes narrow, acute, five- to seven-twelfths the length of corolla: pappus 2.5 mm. long, 2-serial, outer series very short, fimbriate; inner series long, barbellate, 11 to 12 setae, white: anthers with a narrow triangular appendage at apex, at base deeply sagittate: style 2-cleft, pubescent above; achene truncate, slightly pubescent. — Summit of South hill, alt. 300 m., *Miller & Johnston*, no. 254, July 31, 1901, and *Johnston*, no. 329, Aug. 31, 1903.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

ON THE PROPERTIES OF MAGNETS MADE OF HARDENED CAST IRON.

By B. O. Peirce.

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During the last six or seven years a large number of d'Arsonval galvanometers, in which the permanent fields are due to hardened and artificially seasoned cast-iron magnets, have been used in the Physical Laboratory of Harvard University, in competition with similar instruments furnished with hardened forged-steel magnets from the shops of well-known makers. For nearly five years also magnets of the same kind have been employed in standard mirror amperemeters and voltmeters fixed in the laboratory, in cases where it was desirable that the indications of the instruments should be trustworthy within one part in a thousand of their larger deflections, over a considerable range of room temperatures. Besides the cast-iron magnets which we have made ourselves, we have of late used a number of others in moving-coil galvanometers purchased in the market — some of the best of them from the Leeds and Northrup Company.

It early appeared from tests made on these instruments, that whereas good iron castings as they come from the foundry make most unsatisfactory magnets, so far as permanence is concerned, magnets made of castings properly hardened and aged after being machined — if machining is necessary for the purpose to which the magnets are to be put — compare favorably in strength, in permanence, and in the relatively small changes of their moments with room temperature, with the best of tool steel magnets, even if in strength, though not in their other qualities, they fall a little behind magnets made, in a forming press, of steel specially prepared for the purpose.

Although chilled cast-iron bar magnets have been used for a long time in a few forms of telephones, it is usually best to make straight magnets (which do not need to be hammered), of steel; but the forging of steel for permanent magnets of complex forms, without spoiling it, demands a kind of skill which most toolmakers, even in the largest establishments, have not acquired, and it is generally difficult to get satisfactory specimens of any very special shape of curved-steel magnets unless one has access to such facilities as a few of the manufacturers of electrical measuring instruments have provided for themselves. It is true that the hardening of iron castings for magnetic purposes also requires such skill as few persons possess, if the very best results are to be obtained, especially when the pieces to be treated weigh more than a pound or two; but a little practice will enable any good workman, who has a gas forge with blast powerful enough to raise the temperature of the iron uniformly nearly to the melting-point, to make good gray-iron castings of moderate size, hard enough for strong magnets, which will leave little to be desired so far as permanence is concerned. It has been my good fortune to have the help of Mr. G. W. Thompson, the mechanician of the Jefferson Laboratory who has had long experience in treating cast iron, and who has made for me, by a process of his own, massive magnets with extremely low temperature coefficients. It is to be noticed that some of the secret methods of hardening cast iron, used by makers of small parts of machinery, do not fit the castings for making good magnets, and that case hardening, which affects the surface only, is useless. The character of the cold bath into which, while it is kept in violent agitation, the strongly heated castings to be hardened are plunged, seems to have considerable influence upon the result.

At the very high temperature, just under the melting-point, to which the cast iron must be raised before it is suddenly chilled, the metal loses much of its tenacity, and slender pieces must be handled carefully lest they break like chalk. The chilled casting should be hard enough to scratch window glass, if not so readily as hardened tool steel will do it. It is vain to attempt to make any such gray-iron castings as I have used, magnetically hard by chilling them after they have been heated to the comparatively low temperatures that one would use in making steel glass-hard. Everyone who has attempted to harden a thick mass of tool steel uniformly, knows how difficult the task is: it is easy enough to get the outer layers glass-hard, while the interior is much softer; or, sometimes (by overheating the steel), to get the inside hard while the outside is blistered and cracked. If a casting is heated to a very bright red and then plunged into the bath, the outside may become hard to the file, while the interior, as magnetic tests clearly show, remains soft; in this case, however, the material will stand a higher temperature without injury, and if the mass be reheated and when it is just below the melting point be suddenly chilled, the whole interior becomes hard.

It is a good deal easier to harden a lot of straight, round pieces of good gray cast iron, say 20 centimetres long and 1 centimetre in diameter, so that they shall all be nearly alike magnetically, than it is to do the same with an equal number of pieces of drill rod. Six pieces of Crescent Drill Rod each 16 centimetres long and 8 millimetres in diameter, cut from the same specimen, were made glass-hard for me by a skilled worker in steel; these were placed successively in a properly oriented solenoid and exposed, first to the action of an alternating current of intensity gradually decreasing from an initially high value to a very low one, then to a steady field of 147 gausses applied first in one direction and afterwards in the other. As a consequence of the preliminary treatment* with alternating currents, the magnitudes of the moments acquired by the pieces under the action of the steady field were quite independent of the direction of the latter. These moments were approximately 2280, 2395, 2495, 2326, 2325, and 2360, but when the field was removed the residual moments were 1058, 1074, 1136, 1066, 1050 and 1097 respectively. The same pieces were then placed together in a solenoid made of many turns of large wire and the ends of the whole bundle were connected by a massive yoke; when a current of about 45 amperes was sent through the wire the pieces became charged practically to saturation. When they were removed from the solenoid the average moment of the six was about 1240, the highest 1290, and the lowest 1170. Such uniformity as is indicated by these numbers is, I believe, as great as one can expect to get unless one has an elaborate plant; no such agreement can be hoped for from pieces of different rods of the same brand. Some kinds of special magnet steel give rather better results.

Although it is obvious that there is no advantage in using cast iron for straight magnets, I have had a number of such magnets made, of each of three shapes, for purposes of comparison with steel bar magnets of the same dimensions. These were all rather short, because we had no means of treating satisfactorily very long, slender pieces, which are apt to warp if not properly supported. It is, of course, impossible to calculate the demagnetizing effects of the free ends of such pieces as I have used, but it has seemed to me legitimate to draw some inferences from the hysteresis curves and from the temperature coefficients of rods of different materials if they are geometrically alike.

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* J. A. Ewing, Magnetic Induction in Iron and other Metals, 1892, p. 328.

Several years ago, when I had to have a set of the best magnets I could get for measuring purposes, carefully ground into shape after the steel had been hardened, I experimented * upon hundreds of seasoned magnets made of many kinds of steel; ordinary tool steels, self-hardening tool steels, and special magnet steels. In comparing round steels, it appeared that such "Stub's Polished Drill Rod" as I could get made slightly less desirable magnets than did "Crescent Drill Rod," or the common brand of "Jessop's Black Rod," which last two were magnetically indistinguishable. I found nothing better in tool steels than the Crescent Drill Rod, or the Jessop's Black Round Tool Steel, and I have used these as standards in testing my round cast-iron magnets. The subjoined table shows what may be expected of seasoned magnets made of these steels.

Most of my experiments on the strengths of round cast-iron bar magnets have been made with pieces 18 centimetres long and either 0.95 centimetres or 1.25 centimetres in diameter, of which I have a good many, some new and some cast two years ago. These were usually relaxed † after their hardening by being boiled in water for some time; next they were magnetized to saturation in a solenoid, and then they were again boiled and "aged" in the usual manner. The resulting magnets were finally tested in competition with a large number of seasoned tool steel magnets, of different brands but all of the same dimensions as the castings, with the help of a mirror magnetometer. The cast-iron magnets looked, of course, rather rough in comparison with others made of polished rod, but their moments differed among themselves less than those of an equal number of the steel magnets made of any one brand. Just one of the tool steel magnets had a moment sensibly greater (about 4 per cent) than any of the cast-iron magnets, but the average of the moments of the cast-iron magnets was rather greater than those of the steel, even after the records of the two or three steel magnets with abnormally low moments had been rejected.

Two years ago ‡ I measured the temperature coefficients of a good many bar magnets of cast iron and steel. In every instance, as was to be expected, the rate of loss of moment per degree of rise of temperature was much greater at temperatures near the boiling-point of water than at room temperatures, but if for purposes of comparison we used the

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* Peirce, American Journal of Science, 1898, p. 334.

† Barus and Strouhal, Bulletin of the U. S. Geological Survey, No. 14, 1885.

‡ Peirce, These Proceedings, Feb., 1903.

Results of Experiments made upon Pieces of Crescent Drill Rod and Jessop's Black Round Tool Steel.

Length in centimetres	Diameter in centimetres.	Weight in grams per centimetre of length.	Moment per gram of magnetism induced in the specimen when placed lengthwise in a unit field.				Permanent magnetic moment per gram of the seasoned magnet.	Approximate ratio of the induced moment caused by unit field, to the permanent moment in the seasoned magnet.	Mean loss per degree of the magnetic moment when the magnet is heated from about 15° C. to 100° C., expressed in terms of the moment at the lower temperature.	Temperature coefficient of the seasoned magnet at 15° C.
			In the condition in which it was purchased.	When "glass-hard" but still unmagnetized.	When magnetized to saturation while "glass-hard."	In the condition of a seasoned magnet.				
20	1.11	7.6	0.44	0.23	0.28	0.30	30.5	0.010	0.00048	0.00047
18			0.41	0.22	0.26	0.28	25.5	0.011	0.00049	0.00048
16			0.38	0.21	0.24	0.25	21.5	0.012	0.00051	0.00050
14			0.34	0.20	0.22	0.23	17.5	0.013	0.00053	0.00052
12			0.29	0.19	0.19	0.20	15.0	0.014	0.00057	0.00057
10			0.25	0.17	0.17	0.18	12.0	0.015	0.00062	0.00062
8			0.20	0.15	0.14	0.15	10.0	0.015	0.00070	0.00070
6			0.15	0.13	0.11	0.11	8.5	0.015	0.00088	0.00087
4			0.11	0.08	0.08	0.08	5.5	0.017	0.00110	0.00109
20	0.95	5.5	0.50	0.23	0.25	0.26	36.5	0.007	0.00040	0.00036
18			0.48	0.23	0.24	0.25	32.5	0.008	0.00046	0.00041
16			0.45	0.22	0.23	0.24	28.0	0.009	0.00051	0.00046
14			0.42	0.21	0.21	0.22	24.0	0.009	0.00056	0.00051
12			0.37	0.20	0.19	0.20	20.5	0.010	0.00063	0.00056
10			0.31	0.18	0.17	0.18	17.0	0.011	0.00071	0.00063
8			0.25	0.16	0.15	0.16	14.0	0.012	0.00078	0.00070
6			0.19	0.13	0.13	0.13	10.0	0.013	0.00086	0.00078
4			0.12	0.09	0.10	0.10	7.0	0.014	0.00095	0.00087
20	0.80	3.9	0.61	0.24	0.24	0.30	41.0	0.007	0.00040	0.00037
18			0.58	0.23	0.23	0.29	36.5	0.008	0.00047	0.00042
16			0.54	0.22	0.22	0.28	32.5	0.009	0.00053	0.00047
14			0.49	0.20	0.20	0.27	26.5	0.010	0.00061	0.00052
12			0.44	0.18	0.18	0.26	24.0	0.011	0.00066	0.00058
10			0.39	0.16	0.16	0.24	20.0	0.012	0.00073	0.00064
8			0.31	0.14	0.14	0.21	17.0	0.012	0.00080	0.00070
6			0.23	0.12	0.12	0.16	13.0	0.014	0.00087	0.00077
4			0.15	0.09	0.09	0.12	8.5	0.014	0.00094	0.00083
20	0.64	2.4	0.69	0.26	0.23	0.25	43.0	0.006	0.00024	0.00020
18			0.65	0.25	0.23	0.24	38.0	0.006	0.00025	0.00020
16			0.61	0.24	0.22	0.24	33.5	0.007	0.00026	0.00021
14			0.56	0.23	0.21	0.23	28.0	0.008	0.00027	0.00021
12			0.49	0.22	0.20	0.22	23.0	0.009	0.00030	0.00023
10			0.42	0.21	0.19	0.21	19.0	0.010	0.00035	0.00026
8			0.33	0.19	0.17	0.18	15.0	0.012	0.00043	0.00030
6			0.25	0.16	0.13	0.15	11.0	0.014	0.00060	0.00039
4			0.16	0.11	0.10	0.10	7.0	0.014	0.00091	0.00049

mean loss, per degree, of the magnetic moment, when the magnet was heated from about 10° C. to 100° C., expressed in terms of the moment at the lower temperature, it appeared that in the case of rods 16 cm. long and 0.95 cm. in diameter these mean losses were

0.00042 for seasoned, chilled, cast-iron magnets
0.00046 for seasoned magnets made of Crescent Steel Drill Rod
0.00046 for seasoned magnets made of Jessop's Round Black Tool Steel

while they were

0.00070 for seasoned magnets made of Jessop's Square Tool Steel 16 cm. long and of cross-section nearly that of the round rods.

In the case of shorter rods the difference was still more in favor of the castings because, I suppose, they were more uniformly hardened in the interior than the steel could be.

If an iron casting which has been hardened and boiled is magnetized in a solenoid either to saturation or to a degree which falls much short of this, it is practically impossible to decrease the moment by even so little as a tenth of one per cent, by striking the magnet on end with a wooden mallet or with a stone. I have tested many such magnets by dropping each two or three hundred times upon a stone slab, or by giving it hundreds of sharp blows with wooden clubs: the magnets get a little warm during this harsh treatment, but when their temperatures again fall to the original point the moments, which may have fallen a small fraction of one per cent, regain wholly, so far as my observations go, their old values. Some few of the specimens of special magnet steel that I have examined are nearly equal to the castings in this respect.

Prolonged boiling has, however, always reduced the moment of the cast-iron magnets very sensibly, and this loss may be as much as 20 per cent when the magnetizing field has been an extremely strong one and the residual moment is very high; if the casting has not been magnetized to saturation, the loss of moment by boiling is much less. If, after a cast-iron magnet has been seasoned, its temperature be suddenly raised from room temperature to 100° C., and then as suddenly lowered, the magnet may not wholly recover its original strength until after the lapse of several hours; if, however, the upper limit be only 50° C., there seems to be no sensible lag in the attainment of the whole of the original moment after the testing.

Most of the steel made specially for magnets, which I have been able to get, has come in the form of long bars of rectangular cross-section, about 2 cm. wide and 1 cm. thick; of such steel I have had pieces, 18 cm. long, hardened in some of the various ways used by professional magnet-makers, to compare with hardened castings of the same dimensions. One of the hardened but unmagnetized castings was placed in a solenoid, exposed to the action of a demagnetizing alternating current, and then put rapidly two or three times through a hysteresis cycle, using a maximum field of 145.2 gausses. After this preliminary treatment which I used in the case of every piece that I examined — the hysteresis diagram remained unchanged however many times the iron went through the cycle. I first obtained twenty-two points on half the diagram, laid those down carefully upon a piece of coördinate paper on such a scale that the diagram was 41 cm. long, and found that it was possible to draw a smooth curve which would not lie away from any one of the points by so much as a quarter of a millimetre and would apparently pass through almost every point; then I completed the cycle and found that the final reading of the mirror magnetometer did not differ by more than a sixth of one per cent, if by so much, from the initial reading. In the course of its second journey around the cycle, the iron made subsidiary loops on opposite sides of the origin and then returned again sensibly to its original condition. No diagram obtained from a long, slender wire could have been smoother than this one which belonged to this short bar. All the hysteresis curves in this paper are reduced from large drawings; generally about twenty-four points (though sometimes more) were found for each half diagram, and the curve was drawn through practically all of these.

Fig. 1 shows an instance where the points were apparently not so well determined as in other cases. This figure represents hysteresis curves of (A) a piece of hardened cast-iron (18 cm. × 2 cm. × 1 cm.), and (B) of two pieces of Seebohm and Dickstahl magnet steel (of the same dimensions), which makes the strongest saturated magnets of any of the special steels which I have used. The maximum field in the case of the A curve and the larger B curve was nearly 166 gausses, and under this field the cast iron and the special steel acquired moments of about 8900 and 15100 units respectively: when the field was removed the residual moments were about 3120 and 3550. The piece of steel just mentioned was hardened in a water bath; the smaller B diagram was obtained at another time with a similar piece of the same steel chilled in one of the baths used by Mr. Thompson. Although the maximum fields were unfortunately not the same in the two B diagrams, the general shapes of the curves are very similar.

These two pieces of steel and the cast iron were then placed in the solenoid mentioned above, the ends of each piece were connected together outside the coil by a massive iron yoke, and a current of about 45 amperes was sent through the coil; the residual moments of the cast iron, the steel chilled in water, and the steel hardened in the special bath were then about 4390, 5500, and 6220. The weight of each piece of steel was approximately 274 grams. Of three other pieces of steel of a brand used exclusively by well-known makers of magnets for "magnetos," the first was hardened in plain water, the second treated with potassium ferrocyanide, the third chilled in Mr. Thompson's bath. These pieces, like those just described, were 18 cm. long, 2 cm. wide, and 1 cm. thick, and they retained, after being magnetized to saturation,

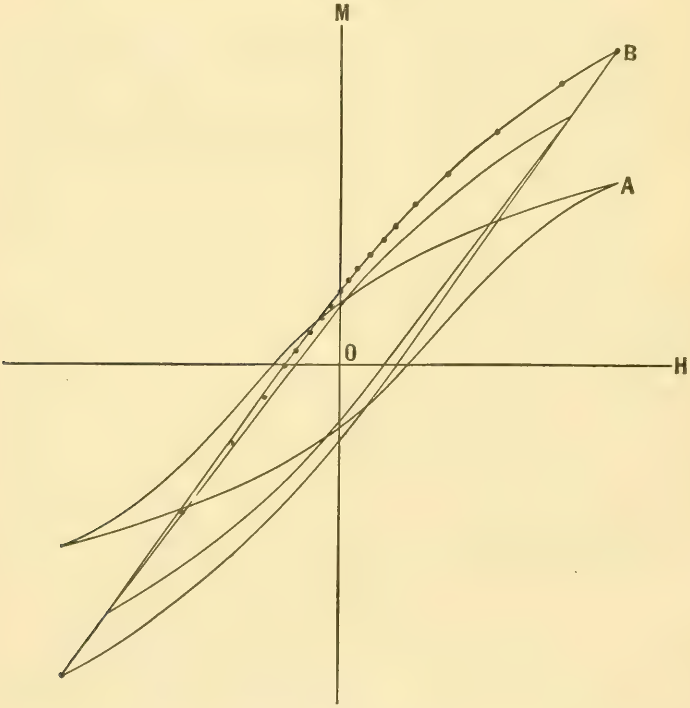

Figure 1.

the moments 3470, 2500, and 4190 approximately: this steel was in our hands, therefore, not so good as the Seebohm and Dickstahl Special. The special magnet steels can be bent or formed, but cannot be heated ho enough for welding without spoiling the material for making magnets.

In Figure 2 the results of observations made by Mr. John Coulson and myself upon round cast-iron rods 18 cm. long and 0.95 cm. in diameter are shown.

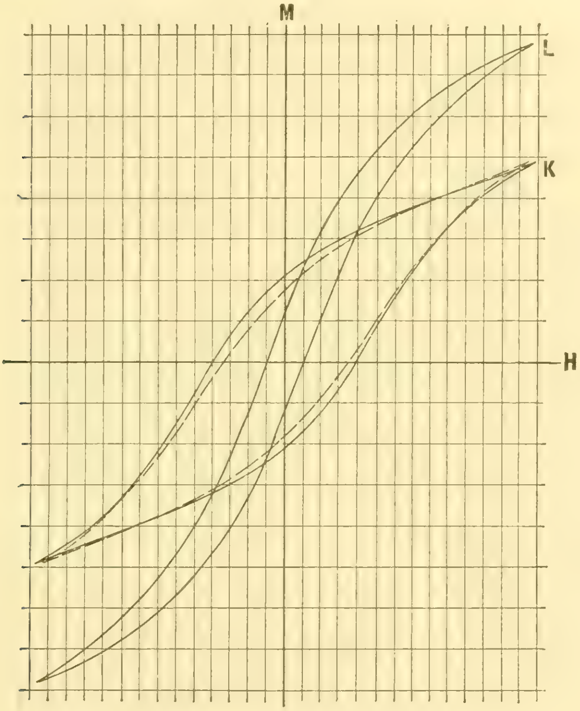

Figure 2.

The rods were magnetized in a solenoid, of 4927 turns in a length of 980 millimetres, placed with its axis horizontal and perpendicular to the meridian, and the relative moments, at different times, of the rod under investigation were determined from the deflection of a mirror magnetometer placed outside the solenoid in the Second Position of Gauss with respect to the magnet. The abscissas represent the field (in tens of gausses) to which the rod is subjected by the current in the solenoid, the ordinates represent the corresponding magnetic moment of the rod in thousands of units. The L-curve was obtained with a soft casting, the heavy K-curve with a casting of the same lot which had been chilled from a high temperature in cold water. At the same time, with this casting, were hardened two or three pieces of Crescent Drill Rod of the same dimensions as it. These were naturally not exactly alike magnetically, but the best of them furnished the dotted K-curve of Figure 2; the others had a very little less retentiveness. The magnetic likeness of the cast iron and the steel is worthy of notice.

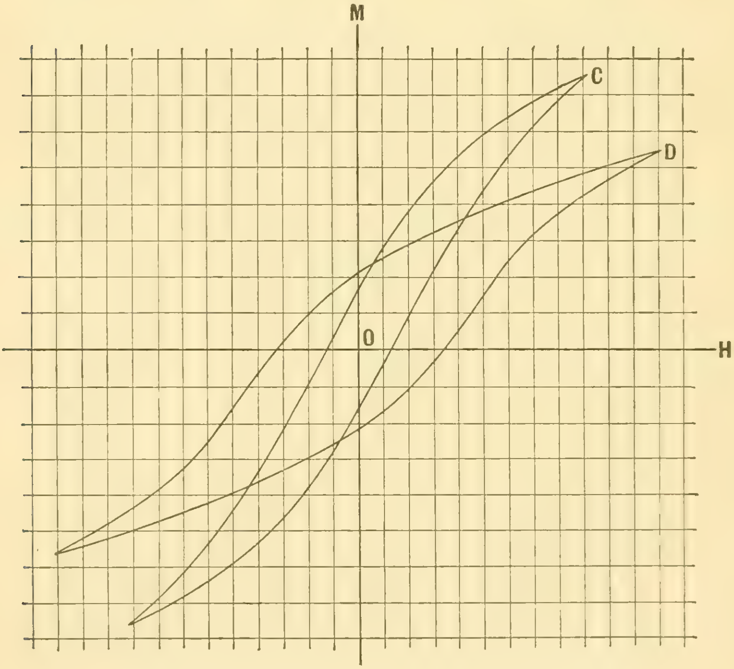

FIGURE 3.

In Figure 3 the horizontal divisions represent, as before, the field in tens of gausses, the vertical divisions the corresponding moments of the specimens in thousands of units.

The C-diagram was obtained with a soft casting 18 cm. long and .95 cm. in diameter, the D-curve with a casting of the same lot hardened by Mr. Thompson's methods. Another casting of the same set, hardened at the same time, gave the continuous curve in Figure 4, while a piece of Crescent Drill Rod, treated with the castings, furnished the dotted curve in the same figure. A careful comparison of the curves of Figures 3 and 4, obtained with the chilled castings — which were chosen from the lot wholly at random — will show that they are magnetically almost indistinguishable; the likeness of the Crescent Drill Rod, chosen also at random, to the castings is again shown by the curves of Figure 4.

We have used cast-iron magnets of many different forms, a few of which only are shown in Figure 5; the shapes marked 1, 2, 3, 6 have been employed, with the long way of the opening between the poles vertical, in d'Arsonval galvanometers, while a number of rather thin plates of the shape marked 8 have been used together in other instruments of the same kind. Magnets of the shapes marked 5 and 7 produce the artificial fields in some mirror needle galvanometers used as voltmeters.

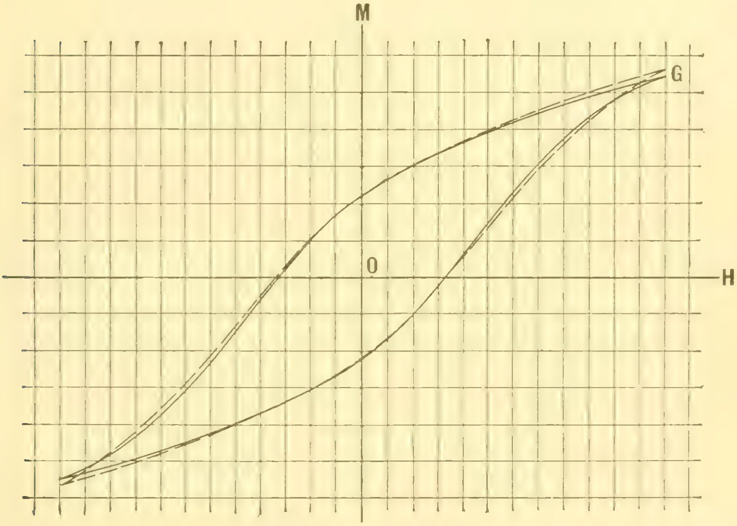

FIGURE 4.

The mean temperature coefficient (K) between 10° C. and 100° C. of hardened magnets of the shape 1, which weigh 1250 grams apiece, is about 0.00036; K is the mean loss per degree of the intensity of the magnetic field at any definite point between the poles — when the magnet is heated from 10° to 100° — expressed in terms of the field intensity at the point at the lower temperature. For magnets of the shapes 3 and 6 — which weigh 260 grams and 500 grams respectively — K has the values

0.00040 and 0.00031. The actual temperature coefficients at room temperature are always less than these mean values, and in the case of the last-mentioned form the coefficient between 10° C. and 40° C. is not greater than 0.00013.

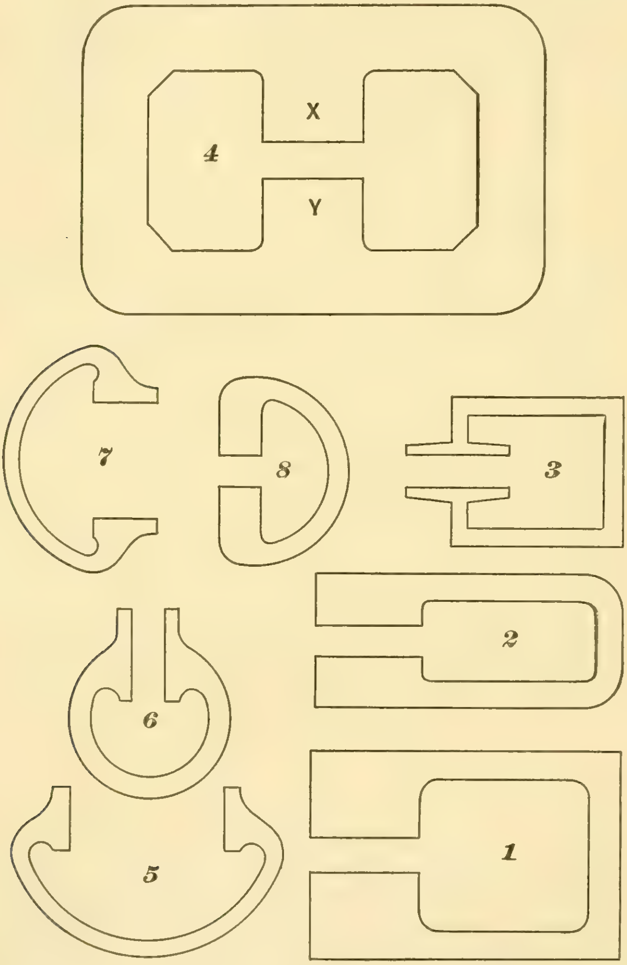

FIGURE 5.

If the slender part of such a casting as No. 1 be wound as uniformly as possible with insulated wire, and if steady currents of different strengths be sent through this wire, the relative values of the whole magnetic induction across a given area between the poles can be determined with sufficient accuracy by pulling out, from a definite position between the jaws, a coil of suitable shape made of very fine manganine wire and connected with a ballistic galvanometer.

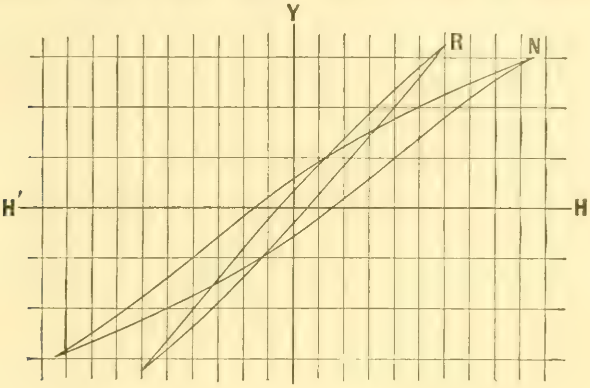

FIGURE 6.

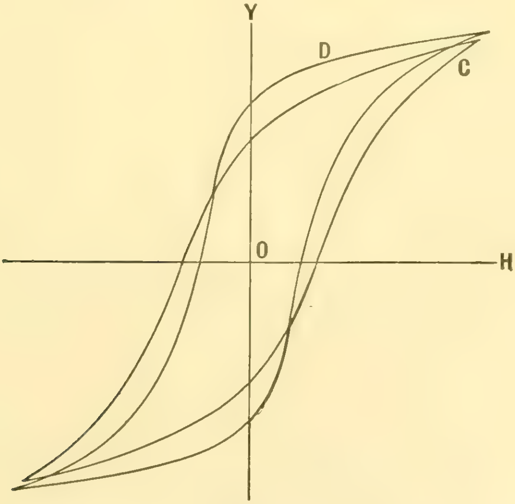

FIGURE 7.

In this way diagrams may be obtained very like the C- and D-curves of Figure 3, according as the casting is soft or hard, and this is a very delicate method of determining whether a casting which is hard on the surface is also hard throughout the interior. The two halves of each of two thick castings of the shape 4, weighing about 4800 grams each, were wound as uniformly as we well could with 156 turns of insulated wire,

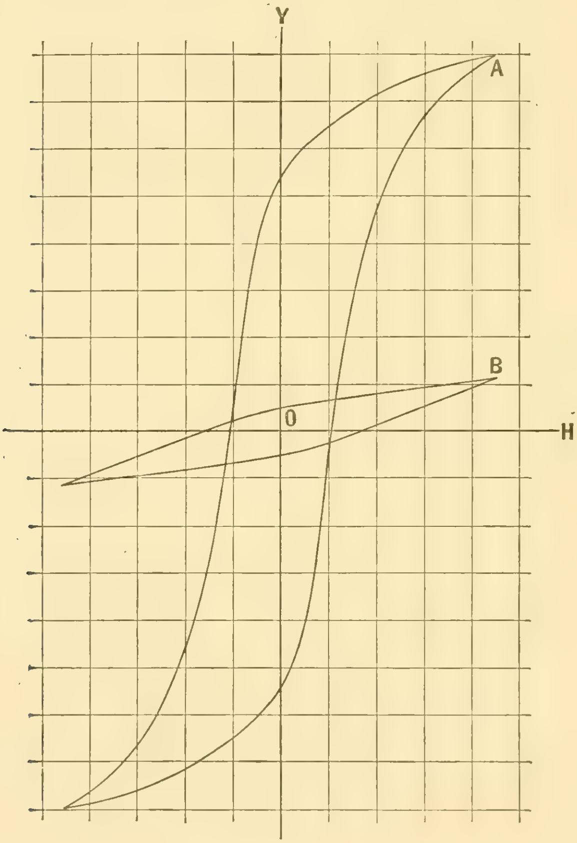

Figure 8.

and the two coils of each casting were so connected in series that when a current was sent through them both conspired to make one of the projections (say X) a north pole and the other (Y) a south pole: one of the castings was soft while the other, as another experiment afterwards proved, had been imperfectly hardened. With each of these castings a rough kind of hysteresis diagram was obtained by measuring for different current strengths the induction flux in the air between the poles; the ordinates in Figure 6 represent, in hundreds of lines per square centimetre, the induction across the centre of the gap X Y, while the abscissas represent the current in the coils measured in amperes; the R-curve belongs to the soft and the N-curve to the supposed hard casting. In a second experiment made with the same castings, one of the coils on each was used as a primary and the other, which was connected with a ballistic galvanometer, as a secondary; this procedure gave the curves of Figure 7, which show clearly that the second casting, which was very hard to the file on the surface, was still soft inside. The diagrams of Figure 8, drawn on a different scale, give the results of a similar experiment on the soft casting just mentioned and a third chilled casting of the same form. These striking curves, which were reduced from a large drawing, represent the observations accurately, and illustrate the fact that it is possible to make the whole inside of a massive casting, like the one here used, magnetically very hard.

When the gap X Y of this third casting had been closed by a piece of soft iron, and a heavy current had been sent through the coils for a few moments, in such a way as to make one of the projections a north pole and the other a south pole, the casting became a fairly strong permanent magnet: the flux per square centimetre across the middle of the air gap was finally 285. The flux per square centimetre across the gap of a similar, though considerably less heavy, hardened cast-iron magnet, belonging to one of a number of excellent d'Arsonval galvanometers furnished by Messrs. Leeds and Northrop, is 187. In this magnet the pole projections are a little shorter and the gap a trifle wider than in the castings 4 described above.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

AUTOCATALYTIC DECOMPOSITION OF SILVER OXIDE.

By Gilbert N. Lewis.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
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AUTOCATALYTIC DECOMPOSITION OF SILVER OXIDE.

BY GILBERT N. LEWIS.

Presented by T. W. Richards, March 8, 1905. Received March 8, 1905.

IN the course of an investigation to determine the equilibrium pressure at a given temperature in the system $2Ag_2O = 4Ag + O_2$, a singular phenomenon appeared. The experiment consisted simply in heating silver oxide at a constant temperature of 335° Centigrade, the stout glass tube which contained the oxide being connected with a manometer. After heating several hours, the manometer gave no evidence that any decomposition had occurred. This occasioned some surprise, since 250° C. is frequently given as the temperature at which silver oxide begins to decompose. As I was about to stop the experiment in order to test the tightness of the joints, I detected a slight deflection of the manometer. This increased considerably during the next hour, and I noticed that the greater the pressure became, the greater became the rate of its increase, until finally the pressure had reached twenty atmospheres, and was increasing at the astonishing rate of between three and four atmospheres per hour. At this point the cement joining the tube and the manometer began to give way, and the experiment had to be discontinued. It had progressed far enough, however, to stimulate considerable curiosity as to the explanation of so unusual a behaviour.

The gradual acceleration in the decomposition which was observed could only be attributed to one of the following causes: 1. The increasing pressure. 2. The increasing concentration of oxygen. 3. The accumulation of metallic silver. 4. Some change in the silver oxide, such as the slow agglomeration of its grains. Of these four, the first two were eliminated at once by a rough experiment carried on at atmospheric pressure, which showed the same phenomenon of accelerating decomposition. It seemed worth while to study more carefully this decomposition under atmospheric pressure, and for this purpose the following apparatus and methods were devised.

Experimental Method.

The apparatus finally adopted for the experiments is sketched in Figure 1. For the thermostat a mixture of sodium and potassium nitrates was used, contained in a cylindrical vessel of enamelled iron ("agate ware"), about 30 cm. in diameter and 25 cm. in height, — A in Figure 1. This vessel rested on a tripod, and was heated from below by two Bunsen burners. The heating efficiency and the constancy of the thermostat were surprisingly increased by a sheath of asbestos (B) surrounding the bath and projecting 10 or 15 cm. below it. This sheath was separated about ½ cm. from the side of the vessel by a number of corrugated brass strips not shown in the figure. An annular space was thus left between the asbestos and the bath through which the hot gases from the burners could pass freely, and thus heat the thermostat from the sides as well as from below. The careful adjustment of this sheath seemed to be almost essential to the satisfactory operation of the thermostat.

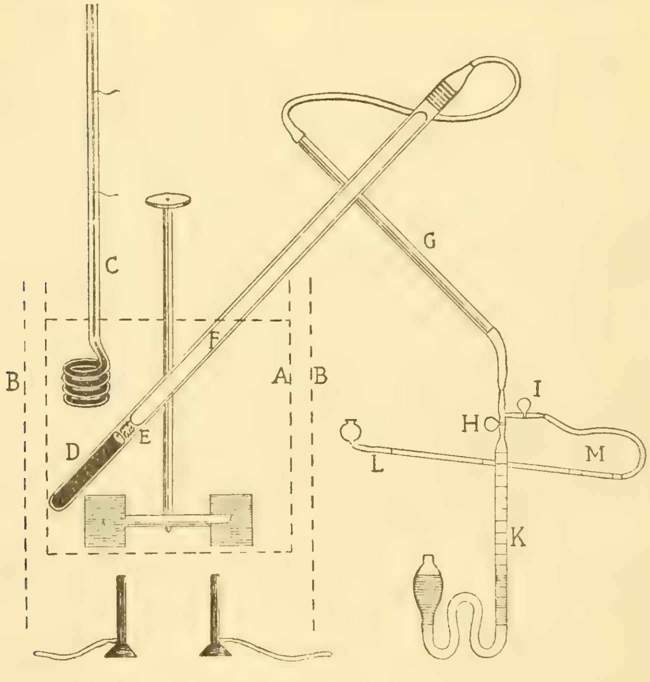

Figure 1.

The stirrer was made very simply without the use of solder, combining the ordinary method of stirring with the centrifugal, and it was so effective that the different parts of the bath never differed by more than one or two hundredths of a degree.

The temperature regulator was simply a large mercury thermometer made with a spiral of thin glass sealed to a vertical capillary (C). Into this capillary two fine platinum wires were sealed, so that when the expansion of the mercury brought it into contact with the upper wire an electric current from a storage battery was made, which, passing through a telegraph instrument, shut off a thin rubber tube supplying gas to one of the burners.

This regulator was, in fact, the only part of the apparatus that caused the slightest inconvenience. The spiral being made of thin glass and holding nearly a kilogram of mercury was very fragile, and, being weakened further by the continued high temperature, broke several times, and had to be replaced. If a regulator of the same type could be made of some suitable metal, which would also have the advantage because of its high conductivity of responding more rapidly to changes in the temperature, a very great constancy might be attained, for a thermostat working at this temperature has two great advantages over the common water thermostat: it is influenced very slightly by changes in the room temperature, and there is no loss of heat by evaporation. It must, however, like all thermostats, be protected from drafts of air.

A mixture of potassium and sodium nitrates proved to be an ideal bath liquid. When pure, the mixture is as clear as water and as mobile. Well-annealed glass apparatus and thermometers may be plunged directly into the liquid again and again without breaking.

The vessel of enamelled ware seemed to suffer no deterioration whatever, even after several months of contact with the molten salt.

The apparatus for containing the silver oxide and measuring its rate of decomposition is also sketched in Figure 1. The glass tube (D), about one meter long, was filled with carefully dried silver oxide to a height of about 10 cm., and the powder was shaken compactly into the end. This was covered with a thin layer of glass wool (E) and the remainder of the tube was filled with a loose-fitting glass rod (F), the object of which was merely to diminish as far as possible all air space subjected to changes of temperature, and thus to avoid fluctuations in volume. The end of the tube containing the oxide was immersed deeply in the thermostat, and the other end was connected by rubber tubing to a long brass tube (G), in passing through which the evolved oxygen came to the temperature of the room. The end of this brass tube was connected with a gas burette (K) and to the recorder (L). This latter was simply a glass tube about 2 mm. in diameter, marked off at such intervals that the volume from one mark to the next was exactly 1 c.c. A small column of water (M) served as index. With stopcock H closed and I open, the small water column advanced along the recording tube, and the time taken by the forward meniscus in moving from one mark to the next was read with a stop-watch. When the reaction was very fast the recording meniscus was allowed to run through several centimeters in order to permit a more accurate measurement of the time. Thus it was possible to measure accurately the rate of evolution of oxygen, whether 1 c.c. in an hour or 1 c.c. in ten seconds, the latter being the greatest velocity measured. After measuring the reaction velocity, the water indicator could be brought back to its initial position by cautiously opening the stopcock (H). I was then closed and H left open, thus allowing the gas to collect in the burette until the velocity was again measured. In this way the velocity at any moment, and the total volume of gas that had been evolved, could be measured independently. After the first few experiments, however, the burette was done away with, and only the velocity was measured, as it was found that in the few cases where a knowledge of the total volume was desired it could be ascertained with sufficient accuracy by a rough measurement of the time-velocity curve.

This simple apparatus gave complete satisfaction, cleanliness in the recording tube being the only precaution necessary for its success.

The Course of the Reaction.

In the first experiment with the apparatus 4.4 grams of silver oxide were used, the temperature was 327.5° C., and the measurements of the reaction velocity were made for twenty-four hours at intervals of one half to one hour. For several hours the decomposition was inappreciable; at the end of ten hours the rate was 1 c.c. of oxygen evolved per hour; after thirteen hours, 1 c.c. in twenty-seven minutes; after fourteen hours, 1 c.c. in eighteen minutes; after sixteen hours, in four minutes; after eighteen hours, in one and one half minutes, and at this point the velocity was increasing rapidly. After twenty hours the rate was 1 c.c. in one minute, and rapidly falling, showing that between the last observation and this a point of maximum velocity had been reached. After twenty and one half hours the rate of evolution was 1 c.c. in two minutes; after twenty-one hours, 1 c.c. in three and one half minutes; after twenty-one and a half hours, in seven minutes; and before twenty-four hours had passed all action had ceased.

This experiment gave a general idea of the course of the reaction. At the beginning the velocity is very small, increasing more and more rapidly to a maximum, then falling, rapidly at first, later more slowly, until finally the reaction is complete. This decomposition has the general character of an explosion, and in fact it is nothing more or less than an explosion which is slow enough so that its velocity can be measured, and differing from the ordinary explosion only in that the latter is a reaction which progresses with accelerating velocity because of increasing temperature due to the heat given off by the reaction, while the decomposition we are studying takes place at constant temperature, and is in fact an endothermic reaction.

To find the cause of the acceleration in this case was the next problem. Two possible explanations were left. The phenomenon might be due to a gradual physical change in the silver oxide, such as to bring it into a more and more reactive condition, or it might be due to the presence of the silver produced in the reaction. The latter supposition could fortunately be tested directly by experiment, for if the silver formed by the reaction were a catalyzer, then unquestionably silver prepared in the same way and mixed with the oxide before the experiment would produce a similar effect. Therefore, the preceding experiment was repeated, using instead of the pure silver oxide a mixture of 5.5 grams of oxide and 0.1 of a gram of the powdered metallic silver obtained from the last experiment. These were rubbed together until very intimately mixed.

Noticeable decomposition began as soon as the mixture was put into the thermostat. The rate of evolution of oxygen after two hours was 1 c.c. in five minutes; after five hours, in three minutes; after seven hours, in one and one half minutes; after about eight hours the maximum of 1 c.c. per minute was reached, and in twelve hours the reaction was over.

In the last experiment the reaction at the end of twelve hours was just beginning to have a measurable velocity. This difference between the two experiments seemed to point conclusively to catalytic action by the silver. Yet, it is not inconceivable that the difference might be due to some difference in the treatment of the two samples of silver oxide. In order to remove this doubt the following experiment was tried later, under new conditions, with a different preparation of silver oxide, and at a different temperature.

Three portions of silver oxide were taken, each weighing 3.3 grams. To the first (A) 1.1 grams of silver powder was added, to the second (B)

.55 gram silver, and the third (C) was left pure. All three were subjected to precisely the same treatment of rubbing and stirring, and introduced in separate tubes simultaneously into the thermostat. At the end of three quarters of an hour 1 c.c. of gas was evolved from A in seventy-two seconds, from B in eighty-five seconds, and from C in two hundred and fifteen seconds; at the end of an hour, from A in forty-two seconds, from B in fifty-six seconds, and from C in two hundred and ten seconds. The velocity reached its maximum for A in eighty-eight minutes, for B in one hundred and ten minutes, and for C in one hundred and sixty minutes.

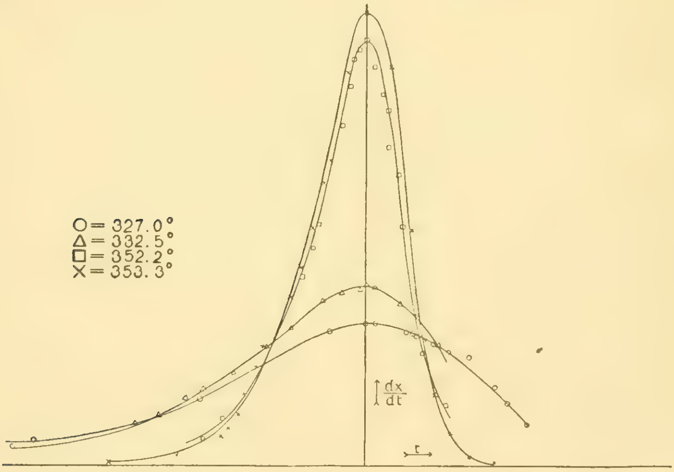

FIGURE 2.

This experiment demonstrated the catalytic influence of the silver and showed that this influence increases with the quantity of silver. More precise quantitative results were not looked for, as it was assumed that the action of the decomposed silver would be dependent upon the degree of fineness, the intimacy of its mixture with the oxide, and possibly upon other conditions which could not be made entirely uniform. It seemed more likely that a careful study of the form of the reaction curve for the pure oxide would yield more trustworthy information as to the law and mode of action of this catalysis.

A series of experiments was therefore carried out, similar to the one already described, but with more frequent and more accurate determinations of the velocity. A considerable number of such experiments were made at different temperatures, and of these I have chosen the four most complete and satisfactory for further discussion. These were made at 327.0° C., 332.5° C., 352.2° C., and 353.3° C., and may be designated A, B, C, and D, respectively.

The results are represented in Figure 2, the reaction velocity being plotted vertically and the time horizontally. Instead of the time when the experiment began, the time of maximum velocity was taken as zero. Thus, the maxima of all the curves fall on the same vertical line. The velocity is expressed as the fraction of the whole quantity of oxide which is decomposed per second. This can, of course, be readily found from the rate of evolution of oxygen when the temperature and pressure are known. If x represents the fraction of the original oxide that has been decomposed, the velocity is $\frac{dx}{dt}$.

An inspection of these curves shows, as was to be expected, that the maximum velocity is greater and the whole time of the decomposition less the higher the temperature of the experiment. Further, we are struck by the great regularity of the curves and by their very evident similarity to one another. They appear to belong to a single family of curves, such as may be represented algebraically by a single equation with one variable parameter. It seemed likely that if these curves were reduced to the same scale they might almost coincide.

This was attempted by plotting each experiment with a different unit of time, so chosen as to make each maximum velocity equal to unity; that is to say, the unit of time in each case is the time which would be required for complete decomposition if the whole reaction progressed at a uniform rate equal to the maximum velocity. In other words, the units of time chosen are inversely proportional to the maximum velocities, which therefore, in the new scale, become numerically equal and fall on a single point.

The four curves thus plotted in Figure 3 come astonishingly near to coincidence; in fact, they do coincide within the limits of experimental error.

Among all the cases in which reaction velocity has been studied, the present phenomenon occupies a wholly unique position. Exceedingly few autocatalytic reactions have been quantitatively studied. In a homogeneous system the study of the reaction of oxalic acid and permanganate by Schilow * may be mentioned, and in heterogeneous systems the study of the action of acids on sodium thiosulphate by Foussereau,† and the

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* Ber. d. deutsch. Chem. Ges., **36**, 2735 (1903).

† Ann. Chim. Phys. (6), **15**, 533 (1888).

decomposition of hydrogen arsenide, hydrogen selenide, and hydrogen antimonide, respectively, by Cohen,[*] Bodenstein,[†] and Stock and Guttmann.[‡] In all these cases in heterogeneous systems a reaction in a liquid or gaseous phase is catalyzed by a solid product. Bodenstein,[§] recalculating the work of Stock and Guttmann, points out the surprising simplicity and regularity of the decomposition studied by them. But how much more surprising it is that in our present case the decomposition of one solid is catalyzed even more smoothly and regularly by the presence of another solid with which, in the very nature of solid substances, it cannot be in very intimate contact. In fact, this decomposition behaves like a simple homogeneous reaction.

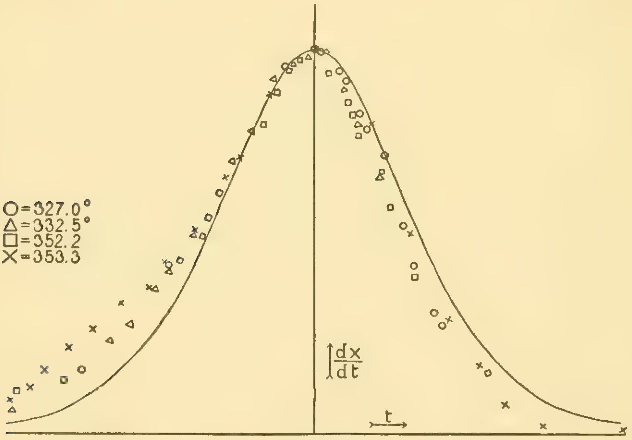

Figure 3.

In a homogeneous catalytic reaction the simplest case is that in which, for a given volume, the reaction velocity is proportional to the quantity of the reacting substance and to the quantity of the catalyzer. Let us see how nearly the decomposition of silver oxide conforms to these conditions.

If we assume that the velocity of reaction is proportional to the quantity of silver oxide and of silver, we may write the equation,

$$\frac{dx}{dt} = Kx(1 - x), \qquad (1)$$

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[*] Z. P. C., **25**, 483 (1898).

[†] Z. P. C., **29**, 429 (1899).

[‡] Ber. d. deutsch. Chem. Ges., **37**, 901 (1904).

[§] Z. P. C., **49**, 41 (1904).

where $\frac{dx}{dt}$ measures the velocity, x is the fraction of oxide decomposed, and therefore proportional to the amount of silver; $1-x$ is the fraction remaining unchanged, and therefore proportional to the amount of silver oxide. K is a constant.

An inspection of the equation shows that it would be represented by a rising and falling curve, with a maximum of velocity at the point where just one half the silver oxide is decomposed. In order to compare the assumptions we are making with our experimental results we must obtain the velocity as a function of t (time) rather than of x. From equation 1, by integration,

$$ln \frac{x}{1-x} = Kt + C,$$

where ln stands for a natural logarithm and C is the integration constant. If we count t from the point of maximum velocity, then at that point $Kt = 0$. Also since

$$x = (1-x), \quad ln \frac{x}{1-x} = 0,$$

and therefore $C = 0$. Therefore our equation stands

$$ln \frac{x}{1-x} = Kt, \quad \text{or} \quad \frac{x}{1-x} = e^{Kt}, \quad \text{or} \quad x = \frac{e^{Kt}}{1+e^{Kt}}, \tag{2}$$

differentiating this equation

$$\frac{dx}{dt} = \frac{Ke^{Kt}}{(1+e^{Kt})^2}. \tag{3}$$

This is the equation sought, and in order to plot its curve on a suitable scale for comparison with those of Figure 3, we must choose in this case also such a unit of time as to make the maximum velocity equal to unity. At this point, then, $\frac{dx}{dt} = 1$, but as previously shown $x = \frac{1}{2}$, and $1-x = \frac{1}{2}$; whence by equation 1, $1 = K \cdot \frac{1}{2} \cdot \frac{1}{2}$, and $K = 4$, whence equation 3 becomes

$$\frac{dx}{dt} = \frac{4e^{4t}}{(1+e^{4t})^2}.$$

The continuous curve shown in Figure 3 is obtained by plotting this equation. Its great similarity to the experimental curves is obvious.

Still, it is not quite identical with them. The experimental curves all coincide more nearly with one another than they do with the theoretical, and all show a certain asymmetry which that does not. In the case of the experimental curves it is evident that the apex comes at a point where more than one half the silver oxide is decomposed. Still the theoretical and experimental curves fall much nearer together than was to have been predicted, and it is possible that the difference that exists may be due to faults in the experimental method; thus, for example, it is not impossible that near the point of maximum velocity the reaction may take place so rapidly that heat cannot be absorbed from the bath rapidly enough to maintain a constant temperature, — a condition which would destroy the symmetry of the curve. Still, it is hardly probable that this was the case, for if it had been there would have been a wide difference between the curves for the highest and the lowest temperatures; since at the latter the maximum velocity is only one third or one fourth as great as at the former.

The Study of Different Varieties of Silver Oxide.

In the experiments described hitherto the same silver oxide was used, namely, a preparation by Merck labelled "highest purity." When this was exhausted the experiments were continued with an oxide furnished by Eimer & Amend. It was soon found, however, that with this substance no results could be obtained comparable with the preceding. The reaction velocity was far more rapid than with the old oxide. Thus, at 340° the maximum velocity was passed in less than an hour, and in three hours all action had ceased. Furthermore, the decomposition was irregular, so that under the same conditions inconstant results were obtained. This sample of oxide was therefore abandoned, and another bottle of Merck's procured. This was of the same grade and bore the same label as that used in the previous experiments, but it, too, decomposed with great rapidity. Thus, the velocity at 352° reached a maximum in thirty-seven minutes, and the reaction was over in three hours, whereas with the original sample at this temperature the maximum came at the end of four hours. The great irregularities shown by the Eimer & Amend samples, however, did not appear. Such irregularities most probably are due to lack of uniformity in the oxide.

The great difference in the rapidity of decomposition of different samples of oxide made it desirable to study the effect of the method of preparation of the oxide upon its decomposition. It seemed probable that the difference between samples was either due to varying quantities of impurities or to variations in the physical state; for example, in the fineness of division of the powder. Pure silver oxide was therefore prepared for study in three different ways. For all these preparations silver nitrate purified by several re-crystallizations was used. In the first case sodium carbonate was added to it. The precipitated carbonate was washed repeatedly by decantation, the last washings occupying twenty-four hours each. It was then dried and heated at 240° for twenty-four hours, to drive off as completely as possible the water and the carbon dioxide. The second and third preparations were made by treating the silver nitrate with a clear solution of barium hydroxide, in one case with both solutions at room temperature, in the other with both at 100°. All the solutions were very dilute, and the reaction was carried on with the exclusion of carbon dioxide. The precipitates were carefully washed, and dried at 240°.

It was expected that the silver carbonate would yield a very finely divided oxide, and therefore one that would rapidly decompose, but as a matter of fact it proved to react much more slowly than any previously tried. The samples prepared with barium hydroxide decomposed more slowly still, — more than four times as slowly as the oxide used in the earlier experiments. Contrary to expectation the oxide prepared in the cold decomposed rather more slowly than the other.

It would appear, therefore, from these few experiments, that the rapid reactions of some of the samples was caused by the presence of impurities. Nevertheless my experiments were hardly sufficient to make this conclusion certain, especially as certain experiments led me to suspect that the rapidity and temperature of drying had some influence on the rate of decomposition. Unfortunately, however, my experiments at this point had to be brought to an abrupt close, and it is therefore necessary to leave this question open.

TEMPERATURE COEFFICIENT OF THE REACTION VELOCITY.

The influence of temperature on the reaction velocity may be found from the four experiments, the results of which are plotted in Figures 2 and 3, since these experiments were all made with the same sample of oxide. We must choose for comparison the reaction velocities at corresponding points in the several experiments. The apices of the curves constitute such a series of corresponding points. The maximum velocities at the four temperatures, 327.0°, 332.5°, 352.2°, 353.3°, were respectively 0.0106, 0.0134, 0.0316, and 0.0335 per cent per second.

If we plot the logarithms of these last numbers on the one hand and the corresponding temperatures on the other, the resulting points lie almost precisely on a straight line whose slant is given by the equation

$$\frac{d \log_{10} v}{d T} = 0.0185,$$

or by the integrated form

$$\log_{10} \frac{v_1}{v_2} = 0.0185 \, (T_1 - T_2).$$

It is customary to express the influence of temperature on reaction velocity by means of the quotient of the velocity at one temperature divided by the velocity at a temperature ten degrees lower. This quotient can be easily found in the present case from the above equation. Since $T_1 - T_2 = 10$,

$$\frac{v_1}{v_2} = \log_{10}^{-1} 0.185 = 1.53.$$

In other words, when the temperature rises 10° the velocity increases by about fifty per cent. This increase is less than that of most reactions, but corroborates the idea of van't Hoff,* who writes concerning this quotient: —

"Bedeutend weniger als Verdoppelung zeigt nur die Zerlegung von Phosphor- und Arsenwasserstoff. Das hängt aber vielleicht mit der hohen Beobachtungstemperatur zusammen, denn in der grossen Mehrheit der bis jetzt beobachteten Fälle nimmt das Geschwindigkeitsverhältnis für 10° mit stiegender Temperatur ab."

The Nature of the Catalytic Action.

At first sight the explanation of the catalytic action of the silver would seem to be like that which is sometimes offered for the development of a photographic plate; namely, that the silver as it is produced becomes supersaturated, and finally reaches such a concentration as to stop the reaction except where silver is already present upon which it may deposit. If this explanation is correct the phenomenon we are studying resembles closely the simple process of crystallization from a supersaturated solution. In fact, I may point out that every case of such crystallization is a case of autocatalysis, and that here also the reaction velocity rises and falls as the crystallization continues; rises as the surface of the crystal increases, falls as the degree of supersaturation decreases.

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* Vorlesungen ueber Theor. u. Phys. Chem.

Striking as the analogy is, it must be borne in mind that we are dealing in this case with the action of one solid upon another, and the great regularity of the reaction seems hard to reconcile with all the other facts known concerning reactions in solid phases.

As a result of other studies I have been inclined to entertain an entirely different explanation of the phenomenon. In the course of researches, some of which will be shortly published, I have been led to believe in the great importance to chemistry of the two following reactions:

$$2H \rightleftarrows H_2$$

$$2O \rightleftarrows O_2.$$

These two reactions have frequently been discussed in connection with the theory of the nascent state, but little attention seems to have been paid to the immense importance of the *velocity* of these reactions in a large number of processes, such as the union of oxygen and hydrogen, combustions in general, reduction by hydrogen, spontaneous decomposition of oxidizing and reducing agents, and especially in the phenomenon of galvanic polarization. Here, however, this larger problem concerns us only as it offers a possible explanation of the autocatalysis of silver oxide. When we consider the probable mechanism of the decomposition, it is evident that instead of the one reaction $2Ag_2O = 4Ag + O_2$, we may write with at least equal probability the two reactions $Ag_2O = 2Ag + O$ and $2O = O_2$. If in fact the decomposition takes place in these two stages the important question is, Which is the reaction of which we are measuring the velocity? that is, Which of the two is the slower?

We have many reasons to believe that the reaction $O_2 = 2O$ is an extraordinarily slow one. For example may be cited the inactivity of oxygen and the preponderating tendency in slow oxidization at ordinary temperatures for the oxygen to enter into the resulting compounds as the radical (O_2) forming the unstable peroxides. We may assume that to an extremely small extent oxgen gas is dissociated, that in ordinary O_2 there is a small concentration of O in equilibrium with it. If when the equilibrium is destroyed by removing some monatomic oxygen it is restored with great slowness, then it will be true also that if the equilibrium is destroyed in the opposite direction the recovery, according to the reaction $2O = O_2$, will likewise be slow.

Therefore, it is not unlikely that this latter reaction is the one whose velocity we have been studying, and that it is the reaction which is catalyzed by the presence of the silver. In fact silver is known to catalyze many gaseous reactions.

The above line of reasoning would apply equally to other reactions in which oxygen is evolved. Let us consider two cases as different from this one as possible, namely, the decomposition of hydrogen peroxide and of potassium chlorate. Both of these reactions may be accelerated by catalyzers; the catalysis of the latter by manganese dioxide, and of the former by manganese dioxide, platinum, and other substances, are familiar to all chemists. If in these cases also we assume that the union of two atoms of oxygen is the reaction, the slowness of which retards the decomposition, then this reaction is catalyzed by platinum and by manganese dioxide. These substances, therefore, if our assumptions are correct, ought to aid the decomposition of silver oxide as silver does. We have, therefore, the means of testing our suppositions by direct experiment.

Two tubes, each containing 5 grams of silver oxide, and one containing besides .25 gram pure platinum black, were put into the thermostat simultaneously. The one containing platinum began to give off oxygen at once. At the end of two hours it was evolving 1 c.c. in twelve seconds, and the other tube 1 c.c. in ten minutes.

A similar experiment was tried with another preparation of silver oxide, in which case the silver oxide containing platinum was entirely decomposed in twenty-four hours and the other not until after three days. In a similar experiment manganese dioxide, which had been previously heated to redness, was used in place of the platinum, and showed also marked catalyzing power.

These results speak strongly in favor of the truth of the assumptions that we have made. It is true that catalysis of silver oxide and hydrogen peroxide by platinum may be specifically different, but let us consider a few of the reactions, where oxygen is a factor or a product, which are catalyzed by platinum: the combustion of gases such as hydrogen, methane, ammonia, vapor of methyl alcohol; the decomposition of oxidizing agents such as hydrogen peroxide; the electrochemical solution of oxygen at the cathode and its separation at the anode. Certainly there seems to be ground for believing that in all these cases, while several reactions occur together, an important and in many cases the only factor in determining the speed of the whole reaction is the speed in one direction or the other of the intermediate reaction, $2O = O_2$, and that this reaction is catalyzed by platinum and by a number of other substances, as well of course in one direction as in the other.

At least we may conclude in regard to the decomposition of silver oxide, that since there are substances besides silver which act as catalyzers the autocatalysis is probably not of the same nature as that of precipitation from a supersaturated solution, and further there is some ground for believing that the velocity which we measured is either that of the reaction $2O = O_2$ or that of the diffusion of the factors or products of this reaction to or from the catalyzing substance.

We should expect the catalytic power of the silver to be proportional to the extent of its surface, while we have found it to be proportional to the amount of silver. But probably these two quantities are very nearly proportional, since the silver is produced as a slightly cohesive mass which readily falls into a fine white powder. If the grains are of fairly uniform size, the surface exposed is proportional to their number, or, in other words, to their mass.

SUMMARY.

A satisfactory thermostat for high temperatures is described, together with a method of measuring the velocity of decomposition of silver oxide.

This velocity of decomposition at constant temperature increases as the reaction proceeds, passes through a maximum, and falls gradually to zero.

The phenomenon is due to autocatalysis, and the catalyzing agent is metallic silver.

The reaction proceeds with great regularity, and the curve of reaction velocity comes very close to the curve of the equation, $v = \frac{Ke^{Kt}}{(1 + e^{Kt})^2}$, which is the equation of the simplest case of autocatalysis.

The influence of the purity and the method of preparation of the silver oxide upon its decomposition is discussed.

The temperature coefficient of the reaction velocity is determined.

Evidence, both experimental and theoretical, is offered in favor of a comprehensive theory of such catalytic action.

Proceedings of the American Academy of Arts and Sciences.

Vol. XL. No. 24. — July, 1905.

RECORDS OF MEETINGS, 1904–1905.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICES.

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Charles Storer Storrow. By Hiram F. Mills.

OFFICERS AND COMMITTEES FOR 1905–1906.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

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(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

Nine hundred and fiftieth Meeting.

JUNE 8, 1904.

The Academy met at the house of Edward Atkinson, Brookline.

The PRESIDENT in the chair.

On the motion of the Recording Secretary, it was

Voted, That the Committee appointed on June 17, 1903, consisting of the President, Treasurer, Librarian, and Recording Secretary, be reappointed to ascertain if more suitable accommodations for the Academy can be obtained.

Dr. Atkinson gave a communication on the following subjects: — I. "Peat, and its Manufacture," illustrated by an apparatus in operation. He exhibited in this connection various kinds of briquettes and of coke made from moss peat and from grass peat in this country and in Europe. He also showed the operation of generating illuminating gas from peat. II. "The Diffusion of Light shown by its Transmission through Various Types of Glass." III. "The Process of Cooking in the Aladdin Oven, with Specimens of Food cooked therein." IV. "Uralite." He called attention to specimens of Uralite, an artificial semi-flexible stone made by a combination of asbestos with silicate of soda, the silicate being subjected to what may be called a double reaction under heat and pressure which renders it insoluble. This material was invented in Russia, where its merits have been tested on a large scale, and is now used for the inside finish of important buildings in England,

and in India as a substitute for slate and thatch for roofing. It is incombustible, water-proof, neither shrinks nor contracts under heat or cold, weather-proof, and cheap. It may be worked with ordinary carpenters' tools, planed, polished, veneered, painted, frescoed, or otherwise treated.

The following papers were presented by title by Professor John Trowbridge: —

"Resonance in Wireless Telegraphy," by G. W. Pierce.

"A Manometric Device for the Air Thermometer," by Harold Edwards.

"Note on the Platinum Thermometer," by Harold Edwards.

"Spectra of Gases at High Temperatures," by John Trowbridge.

Nine hundred and fifty-first Meeting.

OCTOBER 12, 1904. — STATED MEETING.

The PRESIDENT in the chair.

The following letters were read: —

From Herbert Weir Smyth, accepting Resident Fellowship; from Eugene W. Hilgard, James D. Hague, T. Mitchell Prudden, acknowledging election as Associate Fellows; from Adolf Harnach, Gaston Boissier, Pasquale Villari, Felix Klein, accepting Foreign Honorary Membership; from the Prussian Minister, Mr. Studt, presenting a medal commemorating the bi-centennial jubilee of the Royal Prussian Academy of Sciences; from the National Society of Natural Sciences and Mathematics of Cherbourg, announcing the death of its director and founder, M. Auguste-François Le Jolis; from the Belgian Society of Astronomy, soliciting membership; from the National Library of Turin, soliciting aid in replacing its library lost by fire; from a committee soliciting contributions for erecting a monument to Theodor Mommsen.

The Chair announced the following deaths: —

James M. Barnard, Resident Fellow in Class I., Section 3; George F. Hoar, Resident Fellow in Class III., Section I.

On motion of the Recording Secretary, it was

Voted, To meet on adjournment on the second Wednesday in November.

The following communication was given by Professor Charles R. Cross: —

"Description of a Mechanical Model devised by Professor Schwamb to illustrate the Theory of Rotary Polarization of Light."

The following papers were presented by title: —

"A Quantitative Determination of the Anomalous Dispersion of Sodium Vapor in the Visible and Ultra-Violet Regions." By R. W. Wood.

"A Revision of the Atomic Weight of Iodine." By G. P. Baxter.

"Some Additions to the Arc Spectra of the Alkali Metals." By F. A. Saunders.

"The Effect of Heat on the Color Changes in the Skin of Anolis carolinensis Cuv." By G. H. Parker and S. A. Starratt.

"The Achromatization of Approximately Monochromatic Interference Fringes by a highly Dispersive Medium, and the Consequent Increase in the Allowable Path-Difference." By R. W. Wood.

Nine hundred and fifty-second Meeting.

NOVEMBER 9, 1904. — ADJOURNED STATED MEETING.

The Academy met at the house of the President in Cambridge.

The PRESIDENT in the chair.

The following death was announced by the Chair: —

Dr. Samuel Leonard Abbot, Resident Fellow in Class II., Section 4, Recording Secretary of the Academy from 1854 to 1863.

The following letter was read by the Recording Secretary: —

10 June, 1904.

MY DEAR PROFESSOR WATSON, —

In accordance with a vote passed at the meeting of the Academy held on the 8th instant, I have bought, in the name of the American Academy of Arts and Sciences and for the sum of $30,000, the estate No. 28 Newbury Street, Boston, subject to the condition that an examination of the title thereto shall show that there are no restrictions which may prevent said Academy from building a stack-room on the land which lies between the rear of the present house and the alley.

The title to this property will be conveyed in about three weeks.

As I construe the Constitution of the Academy, I do not think that any formal proceedings with reference to this purchase can be taken until the stated meeting in October.

Very truly yours,

FRANCIS BLAKE.

On motion of the Recording Secretary, it was

Voted, That the Academy ratify the action of the Finance Committee in purchasing for thirty thousand dollars ($30,000) the property No. 28 Newbury St.

The Committee on Accommodations for the Academy appointed in June made the following report: —

REPORT OF THE COMMITTEE ON ACCOMMODATIONS FOR THE ACADEMY.

Since the Committee on providing quarters for the Academy was appointed on the 8th of June last, the Treasurer has purchased in the name of the Academy the estate numbered 28 Newbury Street. This comprises a dwelling house on the south side of the street, measuring 25 feet by 56 feet, with an open yard in the rear measuring 25 feet by 34 feet.

This house was built about the year 1870 and is in very good repair. It comprises four stories of good height, besides a basement, in which are the kitchen, furnace, etc.

The Committee now beg to report that they find that very slight changes in the building itself will be needed in order to adapt it to the uses of the Academy, but that for the accommodation of the library, it is desirable to erect a book-stack in the rear, occupying a part of the yard. This will not injure the light in the building.

The Committee have consulted Messrs. Fehmer & Page, the architects of the building, and are informed by them that such a book-stack as is needed will cost about $15,000. It will be fire-proof, composing six stories, each about 7 feet high, with ample staircases, and passageways, and would be convenient of access to the principal stories of the house. It will be exceptionally well lighted, the eastern side being almost entirely of glass. There will be accommodation for about 50,000 volumes and there will be room besides for 5,000 or 10,000 more in book-shelves distributed through the main building.

The two lower stories of the house, containing four large rooms, would be ample for the Office and Reading Rooms, and for the meetings of the Academy. The two upper stories afford ample accommodation for the Assistant Librarian, besides a certain amount of storeroom.

By the expenditure of $2,000 or $3,000 in addition, a larger meeting-room for the Academy, measuring about 25 feet by 30 feet, could be provided in the second story, if at any time it should be deemed desirable.

W. W. Goodwin.
Wm. Watson.
A. Lawrence Rotch.

On motion of Professor W. E. Story, it was

Voted, To accept the report.

After some discussion relating to the library, on the motion of Professor A. G. Webster, it was

Voted, That a committee of five be appointed to consider the whole subject of library accommodations and to report at the next meeting.

The following Committee was then appointed by the President: Francis Blake, William R. Ware, A. Lawrence Rotch, Eliot C. Clarke, Charles P. Bowditch.

On the motion of the Recording Secretary, it was

Voted, To meet on adjournment December 14, 1904.

The following communication was given by Professor M. H. Morgan: —

"The Stature of the Ancient Romans."

Nine hundred and fifty-third Meeting.

Adjourned Stated Meeting. — December 14, 1904.

The President in the chair.

The Chair announced the following deaths: —

Louis Palma di Cesnola, Associate Fellow in Class III., Section 4; Thomas Messinger Drown, Associate Fellow in Class I., Section 3.

The Recording Secretary read the following letter from the Recording Secretary of the Massachusetts Historical Society: —

WALTHAM, Dec. 12, 1904.

DEAR MR. WATSON, —

At the last meeting of the Mass. Historical Society it was *Voted* —

"To allow the American Academy of Arts and Sciences to use the rooms in our building, which they at present occupy, *without charge* till their removal."

Yours truly,

EDWARD J. YOUNG, *Recording Secretary.*

On motion of C. P. Bowditch it was

Voted, That the thanks of the Academy be tendered to the Massachusetts Historical Society in grateful acknowledgment of the favor to the Academy, in allowing the use of the rooms it now occupies, free of charge, until removal.

The following report of the Committee on House and Library Accommodations at 28 Newbury Street was read: —

The Committee appointed on the 9th of November to examine and report upon the house No. 28 Newbury Street, recently bought by the Academy, have visited the house, and consulted with the architects who built it in regard to the nature and cost of the necessary alterations and constructions. They find that even if the building could be made fire-proof, which it cannot, the floors are not strong enough to sustain the weight of the library; but that a fire-proof book-stack can be built in the rear of the house without injuriously darkening it. They find also that the two lower stories are excellently adapted for the uses of the Academy. These afford on the lower floor a room, in front, for the Secretary, and in the rear a reading-room, lighted by south windows and in immediate connection with the book-stack, in which might be placed the most recent additions to the library and current numbers of periodicals. In the second story is a room, handsomely finished in oak, which would serve as a council chamber, and here might be placed, perhaps, encyclopaedias and other works of reference. Behind it, towards the south, is a room for meetings, which by the removal of some closets can be made 30 feet long. The book-stacks would contain the library of the Academy, although a considerable number of books, such as, if destroyed, could easily be replaced, can properly be put upon shelves in the reading-rooms and meeting-room.

The Academy cannot itself use the two upper stories, and these can be occupied by a janitor, or care-taker, or leased, as may prove most advisable.

The cost of the book-stack will be about $15,800; that of altering and furnishing the two lower stories will be about $3,000. To this should be added $2,500 for the cost of moving, architects' fees, and other incidental expenses, making altogether about $21,300.

We accordingly recommend that the Treasurer be authorized to expend for these purposes a sum not exceeding $25,000, under the direction of the President, acting with the advice and consent of the Recording Secretary and Librarian, as soon as the resources of the Academy permit; and that if it is necessary to defer the building of the *book-stack*, the library be meanwhile properly stored, but that the house itself be fitted for occupation as soon as possible. Also that he be authorized to fit up and lease the two upper stories, if it prove best to do so.

The suggestion that the books of the Academy should be deposited with the Boston Public Library, or stored elsewhere, does not commend itself to the judgment of the Committee.

William R. Ware,
Chairman for the Committee.

December 9, 1904.

It was then

Voted, That the foregoing report of the Committee be accepted, placed on file, and that the Committee be discharged.

After a lengthy discussion participated in by Messrs. Bowditch, Clarke, Davis, Denny, Goodwin, Hall, Lanman, Mark, Sabine, Sharples, Watson, and Webster, it was

Voted (by a majority of twenty-three to two), That the Treasurer and Committee of Finance be authorized to expend a sum not exceeding twenty-five thousand dollars ($25,000) under the direction of the President, acting with the advice and consent of the Recording Secretary and Librarian, for the purpose of building a book-stack on the estate No. 28 Newbury Street, and of putting the premises in a suitable condition for occupancy by the Academy, and by others.

It was also

Voted, That the Treasurer and Committee of Finance be authorized to lease the first, third, and fourth floors of the building No. 28 Newbury Street, and such portions of the basement as may not be needed for the use of the Academy.

The following paper was read by Professor W. H. Pickering: "Section of a Crater formed naturally in Slag (with Model)."

The following papers were read by title: —

"A Machine for Averaging Curves for the Purpose of Harmonic Analysis." By H. H. Clayton.

"The Shoal-Water Deposits of the Bermuda Banks." By Henry B. Bigelow. Presented by E. L. Mark.

"Gas Turbine Cycles on the PQ Plane." By Harvey N. Davis. Presented by John Trowbridge.

"The Viscosity of Air." By J. L. Hogg. Presented by John Trowbridge.

"On a Measurement of the Thomson Thermo-Electric Effect in Soft Iron." By Edwin H. Hall.

Nine hundred and fifty-fourth Meeting.

STATED MEETING. — JANUARY 11, 1905.

The Academy met, by invitation of Professor D. G. Lyon, at the Semitic Museum, Cambridge.

The PRESIDENT in the chair.

On motion of C. R. Cross, it was

Voted, That a sum not exceeding two hundred and fifty dollars ($250) be appropriated from the income of the Rumford Fund, subject to the order of the Rumford Committee, for the preparation and distribution of a pamphlet relating to the Rumford Fund and its uses.

A communication, illustrated by lantern projections, on the Semitic Museum, was given by Professor D. G. Lyon.

The following papers were read by title: —

"On Isothermal Systems of Conics with Parallel Axes." By B. O. Peirce.

"On the Magnetic Properties of Hardened Cast Iron." By B. O. Peirce.

"The Atomic Weight of Strontium." By Theodore W. Richards.

Nine hundred and fifty-fifth Meeting.

FEBRUARY 8, 1905.

The Academy met at Ellis Hall.

The PRESIDENT in the chair.

The Corresponding Secretary read the announcement of the opening of the competition for the De Candolle Prize, for the best monograph on a genus, or a family of plants.

The following communications were read: —

Biographical Notice of the late George Frisbie Hoar. By T. W. Higginson.

"Swan Island in the Caribbean Sea, with a List of Plants found upon it." By S. P. Sharples.

"A Machine for Averaging Curves for the Purpose of Harmonic Analysis." By H. H. Clayton.

The following papers were presented by title: —

"Temporal Clauses in Herodotus." By H. D. Brackett. Presented by H. W. Smyth.

Contributions from the Gray Herbarium of Harvard University. New Series, No. XXIX. "New Plants from the Islands of Margarita and Coche, Venezuela." By J. R. Johnston. Presented by B. L. Robinson.

Nine hundred and fifty-sixth Meeting.

STATED MEETING. — MARCH 8, 1905.

The PRESIDENT in the chair.

The Corresponding Secretary read letters from John R. Freeman, "On the Subsidence of the Land in Boston Harbor and

Vicinity," accompanied by a pamphlet and newspaper clippings suggesting the appointment of a committee to investigate the subject; from the Royal Academy of Turin, containing the program of the fifteenth Bressa prize; from the Macon Academy, announcing a competition for prizes, on the occasion of its one hundredth anniversary; from Mme. Cornu, transmitting the collected works of her husband, M. Alfred Cornu.

The Chair announced the following deaths: —

George Sewall Boutwell, Resident Fellow in Class III., Section 4; Alpheus Spring Packard, Associate Fellow in Class II., Section 3.

The Chair appointed the following retiring Councillors to act as Nominating Committee: —

Arthur G. Webster, of Class I.
Edward L. Mark, of Class II.
Arlo Bates, of Class III.

Mr. Eliot C. Clarke proposed that Chapter VII. of the Statutes be amended by adding to it the following clause: —

5. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts as such Assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, Massachusetts, as agent to perform his duties, the compensation of such agent to be paid from the funds of the Academy.

In accordance with the Statutes, this proposed amendment was referred to a Committee of one, — Professor Charles R. Cross.

On the recommendation of the Rumford Committee, it was

Voted, To appropriate the sum of five hundred dollars ($500) from the income of the Rumford Fund to Harry W. Morse in aid of his researches on Fluorescence.

The following papers were read: —

"On Light of very Short Wave-Length." By Theodore Lyman.

"On the First Observations obtained with Registration Balloons in America." By A. Lawrence Rotch.

The following papers were read by title: —

"The Extension to Hyper Complex Number Systems in general of the Quarternion Scalar Function." By Henry Taber.

"Dispersion in Electric Double Refraction." By Howard L. Blackwell. Presented by John Trowbridge.

"New Methods of Determining the Specific Heat and the Reaction-Heat of Solutions." By T. W. Richards and A. B. Lamb.

"The Elimination of Thermometric Lag and Accidental Loss of Heat in Calorimetry." By T. W. Richards, G. S. Forbes, and L. J. Henderson.

"The Coefficient of Expansion of Hydrogen and Carbon Dioxide under Constant Pressure." By T. W. Richards and K. L. Mark.

"Autocatalytic Decomposition of Silver Oxide." By G. N. Lewis. Presented by T. W. Richards.

Nine hundred and fifty-seventh Meeting.

April 8, 1905.

The Academy met in Ellis Hall.

The President in the chair.

The Corresponding Secretary read the following letters: From the Société de Géographie Commerciale de Bordeaux, giving the conditions of the competition for the Saint-Laurent prizes; from the Académie des Sciences, Agriculture, Arts et Belles Lettres d'Aix, relative to the Thiers prize; from the Royal Hungarian Society, inviting the botanists attending the Botanical Congress to be held at Vienna, in June, 1905, to make an excursion in the territory of Hungary; from Arnold Samuelson, of Hamburg, on the subject of the "Resistance of the Air and the Question of Flying"; from the Natural Science Society of Schleswig-Holstein, inviting delegates to take part in the celebration of its fiftieth anniversary, June 17 and 18, 1905; from C. H. Wind, announcing his appointment as Director-in-Chief of the Institut Météorologique royal des Pays-Bas.

The following communications were given: —

Professor William M. Davis. "The Double Unconformity in the Colorado Canyon."

Mr. Hiram F. Mills. Biographical Notice of the late Charles Storer Storrow."

The following papers were read by title: —

"The Relative Positions of the Arc and Spark Lines of the Spectra of Titanium and Zinc." By Norton A. Kent. Presented by Charles R. Cross.

Contributions from the Gray Herbarium of Harvard University. New Series — No. XXX. "A Revision of the Genus Zexmenia." By W. W. Jones. Presented by B. L. Robinson.

Contributions from the Gray Herbarium of Harvard University. New Series — No. XXXI. I. "Descriptions of Spermatophytes from the Southwestern United States, Mexico, and Central America." By J. M. Greenman. II. "Diagnoses and Notes relating to American Eupatorieae." By B. L. Robinson. Presented by B. L. Robinson.

Nine hundred and fifty-eighth Meeting.

ANNUAL MEETING — MAY 10, 1905.

The Academy met in Ellis Hall.

The PRESIDENT in the chair.

The Corresponding Secretary read the following letters: From William F. Ganong, resigning Fellowship; from Professor Curtius and others, soliciting contributions for a monument to Robert Bunsen, at Heidelberg; from the American Chemical Society, transmitting the program of its thirty-second general meeting.

The Chair announced the death of James C. Carter, Associate Fellow in Class III., Section 1.

The annual report of the Council was read.*

The annual report of the Treasurer was read, of which the following is an abstract: —

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* See p. 761.

General Fund.

Receipts.

Balance from last year		$405.09
Investments	$6,045.93	
Assessments	970.00	
Admission fees	20.00	
Sale of publications	75.06	7,110.99
		$7,516.08

Expenditures.

General expenses	$2,622.79
Publication expenses	2,790.89
Library expenses	1,517.01
Balance	585.39
	$7,516.08

Rumford Fund.

Receipts.

Balance from last year		$557.68
Investments	$2,546.98	
Sale of publications	3.75	2,550.73
		$3,108.41

Expenditures.

Researches		$1,650.00	
Medals		339.00	
Publications		697.92	
Special pamphlet		12.00	
Miscellaneous		20.00	
Library. — Subscriptions	$131.10		
Books	50.74		
Binding	16.45	198.29	2,917.21
Balance			191.20
			$3,108.41

WARREN FUND.

Receipts.

Balance from last year	$492.05
Investments	330.08
	$822.13

Expenditures.

Researches	$155.00
Balance	667.13
	$822.13

BUILDING FUND.

Receipts.

Balance from last year	$389.90
Investments	142.33
Balance	$532.23

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

1904–5.

Nothing has been done during the year on the card-catalogue which was begun by Dr. Holden; the present Assistant Librarian having devoted much time and personal interest to making shelf-lists of the books, and to preparing the library for removal with the least disturbance to the present arrangement of the books. When the library is installed at 28 Newbury Street, the catalogue will be continued.

More binding than usual has been done this year in order to put the books into as good condition as possible for removal. The library was never in better condition in this respect.

Many missing parts of volumes or sets have been obtained, and others will be sent for during the ensuing year, thereby making the sets of publications of societies as complete as possible.

The accessions during the year have been as follows: —

	Vols.	Parts of Vols.	Pams	Maps.	Total.
By gift and exchange	188	1884	200	4	2276
By purchase — General Fund	4	717			721
Rumford Fund	20	370	1		391
Total	212	2961	201	4	3388

17 volumes and 1 pamphlet on light and heat, recommended by the Rumford Committee, were purchased at an expense of $50.74.

92 books have been borrowed from the library by 27 persons, including 9 Fellows. The library has been consulted about seventy-five times when no book was taken out.

Of the books borrowed, all have been returned for the annual examination except 8. Of the 11 volumes reported not returned a year ago, one was lost and has been replaced; another lost will be replaced shortly; all others have been returned.

The expenses charged, to the library are as follows: — Miscellaneous, $499.97; Binding, $526.95, General, and $16.45, Rumford Funds; Subscriptions, $490.09, General, and $120.60, Rumford Funds; making a total of $1517.01 for the General, and $147.05 for the Rumford Funds.

Although the appropriation from the General Fund for books and binding was exceeded by $117.01, the appropriation of $100 for cataloguing was not expended, making the net excess but $17.01.

The expense of distributing the Proceedings amounted the past year to $216.40.

The printing bill (not including the Manila envelopes for the Proceedings) was $107.50.

An appropriation of $1500 from the income of the General Fund is requested, $900 of which is to be used for subscriptions and binding, and $600 for miscellaneous expenses. Also the customary appropriation of $150 from the income of the Rumford Fund is requested for the purchase of periodicals and for binding.

A. LAWRENCE ROTCH, *Librarian.*

REPORT OF THE RUMFORD COMMITTEE.

The Chairman of the Rumford Committee respectfully presents the following report upon its work of the last year: —

The following grants have been made by the Committee out of the appropriation of $1000 put at its disposal at the last annual meeting.

Oct. 12, 1904. Three hundred and fifty dollars ($350) to Professor

R. W. Wood in aid of his research on the optical and other physical properties of sodium vapor.

Oct. 12, 1904. One hundred dollars ($100) to Professor N. A. Kent as an addition to a former appropriation in aid of his research on the critical conditions influencing electric spark lines.

Oct. 12, 1904. One hundred and fifty dollars ($150) to Professor A. L. Clark as an addition to a former appropriation in aid of his research on the molecular properties of vapors in the neighborhood of the critical point.

Jan. 11, 1905. Two hundred dollars ($200) to Professor D. B. Brace as an addition of a former appropriation in aid of his research on electric double refraction in gases.

Feb. 8, 1905. One hundred and fifty dollars ($150) to Professor C. B. Thwing in aid of his research on the thermo-electromotive force of metals and alloys.

Feb. 8, 1905, it was voted to request the Academy to appropriate five hundred dollars ($500) from the income of the Rumford Fund to Dr. H. W. Morse of Harvard University in aid of his research on fluorescence. The appropriation was granted by the Academy.

It was voted at the meeting of April 12, 1905, to request the Academy to appropriate from the income of the Rumford Fund the sum of two hundred dollars ($200) to Professor J. Trowbridge, Rumford Professor of the Application of Science to the Useful Arts in Harvard University in aid of a research on the electric double refraction of light.

At the same meeting it was voted to request the Academy to appropriate from the income of the Rumford Fund the sum of two hundred dollars ($200) to Professor E. H. Hall of Harvard University in aid of his research on the thermal and thermo-electric properties of iron and other substances.

In considering possible ways of increasing the usefulness of the Rumford Fund the Committee came to believe that it was desirable to publish a pamphlet regarding its character and the scientific results attained by its aid which could be distributed among scientific men. At the request of the Committee, the Academy has made an appropriation for the purpose from the income of the Fund, and the publication is now in press.

At the meeting of April 12, 1905, the Committee voted: —

(1) To request the Academy to appropriate the sum of $1000 from the income of the Rumford Fund for the immediate needs of the Committee in furtherance of research.

(2) To request the Academy to appropriate from the income of the

Rumford Fund the sum of $150 for the purchase and binding of the usual periodicals.

(3) To recommend to the Academy to appropriate from the income of the Rumford Fund the sum of $50 for the purchase and binding of books relating to light and heat, and to reappropriate the unexpended balance of the appropriation made at the last annual meeting for the same purpose.

(4) To continue the appropriation of $250 made from the income of the Rumford Fund for the preparation and distribution of a pamphlet relating to that Fund.

In accordance with the customary request of the Committee, reports on the progress of their researches have been received from the following persons: C. Barus, F. L. Bishop, D. B. Brace, A. L. Clark, E. B. Frost, G. E. Hale, G. F. Hull, W. J. Humphreys, N. A. Kent, F. A. Laws, R. S. Minor, E. W. Morley, E. C. Pickering, T. W. Richards, R. W. Wood.

Since the last annual meeting papers have been published as follows at the expense of the Rumford Fund : —

R. W. Wood. "A Quantitative Determination of the Anomalous Dispersion of Sodium Vapor in the Visible and Ultra-Violet Regions."

F. A. Saunders. "Some Additions to the Arc Spectra of the Alkali Metals."

R. W. Wood. "The Achromatization of Approximately Monochromatic Interference Fringes by a Highly Dispersive Medium, and the Consequent Increase in the Allowable Path-Difference."

H. N. Davis. "A *PQ* Plane for Thermodynamic Cyclic Analysis."

T. W. Richards and A. B. Lamb. "New Methods of Determining the Specific Heat and the Reaction-Heat of Liquids."

G. N. Lewis. "Autocatalytic Decomposition of Silver Oxide."

The Committee has considered the claims of various persons as suitable candidates to receive the Rumford Premium, but is not prepared to recommend an award.

CHAS. R. CROSS, *Chairman.*

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee beg leave to report that on account of the small amount of money at their disposal during the past year, they deemed it wise not to make any grants during the year for research work in chemistry. The amount of money now at the disposal of the Committee for the current fiscal year is, according to the account of the Treasurer, six hundred sixty-seven and $\frac{13}{100}$ dollars ($667.13); and the Committee ask that six hundred dollars of this sum be placed at their disposal for use in the furtherance of chemical research.

LEONARD P. KINNICUTT, *Chairman.*

REPORT OF THE COMMITTEE OF PUBLICATION.

During the year beginning May 1, 1904, and ending April 30, 1905, there were published four numbers (21 to 24 inclusive) of the Proceedings, Volume XXXIX., and the first twenty-two numbers of Volume XL., in all 885 pages, 7 plates, and 7 maps and charts. Two biographical notices were published. Six numbers of the Proceedings (Volume XXXIX., No. 21; Volume XL., Nos. 6, 9, 16, 19, and 20) were paid for by income from the Rumford Fund.

Of the Memoirs, Volume XIII., Part II, 58 pages, and 2 plates, was issued in November, 1904. By an oversight the publication of Volume XIII., Part I, January, 1904, was not announced in the last annual report.

The unexpended balance of appropriations from the General Fund for publications on April 30, 1904, was $594.38; the appropriation for the year 1904–1905, $2,400; and the receipts from sales of publications, $75.06, making the whole amount available for publications, $3,069.44. The total expenditures for publication during the year now ended have been, from the Rumford Fund $697.92, from the General Fund $2,790.89. The unexpended balance from the General Fund is therefore $278.55.

E. L. MARK, *Chairman.*

May 10, 1905.

On the recommendation of the Committee of Finance, it was

Voted, To make the following appropriation from the income of the General Fund for expenditures during the ensuing year: —

For Publishing		$2,400
For the Library		
Books and Binding	$900	
Miscellaneous	600	1,500
		$3,900

On the recommendation of the Rumford Committee, it was

Voted, To make the following appropriations from the income of the Rumford Fund: —

For the immediate needs of the Committee in furtherance of research, $1,000.

For the purchase and binding of the usual periodicals, $150.

For the purchase and binding of books relating to light and heat, $50.

For the same purpose, the unexpended balance of the appropriation made at the last annual meeting

For the preparation and distribution of a pamphlet relating to that Fund, the continuance of the appropriation of $250.

To Professor John Trowbridge, in aid of a research on the electric double refraction of light, $200.

To Professor E. H. Hall, in aid of his research on the thermal and thermo-electric properties of iron and other substances, $200.

On the recommendation of the C. M. Warren Committee it was

Voted, To make the following appropriation from the income of the C. M. Warren Fund: —

For the use of the C. M. Warren Committee in furtherance of chemical research, $600.

On the motion of the Recording Secretary, it was

Voted, That the annual assessment for the ensuing year be ten dollars ($10).

The report of the Committee to whom the amendment of the Statutes was referred at the meeting of March 8, 1905, was accepted, and in accordance therewith, and on the motion of the Recording Secretary, it was

Voted, That Chapter VII. of the Statutes be amended by adding to it the following clause:

5. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts as such Assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, Massachusetts, as agent to perform his duties; the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

The annual election resulted in the choice of the following officers and committees: —

WILLIAM W. GOODWIN, *President.*
JOHN TROWBRIDGE, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

WALLACE C. SABINE, of Class I.
WILLIAM G. FARLOW, of Class II.
EDWARD ROBINSON, of Class III.

Member of Committee of Finance.

ELIOT C. CLARKE.

Rumford Committee.

CHARLES R. CROSS, ERASMUS D. LEAVITT,
EDWARD C. PICKERING, AMOS E. DOLBEAR,
ARTHUR G. WEBSTER, THEODORE W. RICHARDS,
ELIHU THOMSON.

C. M. Warren Committee.

LEONARD P. KINNICUTT, ROBERT H. RICHARDS,
SAMUEL CABOT, ARTHUR M. COMEY,
ARTHUR MICHAEL, HENRY P. TALBOT.
CHARLES R. SANGER.

The following standing committees were appointed by the Chair: —

Committee of Publication.

SETH C. CHANDLER, of Class I., EDWARD L. MARK, of Class II.,
CRAWFORD H. TOY, of Class III.

Committee on the Library.

HARRY M. GOODWIN, of Class I., SAMUEL HENSHAW, of Class II.,
HENRY W. HAYNES, of Class III.

Auditing Committee.

HENRY G. DENNY, FREDERICK J. STIMSON.

On the motion of the Recording Secretary, the following resolution was unanimously adopted: —

Resolved, That the Fellows of the American Academy desire to place upon record their grateful appreciation of the services of the retiring Treasurer, Francis Blake, during the six years in which he has served them.

On the motion of the Chairman of the Finance Committee it was

Voted, To authorize the expenditure of fifteen hundred dollars ($1500), in addition to the twenty-five thousand dollars ($25,000), which was authorized by the Academy at the meeting of December 14, 1904, to be expended for the purpose of building a book-stack on the estate No. 28 Newbury Street, and of putting the premises in a suitable condition for occupancy by the Academy, and by others.

The following gentlemen were elected members of the Academy: —

Samuel Williston, of Belmont, to be a Resident Fellow in Class III., Section 1 (Philosophy and Jurisprudence). Nominated by James Barr Ames, Jeremiah Smith.

Charles E. Munroe, of Washington, D. C., to be an Associate Fellow in Class I., Section 3 (Chemistry), in place of the late Thomas M. Drown.

The following communication was given, by Professor T. A. Jaggar: — "Some Results of the Study of the Volcano Pelée."

The following papers were presented by title: —

"On the Effects of Sudden Changes in the Inductances and the Resistances of Electrical Circuits." By B. O. Peirce.

"On a Simple Device for Reading the Deflections of a Mirror Galvanometer." By B. O. Peirce.

"The Luminescence of Fluorspar." By Harry W. Morse. Presented by John Trowbridge.

"An Experiment to detect the Fitzgerald-Lorentz Effect." By E. W. Morley and D. C. Miller.

"On Surface Tension at the Boundary between Liquids and Vapors." By Arthur L. Clark. Presented by A. G. Webster.

AMERICAN ACADEMY OF ARTS AND SCIENCES.

Report of the Council. — Presented May 10, 1905.

BIOGRAPHICAL NOTICES.

George Frisbie Hoar

Charles Storer Storrow

Thomas Wentworth Higginson.

Hiram F. Mills.

REPORT OF THE COUNCIL.

The Academy has lost eight members by death since the last report of the Council, May 11, 1904: four Resident Fellows, James M. Barnard, Samuel L. Abbot, George F. Hoar, and George S. Boutwell; four Associate Fellows, T. M. Drown, A. S. Packard, James C. Carter, and L. P. di Cesnola.

One Resident Fellow has resigned during the year.

No new members have been elected.

The roll of the Academy now includes 194 Resident Fellows, 96 Associate Fellows, and 73 Foreign Honorary Members.

GEORGE FRISBIE HOAR.

I BELIEVE it is Howells who somewhere remarks that one may easily hear two Bostonians at a club discussing a third person with the greatest freedom, after which one may be heard casually remarking to the other, "He is a cousin of mine." I might truly say the same either of Senator Hoar or of Senator Lodge, both of whom were descended from Francis Higginson, the early Salem clergyman. In case of Senator Hoar, I was some six years older and knew him first as a boy in college, he reaching Harvard College as I left; then I knew him as a young lawyer in Worcester, where we were fellow townsmen for a dozen years or so, and finally encountered him in relation to public questions, where we alternately agreed and differed for many years. The fact that some months have gone by since his funeral diminishes the peril of that merely good-natured praise which is the general instinct of our friendly nation when a prominent man passes away. Happening to be in England during the exciting days which followed the murder of President McKinley, I was constantly called on to explain how it was that the repeated criticism made upon him in America up to that time was so instantly followed by a chorus of what seemed rather undiscriminating praise, and I could only say that such was our American temperament that with us the applause came first and the more discriminating criticism afterwards.

George Frisbie Hoar was born at Concord, Mass., August 29, 1826, being the son of Samuel Hoar, a prominent lawyer, who was a Whig member of Congress from 1835 to 1837, and was sent in 1844 by the Massachusetts legislature to South Carolina on an unsuccessful mission before the courts of that Southern State. George Frisbie Hoar's eldest brother Ebenezer Rockwood, had also been a member of Congress in 1873–75 and had been attorney-general of the United States. His grandfather and great-grandfather were both in the Concord fight, and his maternal ancestor, Roger Sherman, signed the Declaration of Independence — he who, as Whittier has said, represented "the gentle craft of leather." Senator Hoar became a resident fellow of this academy May 8, 1901 (Class III. Sec. 1. Philosophy and Jurisprudence). He died of disease at Worcester, Mass., September 30, 1904.

The immediate family to which Mr. Hoar belonged has been for several generations the most influential household in the most influential village of eastern Massachusetts. Such an attitude produces a race of men who are the counsellors and friends of their fellow townsmen and the recognized authority as to the character and claims of humbler men. This decision may or may not be infallible. I remember when, after the death of Henry Thoreau, I had applied in vain to his sister for leave to publish something from his numerous volumes of journals and natural observations, now so largely in print, she had refused, and I applied confidently to Judge Hoar. When I got through, the judge said placidly, between puffs of his cigar, "Whereunto? You have not yet unfolded the preliminary question. Why should any one wish to have a sentence of Henry Thoreau's put in print?" I am confident that no one in Concord would have had the daring to ask that question in regard to any member of the Hoar family, and it is clear that the author of the two stout volumes of Senator Hoar's memoirs could at least have felt no such solicitude in regard to himself.

It embarrasses the historic position of a man, however eminent, whenever he has recently published his autobiography. It may vindicate him from some injustice, but is apt to place him at some disadvantage. I knew a man who inscribed in his library, above the department of biography, that fatal saying of Confucius, "How can a man be concealed?" In many cases this unconscious revelation has its merits, but it involves its perils. I think it was Senator Depew who once said with much keenness that every man ought to write his own epitaph, because no one else is usually so familiar with the virtues of the deceased. We recognize this in Senator Hoar's memoirs, when, speaking of his service in the Massachusetts legislature, he says, "I suppose I may say without arrogance that I was the leader of the Free Soil Party in each House when I was a member of it."* He leaves us glad to have the knowledge from high authority, but rather grateful that each one of his three hundred fellow members did not write his own biography in a thousand pages to impose on us the duty of reading them all. It rather reminds us of the young singer in the play who, after making her début, says with undoubted sincerity that she "and the orchestra" were very much applauded.

Those of us who were contemporaries of Senator Hoar in Harvard College turn naturally to the chapters in his book relating to that period, and find at the very outset some of this hasty generalization to which biography tempts. In the very first line of the chapter called "Harvard Sixty Years Ago" is a statement that Harvard had not changed much for nearly a century before he entered it. I graduated the same year that he entered, and can testify, on the other hand, that the basis of the whole transformation from the old prescribed system to the present elective system had been laid during those two or three years, in making mathematics an elective study, with the intention of making almost all other studies elective also, as they now are. His farther judgment in the matter can be estimated from the fact that the one department which he selects for censure and ridicule is that department of English which under Prof. Edward Tyrrel Channing — grand uncle of the present Edward Channing, Professor of History — gave Harvard the lead among all the academic rivals in America in the production of literary men. Channing's list of pupils, beginning with Emerson, included also Winthrop, Hillard, Holmes, Dana, Hildreth, Hedge, James Freeman Clarke, William Henry Channing, Rev. S. F. Smith, Sumner, Motley, the Adamses, the Lowells, the Peabodys, the Hales, the Quincys, President Hill, Francis Parkman, William Story, Fitzedward Hall, and finally Professor Norton; and yet this is the summary of it made by Senator Hoar. I myself, as one of Professor Channing's pupils, have been a literary man by profession for half a century, and it has sometimes seemed to me that forty-eight hours have never passed without there occurring to me, as a piece of second nature, some maxim of that teacher. I do not believe that there has occurred in American educational history a teacher so remarkable, all things considered. It was true, as Mr. Hoar cruelly remarks, that he had a figure somewhat like Punch, though I should say with a noble head above it.

The explanation of Senator Hoar's errors is comparatively simple.

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* Hoar's Autobiography, I. 163.

On looking through the Commencement Day programme for 1845, a programme which gave among the new features lately introduced the departments in which each graduate took honors, we find that Senator Hoar, whose Commencement part lay among the "disquisitions," the lowest grade of rank, was credited only with Latin and Greek as the basis of his rank, such as it was. Now all experience indicates, I believe, that the department of study which a man thinks really important is that one for which he himself has a fancy. Some of you may have known, years ago, the German, Baron Osten Sacken, the celebrated entomologist, stationed in Cambridge for a time, who always complained that he had made a fatal mistake in his career through rashly taking the whole of the diptera, or two-winged insects, for his scientific task, whereas to take charge of a single genus would have been enough, he thought, for the life-work of a judicious man of science; personally he should have preferred the mosquito.

After leaving the Law School, Mr. Hoar established himself in Worcester, Mass., then a city of remarkable activity of mind and independence of thought. Going there myself four years later, I recognized him, of course, as one of the leaders of the young men, though less conspicuously so, I think, than he seems to himself to have been, his narrative still rather omitting the orchestra. I remember clearly that Mr. John Milton Earle, editor of the "Worcester Spy," and then recognized as the leader among the older men in politics and journalism, placed others among the young lawyers, especially the one who afterwards became Judge Nelson, far above Hoar. This of course lies beyond my province. But the defect I find in Senator Hoar's picture of that day is that he overlooks the most remarkable historic transformation, I think, which ever occurred in a Massachusetts town.

Up to about that time Worcester had simply afforded a fine example of the old-fashioned, high-bred, well-organized Massachusetts community. It was ruled with dignity by what was called the Chandler blood, — the Lincolns, the Salisburys, the Waldos, and one or two kindred households. In a short time a change passed over it, almost as if by magic, with the enormous influx of manufacturers. The Abolitionists, the temperance zealots, the social reformers generally, began to congregate there. From being another Quincy it became another Lynn. The Public Library sprang up and a lecture system, a Natural History Society, a gymnasium; the young men of the town became athletic and interchanged cricket games and baseball games with other towns. The neighborhood of those fine old radicals, Stephen and Abby Foster, made it a centre for anti-slavery meetings and fugitive slave cases. Abby Kelly Foster was at that time the pioneer woman-speaker in New England, and Stephen Foster was one of the best known farmers of the neighborhood. His farm was originally of very hard soil and later was made into a fertile one by his efforts, and once when I asked him why he did not take a farm in a Western state instead, he remarked that he should hate to put his spade into the ground and not have it strike against a rock. Yet there is no mention in these two thick volumes of such stalwart neighbors as these. All this social upheaval was going on in the early days of Mr. Hoar's residence there. The peculiarity of the case is that one may look through the two large volumes of his memoirs and scarcely find allusion to all these curious phenomena. Indeed, he says at the outset (I. 159) that there was "little pride of family or wealth in Worcester," when he went there; thus omitting the key to the whole situation.

It was in 1868 that Senator Hoar began his more national public career by being nominated to the lower house of Congress from Worcester. I remember that on meeting his elder brother, Judge Hoar, in a smoking car the next day, I congratulated him on the result of the caucus. He replied that it was peculiarly satisfactory from the fact that all the modes of manœuvring supposed to belong to politics had been entirely prohibited by his brother, whereas Mr. Francis W. Bird, his leading opponent, was well known as a past-master in them. Being somewhat acquainted with the ways of political life in Worcester County, I felt some misgiving at the completeness of this praise, and having an opportunity, not long after, to meet Mr. William W. Rice, George Hoar's brother-in-law, who had personally conducted the campaign, I asked him frankly whether he thought the difference was really so great. He answered with a twinkle in his eye, "Well, I guess there was not much left undone on either side." I must say candidly that I think the same allowance is to be made throughout Senator Hoar's personal memoirs; you can rely on having in full his own point of view on what concerns himself, but you are not quite sure of the rest. He represented, undoubtedly, in this respect the whole tone of older politicians, yet he, too, when he wished to carry a point, did not leave much undone.

A cordial friend and warm admirer and vindicator of all his friends, he was as cordial a hater. To those who took part in the great secession from the Republican Party which put President Cleveland for the first time in office, he held a bitterness of antagonism hardly equalled among Republican leaders, and the slang word "Mugwump," which had but a short-lived run in the newspapers, became to him a permanent weapon. To the latest day of his life he applied it to all who took part in that outbreak, and seemed to regard it as in itself a sufficient condemnation.

This was also true, though not so largely, of his relation to all Democrats. It was the general testimony of General Butler's friends, for instance, that he was helped and not hindered in his political efforts by the unmerciful and undiscriminating manner in which both Senator Hoar and his brother Judge Hoar attacked him in conventions; and one has only to read the long chapter on this subject in Senator Hoar's autobiography to see how largely this was a matter of temperament and an advocate's argument, not a judge's summing up. He does some slight justice to the real services of Butler's prompt adhesion to the war side, carrying the Democratic Party with him in Massachusetts; and, on the other hand, while bringing out, as was fair enough, the general tone of trickiness which spoiled that demagogue's course, he reserves his chief and overwhelming condemnation for Butler's retreat from his first attack on Petersburg, though a little wider inquiry would have satisfied him that this was one of the few occasions where that general was probably right. I know that Col. William Lamb, who commanded Fort Fisher until its fall, always testified that Butler himself was entirely justified in this, that the resources in his hands — especially the naval forces — were wholly inadequate to success, and that the general who followed him was more fortunate simply because he was better provided. Should General Butler ever attain to a statue in the Boston State House, it will be owing largely to the unfairness of a class of bitter opponents of which the Hoar brothers were the leaders.

These were the perils that belonged to Mr. Hoar's temperament. His long and honorable course of independence is nowhere so well exhibited as in an article by Mr. Talcott Williams of Philadelphia, one of the very ablest of American writers, in the "American Monthly Review of Reviews" for November, 1904. Mr. Williams, the first writer to place Mr. Hoar's claims at the highest, is also the first to speak with perfect frankness of his personal faults. He says, "He had many faults. Quick-tempered, he had the impatience over slower and more pliable men frequent in those of high intellectual powers. In his early years of service, he had his share of egotism, not unnatural. He was not at his best in making it easy for his colleagues to get on with him, and he lacked in tact, affronting men by a lofty superiority, to himself unconscious, and to other men sometimes seeming to be self-conscious. He displayed, in short, and particularly before he had reached the full stature of his statesmanship, just the qualities which should have alienated support. He outgrew these faults, as he showed best by his jests about them."

All this is true and candid. Mr. Williams's error comes, I think, in assuming it as the highest merit attributable to a public man that he had invariably stood by his party in the end, however he might criticise it. Not such was the standard of merit of the earlier teachers, the Conscience Whigs, who first taught Senator Hoar the path of freedom. Charles Sumner, Charles Francis Adams, John G. Palfrey, — these were the men who first brought him into public life by showing the higher strength required to forsake one's party. In the first impulse of youth he had done this, but in later life he showed it to be often, in the long run, impossible for him to make the required sacrifice. So far as his speeches went, no one can doubt that they had plenty of pluck in them.

A man, after all, judges himself in his autobiography. It will one day be recognized that in an age of great self-devotion and great sacrifices, Senator Hoar made many small sacrifices, but no large ones; and was amply rewarded by filling in return, for the greater part of his active life, the position held highest by all American public men, — higher than even that of President, as being more permanent. It is absurd to compare his career for an instant with that of Sumner in respect to sacrifices, any more than in regard to scholarship; or with the lifelong sacrifice made by a man like Wendell Phillips, for whom Mr. Hoar frankly expressed his want of sympathy. Wendell Phillips was incomparably the superior of all around him in requisite ability for public life. There were occasions when there was needed only the slightest sacrifice of principle in him to be the head of a successful party and to be launched in that official career for which every attribute in him except conscience longed. I myself know by his own testimony that it was for this public life alone he yearned, but he forbore it. To those who knew the period and place which Mr. Hoar's autobiography describes, there is something almost pathetic in the types of men whom he holds up to admirers in Worcester — second-rate politicians, of whom he makes heroes — and, on the other hand, in the importance attached to the rescue of a frightened slave-catcher by a gathering of men, half in play, who piloted him to the railway station.

The severe test usually came later with Senator Hoar as the moment for voting drew near. Meeting with direct and unequivocal opposition, he was only strengthened by it. Reiterate it, and he sometimes came to resemble his elder brother the judge, who was thus described in one of those piquant analyses in which the late Col. Henry Lee delighted, "strong, but would swear that black was white if contraried." No public man in America ever found himself in a more lamentable position than did Senator Hoar when Senator Foraker began to exhibit the "deadly parallel" between the Massachusetts senator's Panama speech of December 17, 1903, and that of February 22, 1904. It recalled the statement long since attributed to President McKinley, "Senator Hoar is the man who furnishes our opponents with their arguments and us with his votes." "It was," said the "New York Evening Post," speaking of the Senator's change, "a moral eclipse visible all over the United States." "It was," said the New York Commercial Advertiser" (Republican), an exhibition "with the facile grace of lifelong practice of his 'right about face';" or, as the "Providence Journal" (Republican) more tersely said, "The worthy Massachusetts statesman once more went to dinner when the dinner bell rang." Not that he cared so much for the actual dinner, but it came hard to him to see his old table mates sitting down without him.

It sometimes happens that the best test of a public man throughout his life is not to be found in his main or official functions, but in collaterals, in matters where he was entirely free and could be judged by his wholly volunteer action. My own admiration for Mr. Hoar, for I have my share of admiration, has rested largely upon his willingness to take up unpopular causes which could yield him nothing, but rather led to criticism. His loyalty to Woman's Suffrage, for instance, brought him little more than censure or ridicule; it may have cost him votes; it is pretty certain that it yielded him none. The same was true of his unvarying justice for the Roman Catholic church and to the Irish-American race. It was repaid in a warm personal loyalty on the part of those whose cause he espoused, but his political success was in spite of these things, not because of them. It is quite needless, on the other hand, to reproduce the long list of his special public services. Most important of all these is the crowning fact that upon the great national question which dwarfed all others in his very latest years, — that of imperialism as shown in the Philippines, — he held his course with superb courage and was true to the last; being almost alone in the Senate in what he deemed the most momentous of all causes, that of saving the nation from dwindling into an empire. For this he might well be received, as he was, with applauses such as those amid which the ancient Roman senators marched forth from the city to meet the Consul Terentius Varro, after his army had been almost annihilated by Hannibal at the great battle of Cannæ.

The Roman senate went out to do him honor, Livy tells us, because, though he had lost all else, he had not despaired of the republic.

THOMAS WENTWORTH HIGGINSON.

CHARLES STORER STORROW.

CHARLES S. STORROW was my friend, and the friend of three generations of men with whom he was associated, inspiring them with the spirit of gentleness, truthfulness, honorable dealing, thoroughness, and a careful consideration of all sides of a subject that justice might be reached.

Of Massachusetts parentage, he was born March 25, 1809, during the temporary residence of his parents in Montreal. When he was about two years of age they returned to their former home in Boston, where he received his early education until 1818, when his father, Thomas Wentworth Storrow, having established himself in business in Paris, France, took his family there for permanent residence. There he attended private school until 1824, when he returned to America to prepare for Harvard College, where he was graduated first in his class in 1829.

Having determined to adopt the profession of civil engineering, and finding no school in this country where he could obtain the necessary education, he studied for some months in the library of the eminent engineer Loammi Baldwin, but in December he returned to France, and by the influence of General Lafayette, who was a close friend of his father's family, he got permission to take the course in the École des Ponts et Chaussées at Paris. Here he remained two years, taking the course, and also made a study of the important engineering works then in progress in and about Paris, and, before returning to America, made a tour for more than two months throughout Great Britain, mainly to study various works interesting to his profession, including the Liverpool and Manchester Railway, then recently completed by Stephenson.

Returning to Boston in April, 1832, he entered the engineering staff of the Boston and Lowell Railroad, of which Mr. James F. Baldwin was chief engineer. Constructive work was then beginning, and Mr. Storrow served in the engineer corps until the road was completed. When entering upon this work he was the only one engaged who had seen a locomotive.

On May 27, 1835, under his direction, the first train, drawn by the locomotive "Stephenson," carried a party of the leading stockholders from Boston to Lowell and back at the rate of twenty miles an hour. Soon after the road was opened to the public, and in 1836 Mr. Storrow was made the chief official of the road under the title of agent, and managed its affairs for the next nine years.

He had a keen appreciation of justness, and while he insisted that others should be just to the company which he controlled, he was careful that his company should be just to others. Soon after the road was opened to travel, a passenger entered a car after the seats were all occupied. He demanded of the conductor a seat, saying that his ticket entitled him to a seat and he must have one. The conductor told him there would probably be an empty seat a short distance out, but that he could not ask any one to get up and give him a seat; not satisfied, he rushed out of the car as it started, and complained to the depot master that the conductor would not give him the seat to which his ticket entitled him. The depot master took him to Mr. Storrow. The passenger complained excitedly of his treatment by the railroad. Mr. Storrow quietly heard him through, then, turning to the depot master, said, "This railroad was built for the accommodation of the public; this gentleman is entitled to a seat; you may heat up the other engine and take him to Lowell." In this spirit of regarding the rights of others did he begin the railroad business of New England.

While engaged upon the construction of the railroad, his leisure time was devoted to the improvement of his profession in America by preparing a "Treatise on Water Works for Conveying and Distributing Supplies of Water," published in 1835; first giving to English-speaking engineers the results of experiments made by the French engineers and their methods of solving problems on the flow of water in canals and in pipes, and including the discussion, in a very clear and complete manner, of the various problems arising in constructing water works supplied by gravity or by pumping.

On October 3, 1836, Mr. Storrow was married to Miss Lydia Cabot Jackson, daughter of Dr. James Jackson of Boston, and their home for nine years was in Boston.

On March 20, 1845, at the request of Mr. Abbott Lawrence, Mr. Storrow gave up the management of the Boston and Lowell Railroad and took charge of the affairs of the Essex Company, a land and water-power company organized to develop the water power of the Merrimack River for manufacturing purposes, and to lay out and begin the building of a city, which has become the city of Lawrence, with its seventy thousand inhabitants.

Mr. Lawrence was made president, and Mr. Storrow was made treasurer, engineer, and agent of the company. Mr. Storrow had a very broad appreciation of the work before him, and planned wisely for the long future. He designed and built the dam across the river, — an engineering structure of high merit, built in a manner much in advance of the time, consisting of granite masonry laid in cement upon ledge, giving a direct fall of 25 feet, with an overfall 900 feet in length, with wing walls rising 16 feet above the crest, and wing dams to this height extending 1800 feet farther to high land, which have served to protect the city from great freshets.

He designed the canals and laid out the mill sites, both well adapted to the full development of the power of the river. He built several of the large mills, and bore the responsibility of building and carrying on several manufacturing establishments. He laid out the city, planned and built broad streets lined with shade-trees, and provided several public parks, which were afterwards given to the city. He was active in establishing public schools of a high order, and helpful in establishing churches for all of the people, and in 1853, when the charter of the city was obtained, the people made him their first mayor.

Intimate association with Mr. Storrow, and recognition that his education and training qualified him in an unusual degree for directing great and diversified enterprises, caused Mr. Abbott Lawrence to consult with him as to what was needed in a school for educating engineers.

The remarkable letter of Mr. Abbott Lawrence of June 7, 1847, to the treasurer of Harvard College, presents his views of the pressing want in our country "of an increased number of men educated in the practical sciences," and offers a foundation for "a school, not for boys, but for young men whose early education is completed, either in college or elsewhere, and who intend to enter upon an active life as engineers or chemists, or, in general, as men of science, applying their attainments to practical purposes," and goes on to describe as few men, even among engineers then in the world, could have described the essentials of an engineering school having three departments, "1st, Engineering; 2d, Mining in its extended sense, including metallurgy; 3d, the invention and manufacture of machinery." He includes the important provision that "this school of science should number among its teachers men who have practised and are practising the arts they are called to teach." Mr. Lawrence urged Mr. Storrow to take the general direction of the Lawrence Scientific School and to accept its professorship of Engineering; but he was too earnestly engaged in the work in hand to undertake it.

For fifteen years, while details of his work required daily attention, Mr. Storrow resided in Lawrence, but in 1860 he moved his home to Boston.

In 1861 the Governor of the Commonwealth appointed Mr. Storrow one of the commissioners to determine the effect of the dam at North Billerica in flowing the Concord and Sudbury meadows.

In 1862 Mr. Storrow, at the request of the Commissioners of the Hoosac Tunnel, with the approval of the Governor and Council, visited Europe to examine the most important tunnels, and especially that one then in progress under the Alps between France and Italy; and on November 28, 1862, he made to the Commissioners a very complete report upon European tunnels, and bearing especially upon the method of constructing the Mont Cenis tunnel with the help of compressed air in drilling and ventilating, together with his conclusions in regard to the size and method of working to be adopted at the Hoosac Tunnel.

This model report of one hundred and twenty pages was published in Senate Document No. 93 of 1863. Following this, Mr. Storrow was consulting engineer on hydraulic work for this commission.

On December 11, 1876, he was appointed one of the park commissioners of the city of Boston, serving until his resignation on May 19, 1879, when he had become seventy years of age.

Mr. Storrow's generous consideration and kindness to his assistants, and to all in his service of whatever grade, together with his manner of expressing his interest, caused them universally to feel that he was the most gentlemanly of men.

Mr. Storrow continued actively engaged with the affairs of the Essex Company until he was eighty years of age, when he resigned, and the stockholders made up their record as follows:

"The retirement, upon reaching the age of eighty years, of our venerable president, after wisely leading the company from its organization for forty-four years, causes the stockholders to recall the eminent services Mr. Storrow has rendered to the Essex Company.

"For twenty-four years he was treasurer, agent, and chief engineer, carrying the important responsibilities of designing its works, directing their execution, and managing all of its business affairs.

"Thirteen years more he continued as treasurer and agent, directing its business management wisely and with remarkable success; and upon resigning these offices continued to serve the company as director and as president.

"The stockholders desire to express to him their gratitude, as well as their high appreciation of the honor and ability with which he has founded a prosperous city and carried this company through its years of struggle to prosperity."

For ability as an engineer and eminent service to the profession in America, Mr. Storrow was, in December, 1892, elected an Honorary Member of the American Society of Civil Engineers.

Mr. Storrow has been a Fellow of the American Academy of Arts and Sciences sixty-four years, having been elected January 29, 1840.

Always courteous and remarkably vivacious, with helpful cheeriness, happy in seeing his children and grandchildren in honorable and sometimes in distinguished positions in the community, he spent the latter years of his very long life in the quiet enjoyment of his home and his books, until his death on April 30, 1904, in his ninety-sixth year.

HIRAM F. MILLS.

American Academy of Arts and Sciences.

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FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to June 1, 1905.)

RESIDENT FELLOWS. — 194.

(Number limited to two hundred.)

Class I. — *Mathematical and Physical Sciences.* — 78.

Section I. — 18.

Mathematics and Astronomy.

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Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Wellesley Hills.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
James Mills Peirce,	Cambridge.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
Henry S. Pritchett,	Boston.
John Ritchie, Jr.,	Roxbury.
Edwin F. Sawyer,	Brighton.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
Harry W. Tyler,	Boston.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

Section II. — 24.

Physics.

A. Graham Bell,	Washington, D.C.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
George A. Campbell,	Boston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
Amos E. Dolbear,	Somerville.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Boston.
Theodore Lyman,	Brookline.
Benjamin O. Peirce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott
John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

Section III. — 20.

Chemistry.

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Charles L. Jackson,	Cambridge.
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Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	Jamaica Plain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.
Edward S. Wood,	Boston.

SECTION IV. — 16.

Technology and Engineering.

Alfred E. Burton,	Boston.
Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	New York.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Andrew H. Russell,	Manila.
Albert Sauveur,	Cambridge.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
George F. Swain,	Boston.
William Watson,	Boston.

CLASS II. — *Natural and Physiological Sciences.* — 65.

SECTION I. — 15.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
Charles Palache,	Cambridge.
John E. Pillsbury,	Boston.
Nathaniel S. Shaler,	Cambridge.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

SECTION II. — 11.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge.

SECTION III. — 24.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Charles B. Davenport,	Cold Spring Harbor, N. Y.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
John S. Kingsley,	Somerville.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

SECTION IV. — 15.

Medicine and Surgery.

Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
David W. Cheever,	Boston.
Frank W. Draper,	Boston.
Thomas Dwight,	Nahant.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

CLASS III. — *Moral and Political Sciences.* — 51.

SECTION I. — 10.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
John C. Gray,	Boston.
G. Stanley Hall,	Worcester.
Francis C. Lowell,	Boston.
Hugo Münsterberg,	Cambridge.
Josiah Royce,	Cambridge.
Jeremiah Smith,	Cambridge.
Frederic J. Stimson,	Dedham.
Edward H. Strobel,	Cambridge.
Samuel Williston,	Belmont.

SECTION II. — 22.

Philology and Archæology.

Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
Joseph T. Clarke,	Harrow, Eng.
Henry G. Denny,	Roxbury.
William Everett,	Quincy.
J. W. Fewkes,	Washington.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
Morris H. Morgan,	Cambridge.
Bennett H. Nash,	Boston.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	Boston.
Edward S. Sheldon,	Cambridge.
Herbert Weir Smyth,	Cambridge.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.
Edward J. Young,	Waltham.

SECTION III. — 10.

Political Economy and History.

Charles F. Adams,	Lincoln.
Edward Atkinson,	Brookline.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.
A. C. Goodell,	Salem.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 9.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
John Bartlett,	Cambridge.
Arlo Bates,	Boston.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
William R. Ware,	Milton.
Barrett Wendell,	Boston.

ASSOCIATE FELLOWS. — 97.

(Number limited to one hundred. Elected as vacancies occur.)

CLASS I. — *Mathematical and Physical Sciences.* — 37.

SECTION I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay,
S. W. Burnham,	Chicago. [Wis.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	Goshen, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Princeton, N. J.

SECTION II. — 6.

Physics.

Carl Barus,	Providence, R.I.
G. E. Hale,	Williams Bay, Wis.
S. P. Langley,	Washington.
T. C. Mendenhall,	Worcester.
A. A. Michelson,	Chicago.
E. L. Nichols,	Ithaca, N. Y.

SECTION III. — 9.

Chemistry.

Wolcott Gibbs,	Newport, R.I.
Frank A. Gooch,	New Haven.
Eugene W. Hilgard,	Berkeley, Cal.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
Charles E. Munroe,	Washington.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

SECTION IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York. [Va.
W. P. Craighill,	Charlestown, W.
John Fritz,	Bethlehem, Pa.
James D. Hague,	New York.
F. R. Hutton,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	New York.

CLASS II. — *Natural and Physiological Sciences.* — 33.

SECTION I. — 11.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
Samuel F. Emmons,	Washington.
G. K. Gilbert,	Washington.
S. L. Penfield,	New Haven.
R. Pumpelly,	Newport, R.I.
Israel C. Russell,	Ann Arbor.
Charles D. Walcott.	Washington.

SECTION II. — 6.

Botany.

L. H. Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

SECTION III. — 8.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md.
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

SECTION IV. — 8.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
Abraham Jacobi,	New York.
W. W. Keen,	Philadelphia.
William Osler,	Oxford, Eng.
T. Mitchell Prudden,	New York.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

CLASS III. — *Moral and Political Sciences.* — 27.

SECTION I. — 6.

Philosophy and Jurisprudence.

Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

SECTION II. — 7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	New York.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

SECTION III. — 7.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
Arthur T. Hadley,	New Haven.
Henry C. Lea,	Philadelphia.
Alfred T. Mahan,	New York.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

SECTION IV. — 7.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.

FOREIGN HONORARY MEMBERS.—73.

(Number limited to seventy-five. Elected as vacancies occur.)

CLASS I.—*Mathematical and Physical Sciences.*—25.

SECTION I.—8.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
George H. Darwin,	Cambridge.
Sir William Huggins,	London.
Felix Klein,	Göttingen.
Émile Picard,	Paris.
H. Poincaré,	Paris.
Otto Struve,	Karlsruhe.
H. C. Vogel,	Potsdam.

SECTION II.—6.

Physics.

Ludwig Boltzmann,	Vienna.
Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Marburg.
Joseph Larmor,	Cambridge.
Lord Rayleigh,	Witham.
Joseph J. Thomson,	Cambridge.

SECTION III.—6.

Chemistry.

Adolf Ritter von Baeyer,	Munich.
Marcellin Berthelot,	Paris.
J. H. van't Hoff,	Berlin.
D. Mendeleeff,	St. Petersburg.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

SECTION IV.—5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

CLASS II.—*Natural and Physiological Sciences.*—25.

SECTION I.—6.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Freih. v. Richthofen,	Berlin.
Henry C. Sorby,	Sheffield.

SECTION II.—6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

SECTION III. — 6.

Zoölogy and Physiology.

Sir Michael Foster,	Cambridge.
Ludimar Hermann,	Königsberg.
A. von Kölliker,	Würzburg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.

SECTION IV. — 7.

Medicine and Surgery.

Emil von Behring,	Marburg.
Sir T. L. Brunton,	London.
A. Celli,	Rome.
Sir V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.

CLASS III. — *Moral and Political Sciences.* — 23.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Hemel Hempstead.
H. Jackson,	Cambridge.
J. W. A. Kirchhoff,	Berlin.
G. C. C. Maspero,	Paris.

SECTION III. — 5.

Political Economy and History.

James Bryce,	London.
Adolf Harnack,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
John Morley,	London.
Pasquale Villari,	Florence.

SECTION IV. — 6.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Gaston Boissier,	Paris.
Georg Brandes,	Copenhagen.
F. Brunetière,	Paris.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Burwash.

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11 *and May* 10, 1893, *May* 9 *and October* 10, 1894, *March* 13, *April* 10 *and May* 8, 1895, *May* 8, 1901, *January* 8, 1902, *and May* 10, 1905.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; — Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Philosophy and Jurisprudence: — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows shall not exceed two hundred. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual

meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

Of Officers.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the Annual Meeting, on the second Wednesday in May.

2. At the Annual Meeting of 1901, nine Councillors shall be elected by ballot, one from each Class of the Academy to serve for one year, one from each Class for two years, and one from each Class for three years; and at annual meetings thereafter three Councillors shall be elected in the same manner, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several Sections of the same Class, reporting their action to the Academy.

3. If any office shall become vacant during the year, the vacancy shall be filled by a new election, and at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint from the next retiring Councillors a Nominating Committee of three Fellows, one for each class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to summon extraordinary meetings, upon any urgent occasion; and to execute or see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, is empowered to draw upon the Treasurer for such sums of money as the Academy shall direct. Bills presented on account of the Library, or the Publications of the Academy, must be previously approved by the respective committees on these departments.

3. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

4. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have full control and management of the funds and trusts of the Academy, with the power of investing or changing the investment of the same at their discretion. The general appropriations for the expenditures of the Academy shall be moved by this Committee at the Annual Meeting, and all special appropriations from the general and publication funds shall be referred to or proposed by this Committee.

3. The Rumford Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications and claims for the Rumford Premium, also on all appropriations from the income of the Rumford Fund, and generally see to the due and proper execution of this trust.

4. The C. M. Warren Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications for appropriations from the income of the C. M. Warren Fund, and generally see to the due and proper execution of this trust.

5. The Committee of Publication, of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Memoirs and the Proceedings shall be intrusted.

6. The Committee on the Library, of the Librarian ex officio and three other Fellows, one from each class, who shall examine the Library, and make an annual report on its condition and management.

7. An Auditing Committee of two Fellows, for auditing the accounts of the Treasurer.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council for Nomination, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive officially all moneys due or payable, and all bequests or donations made to the Academy, and shall pay such sums as the Academy may direct. He shall keep an account of all receipts and expenditures; shall submit his accounts to the Auditing Committee; and shall report the same at the expiration of his term of office.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. All moneys which there shall not be present occasion to expend shall be invested by the Treasurer, under the direction of the Finance Committee.

5. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts as such Assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, Massachusetts, as agent to perform his duties; the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

CHAPTER VIII.

Of the Librarian and Library.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford or other special Funds of the Academy, for the purchase of books, and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy and shall distribute copies among the Associate Fellows and Foreign Honorary Members, at their request. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings only, or at meetings adjourned from these and regularly notified, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

2. Fifteen Fellows shall constitute a quorum for the transaction of business at a stated meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Fellow residing in Boston and the vicinity; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them, and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council for nomination. No person recommended shall be reported by the Council as a candidate for election, unless he shall have received a written approval, signed at a meeting of the Council by at least five

of its members. All nominations thus approved shall be read to the Academy at a stated meeting, and shall then stand on the nomination list during the interval between two stated meetings, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void ; and if any Resident Fellow shall neglect to pay his annual assessments for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship ; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination of Associate Fellows may take place in the manner prescribed in reference to Resident Fellows. The Council may in like manner originate nominations of Associate Fellows, which must be read at a stated meeting previous to the election, and be exposed on the nomination list during the interval.

4. Foreign Honorary Members shall be chosen only after a nomination made at a meeting of the Council, signed at the time by at least seven of its members, and read at a stated meeting previous to that on which the balloting takes place.

5. Three fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. A majority of any section of the Academy is empowered to present lists of persons deemed best qualified to fill vacancies occurring in the number of Foreign Honorary Members or Associate Fellows allotted to it ; and such lists, after being read at a stated meeting, shall be referred to the Council for Nomination.

7. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship ; and provided that a majority of two thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes or additions to them, shall be referred to a committee, and, on their report at a subsequent meeting, shall require for enactment a majority of two thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded, at any stated meeting, by a majority of two thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

OF LITERARY PERFORMANCES.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice had been given to the Secretary shall take precedence of those not so notified.

2. Resident Fellows who have paid all fees and dues chargeable to them are entitled to receive one copy of each volume or article printed by the Academy, on application to the Librarian personally or by written order, within two years from the date of publication. And the current issues of the Proceedings shall be supplied, when ready for publication, free of charge, to all the Fellows and members of the Academy who desire to receive them.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

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